

1st
PREP.
2024
SECOND TERM



Interactive E-learning Application

Maths

Contents

First

Algebra and Statistics

UNIT **1** Numbers and Algebra.

UNIT **2** Statistics and Probability.



Second

Geometry and Measurement

UNIT **3** Geometry and Measurement.



First | Algebra and Statistics

UNIT **1** Numbers and Algebra — 7

UNIT **2** Statistics and Probability — 45



Numbers and Algebra

Lesson One Repeated multiplication.

Lesson Two Non-negative integer powers.

Lesson Three Negative integer powers.

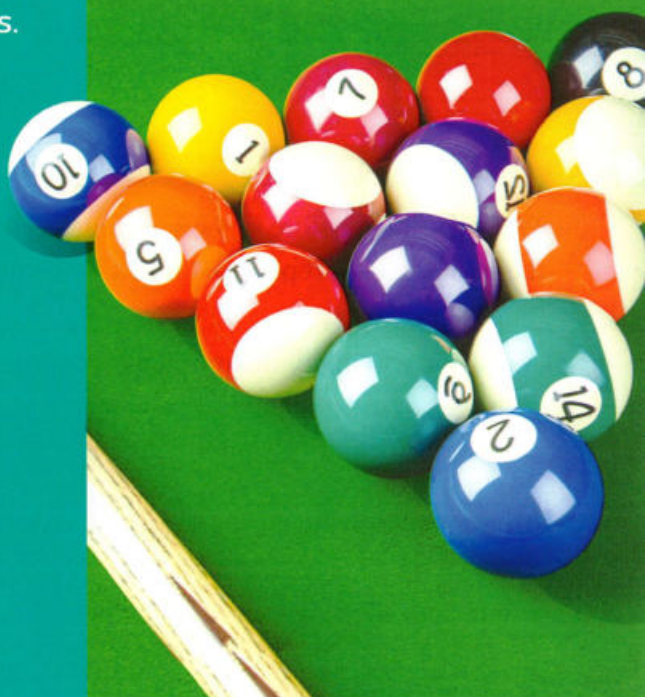
Lesson Four Scientific notation of the rational number.

Lesson Five Order of mathematical operations.

Lesson Six The square root of a perfect square rational number.

Lesson Seven Solving equations in \mathbb{Q} .

Lesson Eight Solving inequalities in \mathbb{Q} .



Unit Objectives : By the end of this unit, student should be able to :

- remember what have been studied on repeated multiplication in \mathbb{Z} .
- multiply repeated multiplication of the rational numbers.
- recognize the laws of powers in \mathbb{Q}
- recognize the negative power of the non-zero rational number.
- recognize the standard scientific notation of the rational number.
- write a rational number in the standard form.
- perform the mathematical operations according to the priority of their performances.
- recognize the square root of a perfect square rational number.
- find the square root of a perfect square rational number.
- solve an equation of the first degree in one unknown in \mathbb{Q} .
- use the equations to solve word problems.
- solve an inequality of the first degree in one unknown in \mathbb{Q} .

Lesson

1

Repeated multiplication



We had known before in the set of integers that : $3^4 = 3 \times 3 \times 3 \times 3$ where we found that the number 3 has repeated 4 times in the multiplication operation and we read it as :

«3 to the power 4»

Also , we can apply the previous on normal fractions :

For example: $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

From multiplying normal fractions , we find that :

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \frac{2^4}{3^4}$$

i.e. $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$

Generally

If $\frac{a}{b}$ is a rational number and n is a positive integer , then : $\left(\frac{a}{b}\right)^n = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \dots$ to n times

It is read as « $\frac{a}{b}$ to the power n » or «the n^{th} power of the number $\frac{a}{b}$ » i.e. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

For example: $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$

$(0.7)^2 = \left(\frac{7}{10}\right)^2 = \frac{7^2}{10^2} = \frac{49}{100}$

! Remark 1

If $\frac{a}{b}$ is a rational number , then : $\left(\frac{a}{b}\right)^0 = 1$ where $a \neq 0$

For example: $\left(\frac{1}{5}\right)^0 = 1$ $\left(-\frac{3}{7}\right)^0 = 1$



! Remark 2

If a is a rational number and m is a positive integer

then

$$(-a)^m = (a)^m$$

when m is an even number.

For example:

$$\left(-\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$(-a)^m = -(a)^m$$

when m is an odd number.

For example:

$$\left(-\frac{1}{2}\right)^3 = -\left(\frac{1}{2}\right)^3 = -\frac{1}{8}$$

Example 1

Find each of the following in the simplest form :

1 $\left(\frac{2}{3}\right)^2 \times \frac{9}{4}$

2 $\left(-\frac{5}{4}\right)^2 \times \left(\frac{2}{5}\right)^4$

3 $\left(3\frac{1}{2}\right)^2 \div \left(-10\frac{1}{2}\right)$

4 $\left(-\frac{2}{5}\right)^2 \times \left(-\frac{5}{2}\right)^3 \times \left(\frac{1}{5}\right)^0$

Solution

1 $\left(\frac{2}{3}\right)^2 \times \frac{9}{4} = \frac{2^2}{3^2} \times \frac{9}{4} = \frac{4}{9} \times \frac{9}{4} = 1$

2 $\left(-\frac{5}{4}\right)^2 \times \left(\frac{2}{5}\right)^4 = \frac{5^2}{4^2} \times \frac{2^4}{5^4} = \frac{25}{16} \times \frac{16}{625} = \frac{1}{25}$

3 $\left(3\frac{1}{2}\right)^2 \div \left(-10\frac{1}{2}\right) = \left(\frac{7}{2}\right)^2 \div \left(-\frac{21}{2}\right) = \frac{7^2}{2^2} \times \left(-\frac{2}{21}\right)$
 $= \frac{49}{4} \times \left(-\frac{2}{21}\right) = -\frac{7}{6}$

4 $\left(-\frac{2}{5}\right)^2 \times \left(-\frac{5}{2}\right)^3 \times \left(\frac{1}{5}\right)^0 = \frac{2^2}{5^2} \times \left(-\frac{5^3}{2^3}\right) \times 1 = \frac{4}{25} \times \left(-\frac{125}{8}\right) = -\frac{5}{2}$

TRY 1 by yourself

Find each of the following in its simplest form :

1 $\left(\frac{1}{5}\right)^2$

2 $\left(-\frac{2}{3}\right)^3$

3 $\left(-\frac{4}{5}\right)^4$

4 $\left(1\frac{1}{2}\right)^4$

5 $\left(-\frac{3}{9}\right)^2 \times \left(\frac{9}{4}\right)^2 \times \left(\frac{81}{16}\right)^0$

Example 2

If $x = -\frac{1}{2}$, $y = \frac{1}{4}$ and $z = 4$

, find the value of : $(x + y)^3 \times z^3$

Solution

$$\begin{aligned}(x + y)^3 \times z^3 &= \left(-\frac{1}{2} + \frac{1}{4}\right)^3 \times 4^3 = \left(-\frac{2}{4} + \frac{1}{4}\right)^3 \times 4^3 \\ &= \left(-\frac{1}{4}\right)^3 \times 4^3 = -\frac{1^3}{4^3} \times 4^3 = -1\end{aligned}$$

TRY
by yourself **2**

If $x = -\frac{2}{3}$, $y = \frac{1}{2}$ and $z = -\frac{4}{3}$

, find the value of : $x^2 - y^2 z$

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Maths & Science

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**Enrich your knowledge****Ghiyath Al-Din Ibn Masoud Al-Kashi**

He was an Arab scientist who had many investigations in mathematics :

- He had invented the decimal fraction.
- He put a theory concerning the sum of the natural numbers that are raised to the fourth power.
- He reached a very accurate rate for the approximate ratio (π) that nearly equates the accuracy of the calculators.



Ghiyath Al-Din
Ibn Masoud Al-Kashi
(1380 A.D. - 1436 A.D.)

Lesson 2

Non-negative integer powers



You have studied in the primary stage the laws of non-negative integer powers in \mathbb{Z} . In this lesson we will illustrate that these laws are also applicable for the rational numbers.

The first law

From the definition of repeated multiplication, you know that :

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}, \quad \left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$$

$$\text{i.e. } \left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^7$$

Generally

If $\frac{a}{b}$ is a rational number, n and m are non-negative integers

$$\text{, then : } \left(\frac{a}{b}\right)^n \times \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n+m}$$

i.e. When multiplying the like bases, we add their powers (indices).

For example:

$$\bullet \left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)^{3+2} = \left(\frac{2}{5}\right)^5$$

$$\bullet \left(-\frac{1}{2}\right)^4 \times \left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)^{4+3} = \left(-\frac{1}{2}\right)^7$$

Example 1 Calculate each of the following, then put the result in its simplest form :

$$1 \quad \frac{2}{3} \times \left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3$$

$$2 \quad \left(-\frac{1}{3}\right)^3 \times \left(\frac{1}{3}\right)^2$$

$$3 \quad \frac{3}{4} \times \left(-\frac{3}{4}\right)^2$$

Solution

$$1 \quad \frac{2}{3} \times \left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)^{1+2+3} = \left(\frac{2}{3}\right)^6 = \frac{2^6}{3^6} = \frac{64}{729}$$

$$\begin{aligned} 2 \quad \left(-\frac{1}{3}\right)^3 \times \left(\frac{1}{3}\right)^2 &= -\left(\frac{1}{3}\right)^3 \times \left(\frac{1}{3}\right)^2 \\ &= -\left(\frac{1}{3}\right)^5 = -\frac{1}{3^5} = -\frac{1}{243} \end{aligned}$$

$$\begin{aligned} 3 \quad \frac{3}{4} \times \left(-\frac{3}{4}\right)^2 &= \frac{3}{4} \times \left(\frac{3}{4}\right)^2 \\ &= \left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64} \end{aligned}$$

Notice that :

$\left(-\frac{1}{3}\right)^3 = -\left(\frac{1}{3}\right)^3$
because the index
is an odd number.

Notice that :

$\left(-\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)^2$
because the index
is an even number.

The second law

According to the first law, you know that : $a^6 = a^2 \times a^4$

, therefore : $a^6 \div a^2 = a^4$, $a^6 \div a^4 = a^2$

Generally

If $\frac{a}{b}$ is a rational number, where $\frac{a}{b} \neq 0$, n and m are non-negative integers, $n \geq m$

$$\text{, then : } \left(\frac{a}{b}\right)^n \div \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n-m}$$

i.e. When dividing like bases, we subtract their powers (indices).

For example:

$$\bullet \left(\frac{3}{8}\right)^5 \div \left(\frac{3}{8}\right)^2 = \left(\frac{3}{8}\right)^{5-2} = \left(\frac{3}{8}\right)^3$$

$$\bullet \left(-\frac{2}{7}\right)^4 \div \left(-\frac{2}{7}\right)^2 = \left(-\frac{2}{7}\right)^{4-2} = \left(-\frac{2}{7}\right)^2$$



Example 2

Calculate each of the following, then put the result in the simplest form :

1 $\left(\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 \div \left(\frac{4}{5}\right)^7$

2 $\frac{2^5 \times 2^4}{2^6}$

Solution

$$\begin{aligned} 1 \quad \left[\left(\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 \right] \div \left(\frac{4}{5}\right)^7 &= \left(\frac{4}{5}\right)^{2+5} \div \left(\frac{4}{5}\right)^7 \\ &= \left(\frac{4}{5}\right)^7 \div \left(\frac{4}{5}\right)^7 \\ &= \left(\frac{4}{5}\right)^{7-7} = \left(\frac{4}{5}\right)^0 = 1 \end{aligned}$$

$$2 \quad \frac{2^5 \times 2^4}{2^6} = \frac{2^{5+4}}{2^6} = \frac{2^9}{2^6} = 2^{9-6} = 2^3 = 8$$

TRY 1 by yourself

Find each of the following in the simplest form :

1 $\left(\frac{1}{5}\right)^2 \times \left(\frac{1}{5}\right)^2$

2 $\left(\frac{3}{7}\right)^8 \div \left(\frac{3}{7}\right)^6$

3 $\left(-\frac{2}{3}\right)^5 \times \left(-\frac{2}{3}\right)^2 \div \left(-\frac{2}{3}\right)^6$

4 $\left(-\frac{1}{4}\right)^7 \div \left(\frac{1}{4}\right)^6 \times \frac{1}{4}$

Remarks

① From the repeated multiplication, notice that :

$$\begin{aligned} \left(\frac{3}{4} \times \frac{5}{7}\right)^3 &= \left(\frac{3}{4} \times \frac{5}{7}\right) \times \left(\frac{3}{4} \times \frac{5}{7}\right) \times \left(\frac{3}{4} \times \frac{5}{7}\right) \\ &= \left(\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right) \times \left(\frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}\right) \\ &= \left(\frac{3}{4}\right)^3 \times \left(\frac{5}{7}\right)^3 \end{aligned}$$

Generally:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, n is a non-negative integer,

then : $\left(\frac{a}{b} \times \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \times \left(\frac{c}{d}\right)^n$

② From the repeated multiplication, notice that :

$$\begin{aligned}\left(\frac{2}{3} \div \frac{5}{11}\right)^4 &= \left(\frac{\frac{2}{3}}{\frac{5}{11}}\right)^4 = \frac{\frac{2}{3}}{\frac{5}{11}} \times \frac{\frac{2}{3}}{\frac{5}{11}} \times \frac{\frac{2}{3}}{\frac{5}{11}} \times \frac{\frac{2}{3}}{\frac{5}{11}} \\ &= \frac{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}}{\frac{5}{11} \times \frac{5}{11} \times \frac{5}{11} \times \frac{5}{11}} \\ &= \left(\frac{2}{3}\right)^4 \div \left(\frac{5}{11}\right)^4\end{aligned}$$

Generally:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, $\frac{c}{d} \neq 0$, n is a non-negative integer

, then : $\left(\frac{a}{b} \div \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \div \left(\frac{c}{d}\right)^n$ (where $\frac{c}{d} \neq 0$)

Example 3 Find the result of each of the following in its simplest form :

1 $\left(\frac{xy}{z}\right)^2$

2 $\left(\frac{2x}{3y}\right)^4$

Solution

1 $\left(\frac{xy}{z}\right)^2 = \frac{(xy)^2}{z^2} = \frac{x^2y^2}{z^2}$

2 $\left(\frac{2x}{3y}\right)^4 = \frac{2^4x^4}{3^4y^4} = \frac{16x^4}{81y^4}$

The third law

You know that : $(a^2)^3 = a^2 \times a^2 \times a^2$ and according to the first law : $a^2 \times a^2 \times a^2 = a^6$

i.e. $(a^2)^3 = a^6$

Generally

If $\frac{a}{b}$ is a rational number, n and m are non-negative integers

, then : $\left[\left(\frac{a}{b}\right)^n\right]^m = \left(\frac{a}{b}\right)^{n \times m}$

For example:

• $\left[\left(\frac{3}{5}\right)^3\right]^2 = \left(\frac{3}{5}\right)^{3 \times 2} = \left(\frac{3}{5}\right)^6$

• $\left[\left(-\frac{1}{2}\right)^4\right]^2 = \left(-\frac{1}{2}\right)^{4 \times 2} = \left(-\frac{1}{2}\right)^8$

**Example 4**

Find each of the following in the simplest form :

1 $\left[(-2\frac{1}{2})^2\right]^2$

2 $\left(\frac{x^2}{y^3}\right)^3$

3 $\frac{(-4x^3y^4)^2}{(-2xy^2)^4}$

Solution

1 $\left[(-2\frac{1}{2})^2\right]^2 = (-2\frac{1}{2})^{2 \times 2} = (-2\frac{1}{2})^4 = (2\frac{1}{2})^4 = (\frac{5}{2})^4 = \frac{5^4}{2^4} = \frac{625}{16}$

2 $\left(\frac{x^2}{y^3}\right)^3 = \frac{(x^2)^3}{(y^3)^3} = \frac{x^{2 \times 3}}{y^{3 \times 3}} = \frac{x^6}{y^9}$

3 $\frac{(-4x^3y^4)^2}{(-2xy^2)^4} = \frac{(-4)^2 \times x^{3 \times 2} \times y^{4 \times 2}}{(-2)^4 \times x^4 \times y^{2 \times 4}} = \frac{16x^6y^8}{16x^4y^8} = x^{6-4} = x^2$

Example 5If $x = \frac{1}{2}$, $y = -\frac{3}{4}$ and $z = \frac{3}{2}$, find the numerical value of each of the following in the simplest form :

1 $\left(\frac{x^2}{z}\right)^3$

2 $\left(\frac{x^2z}{y}\right)^2$

Solution

1 $\left(\frac{x^2}{z}\right)^3 = \left[\left(\frac{1}{2}\right)^2 \div \frac{3}{2}\right]^3 = \left(\frac{1^2}{2^2} \times \frac{2}{3}\right)^3$
 $= \left(\frac{1}{4} \times \frac{2}{3}\right)^3 = \left(\frac{1}{6}\right)^3 = \frac{1^3}{6^3} = \frac{1}{216}$

Notice that :

$$\frac{x^2}{z} = x^2 \div z$$

2 $\left(\frac{x^2z}{y}\right)^2 = \frac{x^{2 \times 2} z^2}{y^2} = \frac{x^4 z^2}{y^2}$
 $= \frac{\left(\frac{1}{2}\right)^4 \times \left(\frac{3}{2}\right)^2}{\left(-\frac{3}{4}\right)^2} = \frac{1^4}{2^4} \times \frac{3^2}{2^2} \times \frac{4^2}{3^2} = \frac{1}{16} \times \frac{1}{4} \times 16 = \frac{1}{4}$

TRY
by yourself **2**

Calculate each of the following, then put the result in the simplest form :

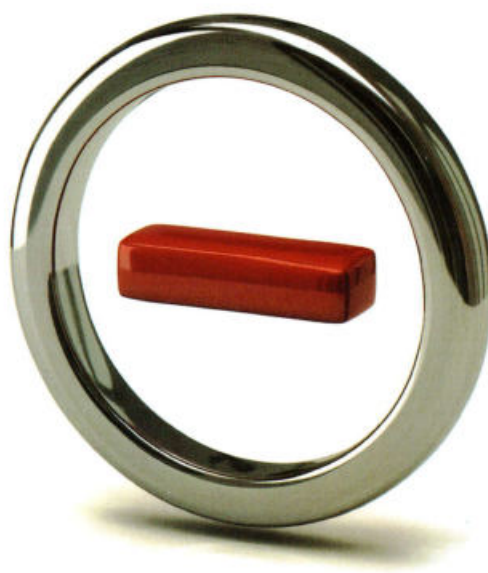
1 $\left[\left(\frac{1}{2}\right)^2\right]^3$

2 $\left(\frac{a^2b^2}{c^3d^4}\right)^2$

3 $\left(\frac{5^2 \times 5^4}{5^5}\right)^2$

Lesson 3

Negative integer powers



You know that, if a is a rational number, $a \neq 0$, then: $a^0 = 1$

therefore, $\frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}$ **i.e.** $a^{-n} = \frac{1}{a^n}$

Definition

If a is a rational number, $a \neq 0$ and n is a positive integer,

then $a^{-n} = \frac{1}{a^n}$ and $a^n = \frac{1}{a^{-n}}$

For example:

$$\begin{aligned} \bullet 3^{-3} &= \frac{1}{3^3} = \frac{1}{27} & \bullet 3 \times 5^{-1} &= 3 \times \frac{1}{5} = \frac{3}{5} & \bullet \frac{2}{5^{-2}} &= 2 \times 5^2 = 2 \times 25 = 50 \\ \bullet 0.1 &= \frac{1}{10} = 10^{-1} & , \quad 0.01 &= \frac{1}{100} = \frac{1}{10^2} = 10^{-2} & , \quad \dots &\text{and so on.} \end{aligned}$$

! Remarks

① If a is a rational number, $a \neq 0$ and n is a positive integer,

then $a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1$ (the multiplicative neutral)

i.e. each of a^n and a^{-n} is the multiplicative inverse of the other

② If $\frac{a}{b}$ is a rational number not equal to zero and n is a positive integer

, then: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

For example: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$



Example 1

Find the value of each of the following in the simplest form :

1 $2^4 \times 2^{-2}$

2 $\frac{5^{-2}}{5^{-3}}$

3 $(3^2)^{-2}$

4 $\frac{6^{-3} \times 6^5}{6^2}$

5 $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2}$

6 $(7^3)^2 \times (7^{-2})^2$

7 $\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3}$

Solution

1 $2^4 \times 2^{-2} = 2^4 \times \frac{1}{2^2} = \frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4$

2 $\frac{5^{-2}}{5^{-3}} = \frac{5^3}{5^2} = 5^{3-2} = 5$

3 $(3^2)^{-2} = \frac{1}{(3^2)^2} = \frac{1}{3^4} = \frac{1}{81}$

4 $\frac{6^{-3} \times 6^5}{6^2} = \frac{6^5}{6^3 \times 6^2} = \frac{6^5}{6^5} = 1$

5 $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2} = \left(\frac{5^3 \times 5}{5^2 \times 5^4}\right)^{-2} = \left(\frac{5^4}{5^6}\right)^{-2}$
 $= \left(\frac{5^6}{5^4}\right)^2 = (5^{6-4})^2 = (5^2)^2 = 5^4 = 625$

6 $(7^3)^2 \times (7^{-2})^2 = (7^3)^2 \times \left(\frac{1}{7^2}\right)^2 = 7^6 \times \frac{1}{7^4} = 7^{6-4} = 7^2 = 49$

7 $\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{3}\right)^3 \div \left(\frac{5}{4}\right)^3 = \left(\frac{5}{3} \div \frac{5}{4}\right)^3$
 $= \left(\frac{5}{3} \times \frac{4}{5}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{4^3}{3^3} = \frac{64}{27}$

Remark

All laws of powers that we have studied in the previous lesson are correct in the case of the negative powers. So , the previous example can be solved by using laws of powers as follows :

1 $2^4 \times 2^{-2} = 2^{4+(-2)} = 2^2 = 4$

2 $\frac{5^{-2}}{5^{-3}} = 5^{-2-(-3)} = 5^{-2+3} = 5$

3 $(3^2)^{-2} = 3^{2 \times (-2)} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

4 $\frac{6^{-3} \times 6^5}{6^2} = 6^{-3+5-2} = 6^0 = 1$

$$5 \left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4} \right)^{-2} = (5^{3+(-2)-(-1)-4})^{-2} = (5^{3-2+1-4})^{-2} \\ = (5^{-2})^{-2} = 5^{(-2) \times (-2)} = 5^4 = 625$$

$$6 (7^3)^2 \times (7^{-2})^2 = (7^3 \times 7^{-2})^2 = (7^{3+(-2)})^2 = 7^2 = 49$$

$$7 \left(\frac{3}{5} \right)^{-3} \div \left(\frac{4}{5} \right)^{-3} = \left(\frac{3}{5} \div \frac{4}{5} \right)^{-3} = \left(\frac{3}{5} \times \frac{5}{4} \right)^{-3} = \left(\frac{3}{4} \right)^{-3} = \left(\frac{4}{3} \right)^3 = \frac{4^3}{3^3} = \frac{64}{27}$$

TRY 1 by yourself

Find the value of each of the following in the simplest form :

$$1 \ 5^{-3}$$

$$2 \left(\frac{3}{7} \right)^{-2}$$

$$3 \ (2^{-3})^2$$

$$4 \left(\frac{2^{-2} \times 2^6}{2^3} \right)^{-3}$$

Example 2

Simplify each of the following to the simplest form where $x \neq 0$:

$$1 \ x^5 \times x^{-2} \times x^{-3}$$

$$2 \ (x^2)^{-3} \div (x^{-1})^2$$

$$3 \ \left(\frac{x^4 \times x^{-3}}{x^{-4} \times x} \right)^{-2}$$

Solution

$$1 \ x^5 \times x^{-2} \times x^{-3} = x^{5+(-2)+(-3)} = x^{5-2-3} = x^0 = 1$$

$$2 \ (x^2)^{-3} \div (x^{-1})^2 = x^{-6} \div x^{-2} = x^{-6-(-2)} = x^{-6+2} = x^{-4} = \frac{1}{x^4}$$

$$3 \ \left(\frac{x^4 \times x^{-3}}{x^{-4} \times x} \right)^{-2} = (x^{4+(-3)-(-4)-1})^{-2} = (x^{4-3+4-1})^{-2} \\ = (x^4)^{-2} = x^{-8} = \frac{1}{x^8}$$

TRY 2 by yourself

Simplify each of the following to the simplest form putting the result in positive integer power where the denominator doesn't equal zero :

$$1 \ (x^{-2})^{-5}$$

$$2 \ \left(\frac{a^4}{a^{-3}} \right)^{-2}$$

$$3 \ (y^5 \times y^{-2})^3$$

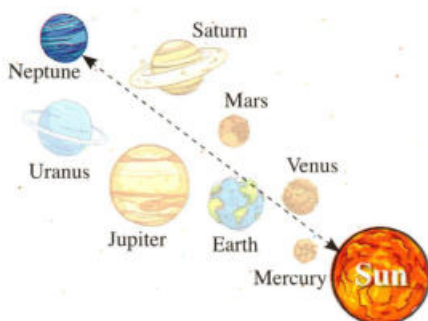
Lesson

4

Scientific notation of the rational number



- The scientific notation of the number is a useful method to deal with the very large numbers or the very small numbers like the numbers in the following examples.



Neptune planet is far from the sun by 2 800 000 000 miles.
(the mile = 1.6 km. approximately).



The diameter length of a virus ≈ 0.0000000025 cm.

- Before explaining how to write the numbers in their scientific notation , we should notice the following :

1 $10 = 10^1$, $100 = 10 \times 10 = 10^2$, $1000 = 10 \times 10 \times 10 = 10^3$ and so on

Hence we find that :

$$2000 = 2 \times 1000 = 2 \times 10^3 \quad , \quad 50\,000 = 5 \times 10\,000 = 5 \times 10^4$$

2 $0.1 = \frac{1}{10} = 10^{-1}$, $0.01 = \frac{1}{100} = \frac{1}{10 \times 10} = 10^{-2}$,

$0.001 = \frac{1}{1000} = \frac{1}{10 \times 10 \times 10} = 10^{-3}$ and so on

Hence we find that :

• $0.03 = \frac{3}{100} = \frac{3}{10 \times 10} = 3 \times 10^{-2}$

• $0.0007 = \frac{7}{10\,000} = \frac{7}{10 \times 10 \times 10 \times 10} = 7 \times 10^{-4}$

The standard scientific notation of a number

The number is written in the standard form as : $a \times 10^n$ where $1 \leq |a| < 10$ and $n \in \mathbb{Z}$

There are examples for some numbers written in its standard form :

- 4.6×10^8
- 5.236×10^{-6}
- -9.6×10^{10}
- -1.001×10^{-5}
- -3×10^{12}
- 1×10^{-7}

Each of the previous numbers is the product of two numbers :

- The first number could be positive or negative and its absolute value must be greater than or equal 1 and less than 10
 - The second number expresses the powers of the number 10
- (These powers could be positive or negative)

* Examples for some numbers not in the standard form :

- 8 200 000 000
- 0.000 000 135
- 45×10^8 (because $45 > 10$)
- 706.4×10^5 (because $706.4 > 10$)
- 0.248×10^{-7} (because $0.248 < 1$)
- -0.0015×10^{-9} (because $|-0.0015| < 1$)

and the next example will show how to write these numbers in the standard form.

Writing the number in the standard form

Example 1 Write each of the following numbers in the standard form :

1 8 200 000 000

2 0.000 000 135

3 45×10^8

4 706.4×10^5

5 0.248×10^{-7}

6 $-0.001\,5 \times 10^{-9}$



Solution

$$1 \quad 8 \, 200 \, 000 \, 000.0 = 8.2 \times 10^9$$

Moving the decimal
point **9** places
towards **left**

Using the power
9 of the
number 10

$$2 \quad 0.000 \, 000 \, 135 = 1.35 \times 10^{-7}$$

Moving the decimal
point **7** places
towards **right**

Using the power
-7 of the
number 10

- 3 To put 45.0×10^8 in the standard form we move the decimal point one place to the left, then we multiply by 10

$$\therefore 45.0 \times 10^8 = 4.5 \times 10^8 \times 10 = 4.5 \times 10^9$$

- 4 To put 706.4×10^5 in the standard form we move the decimal point two places to the left, then we multiply by 10^2

$$\therefore 706.4 \times 10^5 = 7.064 \times 10^5 \times 10^2 = 7.064 \times 10^7$$

- 5 To put 0.248×10^{-7} in the standard form we move the decimal point one place to the right, then we multiply by 10^{-1}

$$\therefore 0.248 \times 10^{-7} = 2.48 \times 10^{-7} \times 10^{-1} = 2.48 \times 10^{-8}$$

- 6 To put -0.0015×10^{-9} in the standard form we move the decimal point three places to the right, then we multiply by 10^{-3}

$$\therefore -0.0015 \times 10^{-9} = -1.5 \times 10^{-9} \times 10^{-3} = -1.5 \times 10^{-12}$$

! Remark

The standard form for 1 is $1 \times 10^{\text{zero}}$

, So is the standard form for 2 is $2 \times 10^{\text{zero}}$, etc ...

TRY
by yourself 1

In the following, determine the numbers that are not in the standard form, then write them in the standard form :

1 8.5×10^{-4}

2 17×10^8

3 0.5×10^{-7}

4 530.5×10^9

5 -0.999×10^{-5}

6 6×10^6

7 650 000 000

8 0.000 001 02

9 -2.5×10^8

Operations on the numbers in the standard form**Example 2**

Write the result of each of the following in the standard form :

1 $(1.2 \times 10^5) \times (4 \times 10^3)$

2 $(6.5 \times 10^4) \times (8 \times 10^2)$

3 $(2.4 \times 10^{11}) \div (1.2 \times 10^{-4})$

4 $(6.6 \times 10^7) \times (3 \times 10)^4$

5 $(2.3 \times 10^6) + (3.7 \times 10^5)$

Solution

1 $(1.2 \times 10^5) \times (4 \times 10^3) = (1.2 \times 4) \times (10^5 \times 10^3) = 4.8 \times 10^8$

2 $(6.5 \times 10^4) \times (8 \times 10^2) = (6.5 \times 8) \times (10^4 \times 10^2)$

$= 52 \times 10^6$

$= 5.2 \times 10^7$

Notice that :

52×10^6 is not in the standard form, then we should put it in the standard form.

3 $(2.4 \times 10^{11}) \div (1.2 \times 10^{-4}) = \frac{2.4}{1.2} \times \frac{10^{11}}{10^{-4}} = 2 \times 10^{15}$

4 $(6.6 \times 10^7) \times (3 \times 10)^4 = (6.6 \times 10^7) \times (3^4 \times 10^4)$

$= (6.6 \times 3^4) \times (10^7 \times 10^4)$

$= 534.6 \times 10^{11} = 5.346 \times 10^{13}$

5 $(2.3 \times 10^6) + (3.7 \times 10^5) = 10^5 (2.3 \times 10 + 3.7)$

$= 10^5 (23 + 3.7) = 10^5 \times 26.7 = 2.67 \times 10^6$

**Example 3**

Write the result of each of the following in the standard form :

1 $30\,000 \times 400\,000$

2 $140\,000 \times 0.005$

3 $0.000\,015 \div 30$

4 $(50\,000)^3$

5 $(0.000\,3)^5$

6 $(-0.001)^6$

Solution

$$1 \quad 30\,000 \times 400\,000 = (3 \times 10^4) \times (4 \times 10^5) = (3 \times 4) \times (10^4 \times 10^5) \\ = 12 \times 10^9 = 1.2 \times 10^{10}$$

$$2 \quad 140\,000 \times 0.005 = (1.4 \times 10^5) \times (5 \times 10^{-3}) \\ = (1.4 \times 5) \times (10^5 \times 10^{-3}) = 7 \times 10^2$$

$$3 \quad 0.000\,015 \div 30 = (1.5 \times 10^{-5}) \div 3 \times 10 \\ = \frac{1.5}{3} \times \frac{10^{-5}}{10} = 0.5 \times 10^{-6} = 5 \times 10^{-7}$$

$$4 \quad (50\,000)^3 = (5 \times 10^4)^3 = 5^3 \times 10^{12} = 125 \times 10^{12} = 1.25 \times 10^{14}$$

$$5 \quad (0.000\,3)^5 = (3 \times 10^{-4})^5 = 3^5 \times 10^{-20} = 243 \times 10^{-20} = 2.43 \times 10^{-18}$$

$$6 \quad (-0.001)^6 = (0.001)^6 = (1 \times 10^{-3})^6 = 1^6 \times 10^{-18} = 10^{-18}$$

TRY
by yourself **2**

Write the result of each of the following in the standard form :

1 $(5.3 \times 10^7) \times (3 \times 10^5)$

2 $0.000\,6 \div 20$

3 $(400\,000)^2$

4 $(3.2 \times 10^9) - (0.2 \times 10^8)$

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Lesson 5

Order of mathematical operations



We know that : addition, subtraction, multiplication and division are the basic mathematical operations which are performed on the numbers.

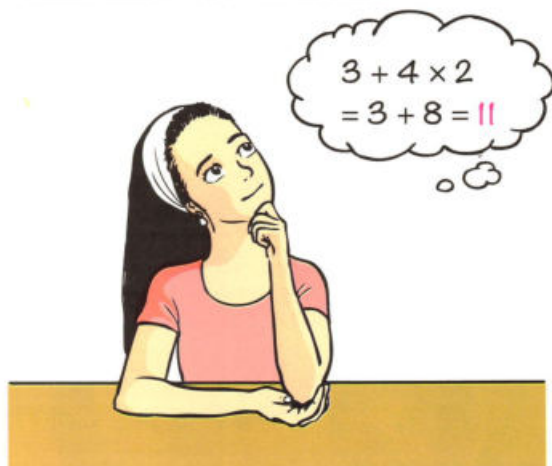
Sometimes one problem contains the four operations or some of them , so it is necessary to be agreed on rules which determine the priority of performance of these operations.

The following situation shows the importance of that :

The following problem was given to each of Heba and Ahmed.

Calculate : $3 + 4 \times 2$

Their answers were as follows :



Heba multiplied at first , then she added.

She got : 11



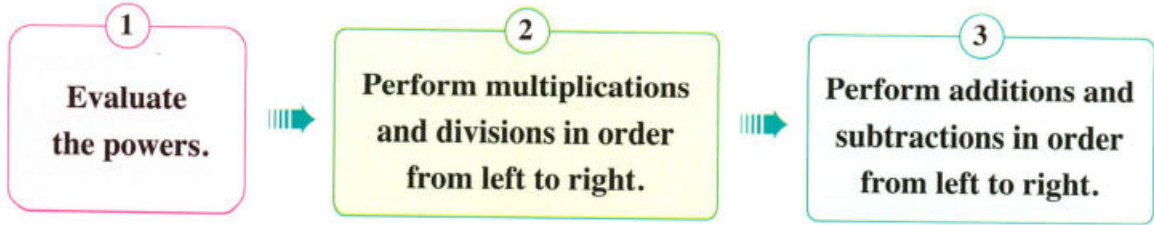
Ahmed added at first , then he multiplied.

He got : 14

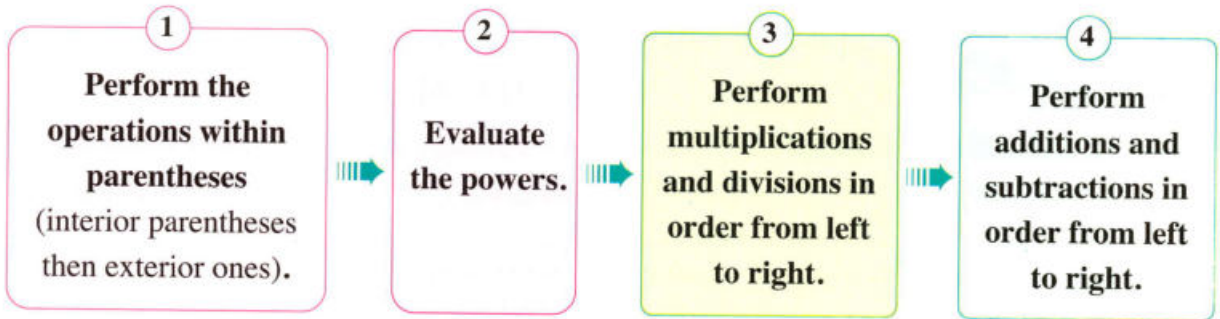
For the results are different, then it is necessary to be agreed on some rules that determine the order of performing the mathematical operations , that are :



First Order of performing the mathematical operations in an expression has no parentheses



Second Order of performing the mathematical operations in an expression has parentheses



- According to these rules , we can determine that Heba held the correct answer because she performed the multiplication operation first , then the addition operation.

Notice that :

The scientific calculators and computers follow the same rules of ordering the operations.

Example 1

Calculate the value of each of the following :

1 $3 + 6 \times (5 + 4) \div 3 - 7$

2 $9 - 5 \div (8 - 3) \times 2 + 6$

Solution

$$\begin{aligned}
 1 \quad 3 + 6 \times (5 + 4) \div 3 - 7 &= 3 + 6 \times 9 \div 3 - 7 && \text{(parentheses)} \\
 &= 3 + 54 \div 3 - 7 && \text{(multiplication)} \\
 &= 3 + 18 - 7 && \text{(division)} \\
 &= 21 - 7 && \text{(addition)} \\
 &= 14 && \text{(subtraction)}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad 9 - 5 \div (8 - 3) \times 2 + 6 &= 9 - 5 \div 5 \times 2 + 6 && \text{(parentheses)} \\
 &= 9 - 1 \times 2 + 6 && \text{(division)} \\
 &= 9 - 2 + 6 && \text{(multiplication)} \\
 &= 7 + 6 && \text{(subtraction)} \\
 &= 13 && \text{(addition)}
 \end{aligned}$$

Example 2

Calculate the value of each of the following :

$$1 \quad 4 - 3 [4 - 2 (6 - 3)] \div 2 \qquad 2 \quad 16 \div [8 - 3 (4 - 2)] + 1$$

Solution

$$\begin{aligned}
 1 \quad 4 - 3 [4 - 2 (6 - 3)] \div 2 &= 4 - 3 [4 - 2 \times 3] \div 2 && \text{(the interior parentheses)} \\
 &= 4 - 3 [4 - 6] \div 2 && \text{(multiplication inside parentheses)} \\
 &= 4 - 3 [-2] \div 2 && \text{(subtraction inside parentheses)} \\
 &= 4 + 6 \div 2 && \text{(multiplication by parentheses)} \\
 &= 4 + 3 && \text{(division)} \\
 &= 7 && \text{(addition)}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad 16 \div [8 - 3 (4 - 2)] + 1 &= 16 \div [8 - 3 \times 2] + 1 && \text{(the interior parentheses)} \\
 &= 16 \div [8 - 6] + 1 && \text{(multiplication inside parentheses)} \\
 &= 16 \div 2 + 1 && \text{(subtraction inside the parentheses)} \\
 &= 8 + 1 && \text{(division)} \\
 &= 9 && \text{(addition)}
 \end{aligned}$$

Example 3

Calculate the value of each of the following :

$$1 \quad 8 \times 2^2 - 7 \times (4 + 1) \qquad 2 \quad 2 + 3 [5 + (4 - 1)^2]$$

$$3 \quad 3 [(3^2 + 1) - (2^3 - 2)]$$

Solution

$$\begin{aligned}
 1 \quad 8 \times 2^2 - 7 \times (4 + 1) &= 8 \times 2^2 - 7 \times 5 && \text{(addition inside parentheses)} \\
 &= 8 \times 4 - 7 \times 5 && \text{(powers)} \\
 &= 32 - 35 && \text{(multiplication)} \\
 &= -3 && \text{(subtraction)}
 \end{aligned}$$



$$\begin{aligned}
 2 \quad 2 + 3 [5 + (4 - 1)^2] &= 2 + 3 [5 + 3^2] && \text{(subtraction inside interior parentheses)} \\
 &= 2 + 3 [5 + 9] && \text{(powers inside parentheses)} \\
 &= 2 + 3 \times 14 && \text{(addition inside parentheses)} \\
 &= 2 + 42 && \text{(multiplication)} \\
 &= 44 && \text{(addition)} \\
 3 \quad 3 [(3^2 + 1) - (2^3 - 2)] &= 3 [(9 + 1) - (8 - 2)] && \text{(powers)} \\
 &= 3 [10 - 6] && \text{(the interior parentheses)} \\
 &= 3 \times 4 && \text{(subtraction inside parentheses)} \\
 &= 12 && \text{(multiplication)}
 \end{aligned}$$

! Remark

In the problems containing fractions, we should perform the operations in the numerator and denominator before division.

Example 4

Calculate the value of each of the following :

$$1 \quad \frac{36 - 6}{3 + 12}$$

$$2 \quad \frac{11 - (5 - 4)}{5^2 - 10 \times 2}$$

$$3 \quad 7 + 8 \div \frac{4 + 12 - 2}{3^2 - 2} - (2^3 + 2)$$

Solution

$$1 \quad \frac{36 - 6}{3 + 12} = \frac{30}{15} = 2$$

$$2 \quad \frac{11 - (5 - 4)}{5^2 - 10 \times 2} = \frac{11 - 1}{25 - 20} = \frac{10}{5} = 2$$

$$\begin{aligned}
 3 \quad 7 + 8 \div \frac{4 + 12 - 2}{3^2 - 2} - (2^3 + 2) &= 7 + 8 \div \frac{14}{7} - (2^3 + 2) \\
 &= 7 + 8 \div 2 - 10 = 7 + 4 - 10 = 1
 \end{aligned}$$

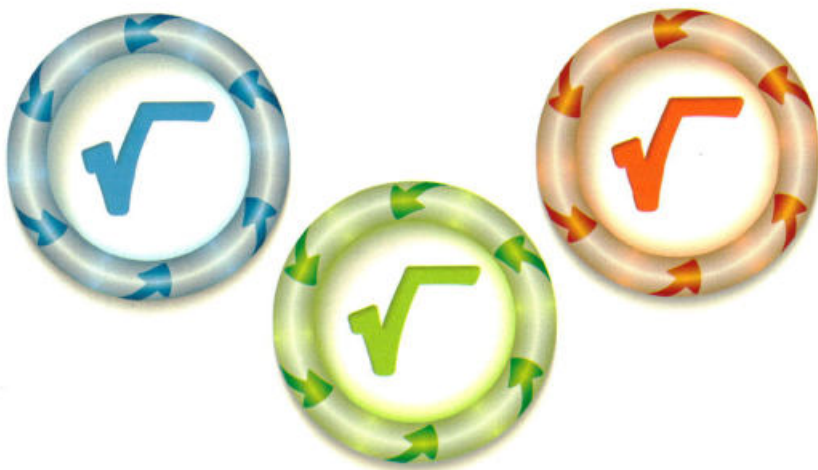
TRY
by yourself

Calculate the value of each of the following :

$$1 \quad 20 \div (12 - 2) \times 3^2 - 2$$

$$2 \quad \frac{6 \times 3 + 10 \div 5}{2 - (10 - 2^2)}$$

The square root of a perfect square rational number



Definition

The square root of the perfect square rational number “ a ” is the number whose square equals “ a ”

For example:

- The number 6 is a square root of the number 36 because : $6^2 = 36$
- Also , the number -6 is a square root of the number 36 because : $(-6)^2 = 36$

i.e. Finding the square root is the inverse operation of finding the square of a number.

It means that , to find the square root of a number , we search a number which , when multiplied by itself , equals this number.

Generally

- The **positive** square root of the number a is symbolized by \sqrt{a}
- The **negative** square root of the number a is symbolized by $-\sqrt{a}$
- The two square roots of the number a is symbolized by $\pm\sqrt{a}$ which means \sqrt{a} , $-\sqrt{a}$ and each of them is the additive inverse of the other.

Examples :

The positive square root of 25 is $\sqrt{25} = 5$

The negative square root of 16 is $-\sqrt{16} = -4$

The two square roots of 49 are $\pm\sqrt{49} = \pm 7$



! Remarks

1 $\sqrt{0} = 0$

2 In the set of rational numbers it is meaningless to find \sqrt{a} if a is a negative rational number because there is no rational number if it is multiplied by itself, the result will be negative.

3 $\sqrt{a^2} = |a|$

For example: $\bullet \sqrt{(-3)^2} = |-3| = 3$ $\bullet \sqrt{\left(-\frac{4}{5}\right)^2} = \left|-\frac{4}{5}\right| = \frac{4}{5}$

4 $\sqrt{a^2 b^2} = \sqrt{(ab)^2} = |ab|$ For example: $\sqrt{a^4 b^6} = \sqrt{(a^2 b^3)^2} = |a^2 b^3|$

5 If $X^2 = a$ where $a \geq 0$, then $X = \pm\sqrt{a}$

Example 1

Find each of the following in the simplest form :

1 $\sqrt{36}$

2 $-\sqrt{\frac{16}{25}}$

3 $\pm\sqrt{2\frac{1}{4}}$

4 $\sqrt{\left(-\frac{2}{7}\right)^2}$

5 $-\sqrt{0.25}$

6 $\pm\sqrt{\frac{3.6}{10}}$

7 $\sqrt{16+9}$

8 $\sqrt{100-36}$

9 $\sqrt{\frac{36a^8}{49d^4}}$

Solution

1 $\sqrt{36} = 6$ because $6^2 = 36$

2 $-\sqrt{\frac{16}{25}} = -\frac{4}{5}$ because $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$

3 $\pm\sqrt{2\frac{1}{4}} = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$

4 $\sqrt{\left(-\frac{2}{7}\right)^2} = \left|-\frac{2}{7}\right| = \frac{2}{7}$

5 $-\sqrt{0.25} = -\sqrt{\frac{25}{100}} = -\frac{5}{10} = -\frac{1}{2}$

6 $\pm\sqrt{\frac{3.6}{10}} = \pm\sqrt{\frac{36}{100}} = \pm\frac{6}{10} = \pm\frac{3}{5}$

7 $\sqrt{16+9} = \sqrt{25} = 5$

8 $\sqrt{100-36} = \sqrt{64} = 8$

9 $\sqrt{\frac{36a^8}{49d^4}} = \frac{6a^4}{7d^2}$

Notice that :

When there is an addition or a subtraction operation under the square root, it must be performed first before finding the square root.

TRY
by yourself **1**

Complete the following :

1 $\sqrt{64} = \dots\dots\dots$

2 $-\sqrt{900} = \dots\dots\dots$

3 $-\sqrt{\frac{36}{25}} = \dots\dots\dots$

4 $\pm \sqrt{6\frac{1}{4}} = \dots\dots\dots$

5 $\sqrt{0.64} = \dots\dots\dots$

6 $\sqrt{100 - 64} = \dots\dots\dots$

Remark

In some cases , it is more easy to use factorization in finding the square root of a number , to do that we factorize the given number into its prime factors , then we take one factor from each two equal factors , then the product of these taken factors is the square root of this number.

Example 2Find : $\sqrt{441}$ **Solution**

$$\begin{aligned} \therefore 441 &= 3 \times 3 \times 7 \times 7 \\ \therefore \sqrt{441} &= 3 \times 7 \\ &= 21 \end{aligned}$$

$$\begin{array}{r|l} \textcircled{3} & 3 & 441 \\ & 3 & 147 \\ \textcircled{7} & 7 & 49 \\ & 7 & 7 \\ & & 1 \end{array}$$

Example 3

Simplify each of the following to the simplest form :

1 $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2$

2 $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0$

3 $\left(2\frac{7}{9}\right)^2 \div \sqrt{\frac{25}{9}}$

Solution

1 $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2 = -\frac{2}{7} \times \frac{7}{2} \times \frac{4}{49} = -\frac{4}{49}$

2 $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$

$$\begin{aligned} \textbf{3} \quad \left(2\frac{7}{9}\right)^2 \div \sqrt{\frac{25}{9}} &= \left(\frac{25}{9}\right)^2 \div \frac{5}{3} = \left(\left(\frac{5}{3}\right)^2\right)^2 \div \frac{5}{3} \\ &= \left(\frac{5}{3}\right)^4 \div \frac{5}{3} = \left(\frac{5}{3}\right)^{4-1} = \left(\frac{5}{3}\right)^3 = \frac{125}{27} \end{aligned}$$


TRY 2
 by yourself

Simplify to the simplest form :

$$\textcircled{1} \left(\frac{2}{3}\right)^2 \times \sqrt{\frac{81}{16}} \times \left(\frac{7}{9}\right)^0$$

$$\textcircled{2} \frac{5}{7} \times \sqrt{\frac{49}{36}} \div \left(-\frac{5}{3}\right)^2$$

Example 4

The base length of a triangle is 16 cm. and its corresponding height is 8 cm. Find the side length of a square having the same area of that triangle.

Solution

\therefore The area of the triangle = $\frac{1}{2}$ of the base length \times its corresponding height.

\therefore The area of the triangle = $\frac{1}{2} \times 16 \times 8 = 64 \text{ cm}^2$

\therefore The area of the square = 64 cm^2

\therefore The side length of the square = $\sqrt{64} = 8 \text{ cm}$.

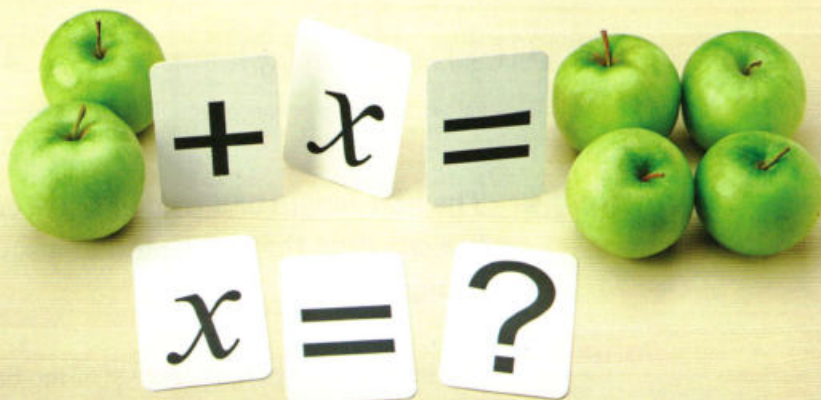
TRY 3
 by yourself

The area of a square is 1.44 cm^2 . Find its perimeter.

Lesson

7

Solving equations in \mathbb{Q}



The equation

is a mathematical statement which contains one variable as x (or more as x and y) and contains equality relation « = »

as : $2x = 6$, $x + 3 = 5$, $2x - y = 3$ and $x^2 = 25$

The degree of the equation

is determined by the highest degree of the terms forming the equation.

For example:

- $5x + 2 = 7$ is an equation of **the first** degree in one unknown x
- $x^2 + x - 3 = 0$ is an equation of **the second** degree in one unknown x
- $2x + 3y = 5$ is an equation of **the first** degree in two unknowns x and y

The substitution set

is the set that contains the probable values of the unknown.

The solution set (The S.S.)

is the set whose elements satisfy the equality of the equation and it is a subset of the substitution set.



For example:

If $X + 3 = 5$ and the substitution set is $\{2, 3\}$

- Putting $X = 2$, **we get** that the left side = $2 + 3 = 5$ = the right side.

i.e. $X = 2$ is a solution of the equation.

- Putting $X = 3$, **we get** that the left side = $3 + 3 = 6 \neq$ the right side.

i.e. $X = 3$ is not a solution of the equation.

\therefore The S.S. = $\{2\}$ and it is a subset of the substitution set $\{2, 3\}$

- The previous method for solving the equation is called substitution method and we notice that it is a long method and it may be impossible if the number of elements of the substitution set is infinite as we see in $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$

Therefore, we will use another easier method that will need studying the properties of the equality relation to enable us to put the unknown X in one side of the equation alone.

The properties of the equality relation

- We can **add** any rational number to both sides of the equation.

For example:

If $X - 1 = 5$, then $X - 1 + 1 = 5 + 1$

i.e. $X = 6$

- We can **subtract** any rational number from both sides of the equation.

For example:

If $X + 3 = 2$, then $X + 3 - 3 = 2 - 3$

i.e. $X = -1$

- We can **multiply** both sides of the equation by the same rational number.

For example:

If $\frac{1}{5} X = 2$, then $\frac{1}{5} X \times 5 = 2 \times 5$

i.e. $X = 10$

- We can **divide** both sides of the equation by the same rational number not equal to zero.

For example:

If $7 X = 14$, then $\frac{7 X}{7} = \frac{14}{7}$

i.e. $X = 2$

Then by applying any of the previous properties on any equation, then we will get an **equivalent equation** to the origin equation that **has the same solution**.

Generally

If a , b and c are three rational numbers, then these numbers have the following properties :

1 If $a = b$	then $a + c = b + c$
2 If $a + c = b + c$	then $a = b$
3 If $a = b$	then $a \times c = b \times c$
4 If $a \times c = b \times c$, $c \neq 0$	then $a = b$

The following examples show how to use the equality properties to solve an equation of the first degree in one unknown.

Example 1

Find the solution set of the equation $x + 5 = 4$ if the substitution set is :

1 \mathbb{Z}

2 \mathbb{N}

Solution

1 If the substitution set is \mathbb{Z}

$$\therefore x + 5 = 4$$

Adding -5 to both sides

(-5 is the additive inverse of 5)

$$\therefore x + 5 + (-5) = 4 + (-5)$$

$$\text{i.e. } x + 0 = -1$$

$$\text{i.e. } x = -1$$

You can check the truth of your solution by substituting by $x = -1$ in the origin equation, you will get the left side $= -1 + 5 = 4 =$ the right side.

$$\therefore \text{The S.S.} = \{-1\}$$

2 If the substitution set is \mathbb{N}

$$\therefore x + 5 = 4$$

Subtracting 5 from both sides

$$\therefore x + 5 - 5 = 4 - 5$$

$$\therefore x = 4 - 5$$

\therefore The subtraction $(4 - 5)$ is impossible in \mathbb{N}

\therefore The S.S. in \mathbb{N} is \emptyset

Another method :

You can imagine that 5 is moved from the left side to the right side and became -5

$$x + 5 = 4 \Rightarrow x = 4 - 5$$

Example 2

Find the solution set of each of the following equations in \mathbb{Q} :

1 $2x - 5 = 13$

2 $2\frac{1}{2} - \frac{3}{2}x = 5$

Solution

1 $\therefore 2x - 5 = 13$

Adding 5 to both sides (5 is the additive inverse of (-5))

$$\therefore 2x - 5 + 5 = 13 + 5 \quad \text{i.e. } 2x = 18$$



Dividing both sides by 2

$$\therefore \frac{2x}{2} = \frac{18}{2}$$

$$\therefore x = 9 \quad \therefore \text{The S.S.} = \{9\}$$

“Check the truth of the solution”

Another method :

You can imagine that 2 is moved from the left side to the right side and it became divisor

$$2x = 18 \Rightarrow x = \frac{18}{2}$$

Subtracting $2\frac{1}{2}$ from both sides

$$2 \therefore 2\frac{1}{2} - \frac{3}{2}x = 5$$

$$\therefore 2\frac{1}{2} - \frac{3}{2}x - 2\frac{1}{2} = 5 - 2\frac{1}{2}$$

$$\therefore -\frac{3}{2}x = \frac{5}{2}$$

$$\therefore -\frac{3}{2}x = 2\frac{1}{2}$$

Multiplying both sides by $(-\frac{2}{3})$ ($-\frac{2}{3}$ is the multiplicative inverse of $-\frac{3}{2}$)

$$\therefore -\frac{3}{2}x \times (-\frac{2}{3}) = \frac{5}{2} \times (-\frac{2}{3}) \quad \therefore x = -\frac{5}{3}$$

$$\therefore \text{The S.S.} = \{-\frac{5}{3}\}$$

“Check the truth of the solution”

Example 3

Find the S.S. of each of the following equations :

$$1 \quad 2(x+3) = 4, \text{ where } x \in \mathbb{Z}$$

$$2 \quad 5(x+2) - 1 = 19, \text{ where } x \in \mathbb{Q}$$

Solution

$$1 \therefore 2(x+3) = 4$$

$$\therefore \frac{2(x+3)}{2} = \frac{4}{2}$$

Adding (-3) to both sides

$$\therefore x = -1$$

Dividing both sides by 2

$$\therefore x+3 = 2$$

$$\therefore x+3-3 = 2-3$$

$$\therefore \text{The S.S.} = \{-1\}$$

$$2 \therefore 5(x+2) - 1 = 19$$

Using the distribution property

$$\therefore 5x + 10 - 1 = 19$$

$$\therefore 5x + 9 = 19$$

Adding (-9) to both sides

$$\therefore 5x = 10$$

$$\therefore \frac{5x}{5} = \frac{10}{5}$$

Notice that :

$5(x+2) - 1 = 19$, $5x + 9 = 19$
and $5x = 10$ are equivalent equations.

$$\therefore 5x + 9 - 9 = 19 - 9$$

Dividing both sides by 5

$$\therefore x = 2 \quad \therefore \text{The S.S.} = \{2\}$$

Example 4 Find in \mathbb{Q} the solution set of each of the following equations :

1 $3x + 4 = 2(x + 1)$

2 $2(x + 3) - (x - 2) = 4(x - 1) + 3$

Solution

Notice that the variable (x) exists in the two sides, then we try to collect it in one side (say the left side)

1 $\therefore 3x + 4 = 2(x + 1)$

Using the distribution property

$$\therefore 3x + 4 = 2x + 2$$

Subtracting $2x$ from both sides

$$\therefore 3x - 2x + 4 = 2x - 2x + 2$$

$$\therefore x + 4 = 2$$

Subtracting 4 from both sides

$$\therefore x + 4 - 4 = 2 - 4$$

$$\therefore x = -2$$

$$\therefore \text{The S.S.} = \{-2\}$$

2 $\therefore 2(x + 3) - (x - 2) = 4(x - 1) + 3$

Using the distribution property : $\therefore 2x + 6 - x + 2 = 4x - 4 + 3$

$$\therefore x + 8 = 4x - 1$$

Subtracting x from both sides : $\therefore x - x + 8 = 4x - x - 1$

$$\therefore 8 = 3x - 1$$

Adding 1 to both sides :

$$\therefore 8 + 1 = 3x - 1 + 1$$

$$\therefore 9 = 3x$$

Dividing both sides by 3 : $\therefore \frac{9}{3} = \frac{3x}{3}$

$$\therefore 3 = x \quad \therefore \text{The S.S.} = \{3\}$$

Another method :

$$\begin{array}{c} \xrightarrow{(-4)} \\ 3x + 4 = 2x + 2 \\ \xrightarrow{(-2x)} \end{array}$$

$$\therefore 3x - 2x = 2 - 4$$

$$\text{i.e. } x = -2$$

Another method :

$$\begin{array}{c} \xrightarrow{(+1)} \\ x + 8 = 4x - 1 \\ \xrightarrow{(-x)} \end{array}$$

$$\therefore 8 + 1 = 4x - x$$

$$\text{i.e. } 9 = 3x$$

TRY
by yourself 1

Find the solution set of each of the following equations :

1 $x - 5 = 2$, where $x \in \mathbb{N}$

2 $2x + 11 = 3$, where $x \in \mathbb{Z}$

3 $2x - 3 = 5x + 6$, where $x \in \mathbb{Q}$



Using equations in solving word problems

To solve the word problems in algebra, we translate the verbal statements into algebraic symbols and expressions and the following table shows some examples for that.

Verbal statement	Algebraic expression
• Two numbers, their sum is 9	$x, 9 - x$
• Two numbers, the difference between them is 4	$x, x - 4$ (or $x, x + 4$)
• Two numbers, their product is 10	$x, \frac{10}{x}$
• Two numbers, one of them is twice the other.	$x, 2x$ (or $x, \frac{1}{2}x$)
• Two numbers, one of them is third of the other.	$x, \frac{1}{3}x$ (or $x, 3x$)
• Eight subtracted from three times of a number.	$3x - 8$
• Two numbers, one of them increases than twice of the other by 5	$x, 2x + 5$
• Three consecutive integers.	$x, x + 1, x + 2$
• Three consecutive even numbers.	$x, x + 2, x + 4$
• Three consecutive odd numbers.	$x, x + 2, x + 4$

Example 5

Two natural numbers, one of them is thrice of the other. If the sum of them is 16, find the two numbers.

Solution

- We give one of the two number the symbol x
- Using the information given, we form a first degree equation in one unknown.
 - \therefore The other number is thrice of the number x
 - \therefore The other number $= 3x$
 - \therefore The sum of the two numbers $= 16$
 - \therefore The equation is $x + 3x = 16$
- We solve the equation we get to find the value of the unknown.
 - $\therefore x + 3x = 16 \quad \therefore 4x = 16 \quad \text{Dividing by 4: } \therefore x = 4$
- i.e. one of the two numbers $= 4$, the other number $= 3 \times 4 = 12$
- We make sure that the solution is right by using the problem itself, not by using the equation.
 - $\therefore 12$ is the thrice of 4 , $12 + 4 = 16 \quad \therefore$ The solution is true.

Example 6

Three natural consecutive odd numbers whose sum is 27, find these numbers.

Solution

Let the smallest odd number = X

\therefore Each odd number exceeds the odd number just before it by 2

\therefore The next odd number = $X + 2$ and the third odd number = $X + 4$

\therefore The sum of the numbers = 27

$\therefore X + (X + 2) + (X + 4) = 27$

$\therefore 3X + 6 = 27$

$\therefore 3X = 27 - 6$

$\therefore 3X = 21$

$\therefore X = \frac{21}{3}$

$\therefore X = 7$

i.e. The numbers are 7, 9 and 11

To check the solution :

The numbers 7, 9 and 11 are natural consecutive odd numbers

, $7 + 9 + 11 = 27$

\therefore The solution is true.

**Remember that**

- The perimeter of a rectangle = 2 (length + width)
- The perimeter of a square = side length \times 4
- The perimeter of the triangle = the sum of its side lengths
- The area of the triangle = $\frac{1}{2}$ the base length \times the height
- The sum of measures of the interior angles of the triangle = 180°

Example 7

A rectangle with length equals twice its width and its perimeter = 18 cm. Find the dimensions of the rectangle.

Solution

Let the width of the rectangle = X cm.

\therefore Its length = $2X$ cm.

\therefore The perimeter of the rectangle = 2 (length + width)

$\therefore 18 = 2(2X + X)$

$\therefore 18 = 2 \times 3X$

$\therefore 18 = 6X$

$\therefore X = 3$

i.e. The width of the rectangle = 3 cm. and its length = 6 cm.

To check the solution :

\therefore The length of the rectangle = 6 cm. equals twice its width 3 cm.

, the perimeter of the rectangle = $2(6 + 3) = 2 \times 9 = 18$ cm.

\therefore The solution is true.

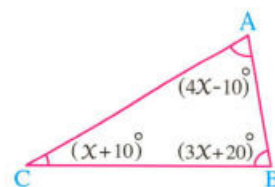
**Example 8**

In the opposite figure :

ABC is a triangle in which $m(\angle A) = (4x - 10)^\circ$

, $m(\angle B) = (3x + 20)^\circ$, $m(\angle C) = (x + 10)^\circ$

Find the measures of the angles of the triangle.

**Solution**

\therefore The sum of measures of the interior angles of the triangle = 180°

$$\therefore (4x - 10) + (3x + 20) + (x + 10) = 180$$

$$\therefore 8x + 20 = 180$$

Subtracting 20 from both sides

$$\therefore 8x = 180 - 20$$

$$\therefore 8x = 160$$

$$\therefore x = \frac{160}{8}$$

$$\therefore x = 20$$

$$\therefore m(\angle A) = (4 \times 20) - 10 = 80 - 10 = 70^\circ$$

$$\therefore m(\angle B) = (3 \times 20) + 20 = 60 + 20 = 80^\circ$$

$$\therefore m(\angle C) = 20 + 10 = 30^\circ$$

To check the solution :

$$\therefore m(\angle A) + m(\angle B) + m(\angle C) = 70^\circ + 80^\circ + 30^\circ = 180^\circ$$

\therefore The solution is true.

TRY
by yourself **2**

The difference between two integers is 4 and their sum is 14 , find the two numbers.

Solving inequalities in \mathbb{Q}



- We had studied before some concepts as the substitution set and the solution set in equations which are also the same concepts for inequalities.
- The solution set of the inequality is the set whose elements satisfy the inequality and it is a subset of the substitution set.

And before studying how to solve inequalities we will study the properties of inequality :

Properties of inequalities

We know that $6 > -9$ is a true inequality.

But do the following operations lead to true inequalities ?

- 1 Add 2 to the two sides of the inequality

$$\therefore 6 + 2 > -9 + 2 \longrightarrow 8 > -7 \text{ (true inequality)}$$

Generally We can add a constant number to both sides of the inequality without change in the inequality relation.

- 2 Subtract 7 from the two sides of the inequality

$$\therefore 6 - 7 > -9 - 7 \longrightarrow -1 > -16 \text{ (true inequality)}$$

Generally We can subtract a constant number from the two sides of the inequality without change in the inequality relation.



③ Multiply the two sides of the inequality by 5 (positive number)

$$\therefore 6 \times 5 > -9 \times 5 \longrightarrow 30 > -45 \text{ (true inequality)}$$

Generally Multiplying the two sides of the inequality by a positive number does not change the inequality relation.

④ Divide the two sides of the inequality by 3 (positive number)

$$\therefore \frac{6}{3} > -\frac{9}{3} \longrightarrow 2 > -3 \text{ (true inequality)}$$

Generally Dividing the two sides of the inequality by a positive number does not change the inequality relation.

⑤ Multiply the two sides of the inequality by -1 (negative number)

$$\therefore 6 \times (-1) > -9 \times (-1) \longrightarrow -6 > 9 \text{ (false inequality) because } -6 < 9$$

Generally If we multiply the two sides of the inequality by a negative number , then we change the sign of the inequality to the opposite sign.

⑥ Divide the two sides of the inequality by -3 (negative number)

$$\therefore \frac{6}{-3} > \frac{-9}{-3} \longrightarrow -2 > 3 \text{ (false inequality) because } -2 < 3$$

Generally If we divide the two sides of the inequality by a negative number , then we change the inequality sign to the opposite sign.

We can summarize the properties of inequality that noticed before as follows :

Assuming that a , b , c are three rational numbers , then :

① If $a < b$, then $a + c < b + c$
② If $a < b$, then $a - c < b - c$
③ If $a < b$, c is a positive number	, then $ac < bc$
④ If $a < b$, c is a positive number	, then $\frac{a}{c} < \frac{b}{c}$
⑤ If $a < b$, c is a negative number	, then $ac > bc$
⑥ If $a < b$, c is a negative number	, then $\frac{a}{c} > \frac{b}{c}$

Remark

If a and b are two non-zero rational numbers have the same sign and $a > b$, then : $\frac{1}{a} < \frac{1}{b}$

Example 1

Find the solution set of the inequality : $x + 2 < 5$, in each of the following cases :

1 If $x \in \mathbb{Z}$

2 If $x \in \mathbb{N}$

, then represent the solution set on the number line in each case.

Solution

$$\therefore x + 2 < 5$$

Subtracting 2 from the two sides

$$\therefore x + 2 - 2 < 5 - 2$$

$$\text{i.e. } x < 3$$

1 When $x \in \mathbb{Z}$

The solution set is all the integers which are less than 3

$$\text{i.e. The S.S.} = \{2, 1, 0, -1, \dots\}$$



2 When $x \in \mathbb{N}$

The solution set is all the natural numbers which are less than 3

$$\text{i.e. The S.S.} = \{2, 1, 0\}$$



We notice from the previous example that :

The solution set in \mathbb{N} differs from the solution set in \mathbb{Z} , because the solution set of the inequality depends on the substitution set.

Example 2

Find the solution set of the inequality : $2x - 5 > 5$, in each of the following cases :

1 If $x \in \mathbb{Q}$

2 If $x \in \mathbb{Z}$

Solution

$$\therefore 2x - 5 > 5$$

Adding 5 to both sides

$$\therefore 2x - 5 + 5 > 5 + 5$$

$$\therefore 2x > 10$$

Multiplying both sides by $\frac{1}{2}$

$$\therefore \frac{1}{2} \times 2x > \frac{1}{2} \times 10$$

$$\text{i.e. } x > 5$$



1 When $x \in \mathbb{Q}$

The S.S. is all the rational numbers which are greater than 5, then we write it by characterized property method because it is difficult to list all its members.

i.e. The S.S. = $\{x : x \in \mathbb{Q}, x > 5\}$

2 When $x \in \mathbb{Z}$

The solution set is all the integers which are greater than 5

i.e. The S.S. = $\{6, 7, 8, \dots\}$

Example 3

Find in \mathbb{Q} the solution set of each of the following inequalities :

1 $4 - 2x \leq 2$

2 $7(x - 1) > 9x - 6$

Solution

1 $\therefore 4 - 2x \leq 2$

Adding -4 to both sides

$$\therefore -4 + 4 - 2x \leq -4 + 2 \quad \therefore -2x \leq -2$$

Dividing both sides by (-2)

$$\therefore \frac{-2x}{-2} \geq \frac{-2}{-2} \quad (\text{Notice the change of inequality sign})$$

$$\therefore x \geq 1 \quad \text{i.e. The S.S.} = \{x : x \in \mathbb{Q}, x \geq 1\}$$

2 $\therefore 7(x - 1) > 9x - 6 \quad \therefore 7x - 7 > 9x - 6$

Subtracting $(9x)$ from both sides

$$\therefore 7x - 9x - 7 > 9x - 9x - 6 \quad \therefore -2x - 7 > -6$$

Adding 7 to both sides

$$\therefore -2x - 7 + 7 > -6 + 7 \quad \therefore -2x > 1$$

Dividing both sides by (-2)

$$\therefore \frac{-2x}{-2} < \frac{1}{-2} \quad (\text{Notice the change of the inequality sign})$$

$$\therefore x < -\frac{1}{2} \quad \text{i.e. The S.S.} = \{x : x \in \mathbb{Q}, x < -\frac{1}{2}\}$$

Example 4

Find in \mathbb{Z} the solution set of the inequality $-11 \leq 3x - 5 < 4$, then represent it on the number line.

Solution

$$\therefore -11 \leq 3x - 5 < 4$$

Adding 5 to the three sides

$$\therefore -11 + 5 \leq 3x - 5 + 5 < 4 + 5 \qquad \therefore -6 \leq 3x < 9$$

Dividing all sides by 3

$$\therefore \frac{-6}{3} \leq \frac{3x}{3} < \frac{9}{3} \qquad \therefore -2 \leq x < 3$$

i.e. The S.S. = $\{-2, -1, 0, 1, 2\}$

**TRY**
by yourself

Find the solution set of each of the following inequalities :

1 $2x - 3 \geq 5$, where $x \in \mathbb{Q}$

2 $5x - 10 < 2x - 1$, where $x \in \mathbb{N}$

2 Statistics and Probability

Lesson One Samples :

- Systematic sample.
- Random sample.

Lesson Two Probability :

- Experimental probability.
- Theoretical probability.



Unit Objectives : By the end of this unit, student should be able to :

- recognize the sample and how to select it.
- classify the samples according to the selecting method of its elements.
- select a random sample from a society distributed randomly.
- use a calculator to select a random sample.
- perform a random experiment and write the sample space.
- recognize the concept of the event.
- calculate the probability of an event.
- recognize the impossible event.
- recognize the certain event.

Lesson

1

Samples



Prelude

When we check the production of a factory to know if its products are manufactured according to the certain specifications , usually we don't check the whole production of this factory , but it is sufficient to check a part of this production under certain conditions , in order to represent the whole production , then we generalize the results for all the production.

This part is called “a sample”.



Definition

A sample is a small part from a large society that looks like this society and represents it well and is selected randomly.

- And notice that the selected sample should wholly represent the society (the object of study) and it shouldn't be based on a certain group and neglect the other , so that the results of the study can be near reality and we can make decisions according to these results , so we can generalise these results on the society as a whole.



Types of samples

Samples are classified according to the way used in selecting its items and in this lesson , we introduce two types of samples :

1 Systematic sample.

2 Random sample.

1 Systematic sample

Systematic sample is the sample whose elements are selected from the elements of a society distributed randomly by following **a certain system or method** in selection.

For example:

To select a systematic sample representing 10% of the marks of students in a preparatory school in the mid-year exam in maths , to study the standards of students , we will do as follows :

1 Students must be distributed randomly in a numbered list **i.e.** selection shouldn't be from certain classes as excellent students' classes or selecting certain classes and neglecting others.

2 We select in a regular way the tenth student in each 10 student from the students list **i.e.** we select the mark of the tenth , twentieth , thirtieth , ... students is the list.



! Remark

If the society (the object of study) is already divided into classes or groups as the school is divided into classes for boys and others for girls , then we select a part of each group to be represented in the sample so that the sample represents the society as a whole.

2 Random sample

Random sample is the sample whose elements are selected from the elements of a society distributed randomly by following **a random and irregular method of selecting**.

In this sample , each individual must get the same chance of selecting.

So , we can select its elements by two methods :

- Manual method.
- Using the scientific calculator.

The first method (manual method) :

It is done as follows :

- 1** Every member of the society is given a number , then this number is written on a piece of paper such that all pieces of paper are of the same colour and size.
- 2** Each piece of paper is folded perfectly such that the written number does not appear and is put in a bag or a box and mixed together very well.
- 3** The selection is carried out by drawing a piece after piece without looking inside the paper till the operation of drawing is finished when the required number of selected members is done.



The second method (using the scientific calculator) :

This method depends on using the random number function on the scientific calculator shown in the opposite figure by pressing the following keys successively from the left :



Then a random number in the range 0.000 to 0.999 will appear every time. Take the apparent numbers without the decimal point. The numbers which are greater than the whole number of the society under study should be ignored and also if a number is repeated it must be ignored and taken once only.



In any survey , a 10% sample is considered adequate to provide reliable information about the whole society.

Example

A factory has 300 workers. The people in charge of the monthly magazine of this factory want to develop this magazine by doing a survey of a sample representing 10% of the total number of the workers in this factory. Show how the selection of this sample can be carried out using the calculator.





Solution

∴ The number of workers in the factory = 300 workers.

∴ The number of the random sample = $\frac{10}{100} \times 300 = 30$ workers.

Then we want to select 30 workers to do this survey. The selection operation can be carried out as follows :

- 1 Each worker in the factory is given a number from 1 to 300
- 2 Use the calculator to select 30 numbers randomly by the method mentioned before and the number that is more than 300 should be ignored.

For example:

By pressing the keys  →  →  →  successively.

- If we get the decimal 0.56 , then the number of the selected person is 56
- If we get the decimal 0.049 , then the number of the selected person is 49
- If we get the decimal 0.132 , then the number of the selected person is 132
- If we get the decimal 0.453 , it must be ignored because 453 is more than 300 and so on till we get 30 numbers.

Assuming that the calculator gave us the shown numbers in the opposite table , then the workers who carry these numbers are the selected sample to carry out this survey.

56	49	132	141	249	272
254	256	4	213	74	198
131	2	156	47	172	13
8	3	85	82	9	38
41	14	34	279	118	103

Enrich your knowledge

Pierre Simon Marquis de Laplace

He was a French mathematician and astronomer.

His first work was published in 1771 starting with differential equations however he had already started to think about the mathematical and philosophical concepts of probability and statistics.



Pierre Simon Marquis
de Laplace
(1749 - 1827)

Lesson 2

Probability



Prelude

In our daily life , many times we ask ourselves about some affairs that may happen in the future and that we cannot give a certain result for them.

For example:

- If the Egyptian football team get to the finals of African nations championship , what is its chance for winning the cup ?
- If an Egyptian citizen puts himself up for parliamentary elections in one of elections zones , what is his chance for winning in the elections ?

All these questions' answers are expectations to what may happen (occur) in the future referring to previous experience , studies or observations.



When we answer these questions , we use some words as «may be , chance or probable».

In mathematics , we call this probability. In this lesson , we will study :

1 Experimental probability.

2 Theoretical probability.



1 Experimental probability

- If one of the Olympic swimmers wants to achieve a new record in the next Olympic Games, what is the probability that this swimmer achieves this record?

The answer to this question cannot be got by expecting, hoping or by doing a survey of the opinions of the trainers or by asking the swimmer himself, but by trying.



i.e. The swimmer covers the needed distance in the race several times, then we record the number of times in which he could achieve the requested number and divide it by the total number of times, so the quotient is the probability of achieving the new record in the next Olympics.

The experimental probability depends on performing an experiment, then we record the results and use them to calculate the value of probability of an event occurrence using the rule:

$$\text{Experimental probability} = \frac{\text{Number of trials in which the outcome occurs}}{\text{Total number of trials}}$$

It is noticed that the more we carry out the experiment, the more we obtain an accurate value for the probability.

Example 1

If we tossed a piece of coin with double face 200 times and the results of appearance of a head or a tail in each toss were recorded in a table as shown:



	Heads (H)	Tails (T)	Total
Statistics Tallies	### /	### ### ### ### ### ### ### ### ### ### ### ### ### ### ### ### ### ### ////	
Frequency	106	94	200

Calculate:

- 1 The probability of appearance of a head.
- 2 The probability of appearance of a tail.

Solution

- 1 The probability of appearance of a head = $\frac{\text{Number of getting heads}}{\text{Total number of tosses}}$
 $= \frac{106}{200} = 0.53$
- 2 The probability of appearance of a tail = $\frac{\text{Number of getting tails}}{\text{Total number of tosses}}$
 $= \frac{94}{200} = 0.47$

TRY
by yourself **1**

Roll a fair die 25 times and record the results of appearance of a number on the upper face in a table, then calculate :

- 1 The probability of appearance of the number 4
- 2 The probability of appearance of the number 3

2 Theoretical probability

In the previous , we carried out the experiment of tossing a piece of coin and we found that :

- The probability of appearance of a head = 0.53
- The probability of appearance of a tail = 0.47



But when we study this experiment theoretically , we find that :

If we tossed the coin piece once , then we obtain either a head or a tail.

i.e. the number of possible outcomes = 2

and there is one chance to obtain a head and also one chance to obtain a tail.

i.e. all outcomes of the experiment have the same chance to happen.

i.e. the probability of appearance of a head = $\frac{1}{2} = 0.50$
 and the probability of appearance of a tail = $\frac{1}{2} = 0.50$

Notice that :

We can express the probability by percentage.

i.e. We write the probability of appearance of a head = 50%



Remark

Notice the difference between the experimental probability of appearance of a head (0.53) and the theoretical probability of appearance of a head (0.50)

i.e. When the number of times of carrying out the experiment increases, the value of experimental probability approaches the value of theoretical probability.

Definition of random experiment

Random experiment is an experiment in which we can specify all its possible outcomes before carrying it out but we cannot determine certainly which of them will occur.

Sample space

Sample space is the set of all possible outcomes of a random experiment and it is denoted by S

For example:

- When we toss a piece of coin once, then the sample space is $S = \{H, T\}$
- When we roll a fair die once observing the apparent number on the upper face, then the sample space is $S = \{1, 2, 3, 4, 5, 6\}$



Event

Event is a subset of the sample space.

For example:

If A is the event of appearance of an odd number when rolling a fair die once and observing the apparent number on the upper face, then $A = \{1, 3, 5\}$, $A \subset S$



Generally

The probability of any event occurrence $A \subset S$ is denoted by $P(A)$ and it is given by using the relation :

$$P(A) = \frac{\text{The number of elements of the event « A »}}{\text{The number of elements of sample space « S »}} = \frac{n \ll A \gg}{n \ll S \gg}$$

Example 2

If a fair die is rolled once and we observe the apparent number on the upper face, find the probability of each of the following events :

- 1 A is the event of appearance of a number more than 4
(Approximating the result to the nearest hundredth)
- 2 B is the event of appearance of an even number.
- 3 C is the event of appearance of a number equal to 5
(Approximating the result to the nearest tenth)
- 4 D is the event of appearance of a number equal to 7
- 5 E is the event of appearance of a number less than 7

**Solution**

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$$

$$1 \quad A = \{5, 6\}, n(A) = 2$$

$$\therefore P(A) = \frac{2}{6} = \frac{1}{3} \approx 0.33 \quad (\text{to the nearest hundredth})$$

$$2 \quad B = \{2, 4, 6\}, n(B) = 3 \quad \therefore P(B) = \frac{3}{6} = 0.5$$

$$3 \quad C = \{5\}, n(C) = 1$$

$$\therefore P(C) = \frac{1}{6} \approx 0.2 \quad (\text{to the nearest tenth})$$

$$4 \quad D = \{ \} \text{ or } \emptyset, n(D) = \text{zero}$$

$$\therefore P(D) = \frac{0}{6} = \text{zero} \quad (\text{the impossible event})$$

$$5 \quad E = \{1, 2, 3, 4, 5, 6\}, n(E) = 6$$

$$\therefore P(E) = \frac{6}{6} = 1 \quad (\text{the certain event})$$

! Remarks

- 1 **The impossible event** : is the event that has no chance for occurring.

i.e. The probability of the impossible event = Zero

- 2 **The certain event** : is the event that has all the possible outcomes.

i.e. The probability of the certain event = 1

- 3 The value of probability of any event is not less than zero and not more than one

i.e. $0 \leq \text{the probability of an event occurrence} \leq 1$



Example 3

From the set of digits $\{3, 4, 5\}$, form a two-digit number, then find the probability of each of the following events :

- 1 A « the event that the units digit is odd »
- 2 B « the event that the tens digit is even »
- 3 C « the event that the two digits are odd »
- 4 D « the event that the sum of the two digits = 8 »
- 5 E « the event that the product of the two digits = 20 »

Solution

$$S = \{33, 43, 53, 34, 44, 54, 35, 45, 55\}, n(S) = 9$$

- | | |
|--|---|
| 1 $A = \{33, 43, 53, 35, 45, 55\}, n(A) = 6$ | $\therefore P(A) = \frac{6}{9} = \frac{2}{3}$ |
| 2 $B = \{43, 44, 45\}, n(B) = 3$ | $\therefore P(B) = \frac{3}{9} = \frac{1}{3}$ |
| 3 $C = \{33, 53, 35, 55\}, n(C) = 4$ | $\therefore P(C) = \frac{4}{9}$ |
| 4 $D = \{53, 44, 35\}, n(D) = 3$ | $\therefore P(D) = \frac{3}{9} = \frac{1}{3}$ |
| 5 $E = \{54, 45\}, n(E) = 2$ | $\therefore P(E) = \frac{2}{9}$ |

Example 4

A bag has an amount of marbles of the same size and touch. If 2 marbles are red, 3 are blue and 5 are white and a marble is drawn randomly, calculate :



- 1 The probability of that (the drawn marble is red)
- 2 The probability of that (the drawn marble is blue)
- 3 The probability of that (the drawn marble is white)
- 4 The probability of that (the drawn marble is not blue)

Solution

The probability of occurrence of a certain outcome

$$= \frac{\text{The number of possible chances to get this outcome}}{\text{The total number of chances}}$$

$$\therefore \text{The total number of marbles} = 2 + 3 + 5 = 10$$

- 1 The probability of that (the drawn marble is red)

$$= \frac{\text{The number of red marbles}}{\text{The total number of marbles}} = \frac{2}{10} = \frac{1}{5}$$

- 2 The probability of that (the drawn marble is blue)

$$= \frac{\text{The number of blue marbles}}{\text{The total number of marbles}} = \frac{3}{10}$$

- 3 The probability of that (the drawn marble is white)

$$= \frac{\text{The number of white marbles}}{\text{The total number of marbles}} = \frac{5}{10} = \frac{1}{2}$$

- 4 The probability of that (the drawn marble is not blue)

$$= \frac{\text{The number of marbles which aren't blue}}{\text{The total number of marbles}} = \frac{10 - 3}{10} = \frac{7}{10}$$

! Remark

In the previous example , notice that :

$$P(\text{red marble}) = \frac{2}{10}, P(\text{blue marble}) = \frac{3}{10},$$

$$P(\text{white marble}) = \frac{5}{10} \therefore \frac{2}{10} + \frac{3}{10} + \frac{5}{10} = 1$$

\therefore The sum of probabilities of all outcomes of a random experiment = 1

So , if the probability of occurrence of an event is a , then the probability that it doesn't occur = $1 - a$

So , we can find the probability that the drawn marble is not blue as follows :

The probability that the drawn marble is not blue

$$= 1 - \text{the probability that it is blue} = 1 - \frac{3}{10} = \frac{7}{10}$$

Example 5

A Class has some students who wear glasses and other students who don't wear glasses. If a student is chosen randomly from this class and the probability that this student wears glasses is 0.1



- 1 Find the probability that this student does not wear glasses.
- 2 If the number of students in this class is 30 students , find the expected number of students who wear glasses.



Solution

- 1 The probability that this student does not wear glasses
 $= 1 - \text{the probability that the student wears glasses} = 1 - 0.1 = 0.9$
- 2 \therefore The expected number of outcomes of an event = the probability of occurrence of this event \times the total number of all possible outcomes.
 \therefore The expected number of students who wear glasses $= 0.1 \times 30$
 $= 3$ students.

Example 6

A spinner game was divided into some equal sectors. 2 of them are green, 4 are blue and the rest are red. If the probability that the pointer stops pointing at a green sector is $\frac{1}{6}$, then find the number of red sectors.

Solution

\therefore The probability that the pointer stops pointing at a green sector

$$= \frac{\text{The number of green sectors}}{\text{The number of all sectors}}$$

$$\therefore \frac{1}{6} = \frac{2}{\text{The number of all sectors}}$$

\therefore The number of all sectors $= 2 \times 6 = 12$ sectors.

\therefore The number of red sectors $= 12 - (2 + 4) = 6$ sectors.

TRY by yourself 2

- 1 A box contains cards numbered from 1 to 15. If a card is drawn randomly, what is the probability that the written number on the card is divisible by 5?
- 2 An experiment has 3 outcomes. If the probability of occurrence of the first outcome is 0.3 and the probability of the second is 0.45, calculate the probability of the third outcome.
- 3 A farm has 2000 cows. If the probability of cow madness infection in this farm is 0.17, what is the expected number of infected cows?

Second | Geometry and Measurement

UNIT 3 Geometry and Measurement — 59



3 Geometry and Measurement

- Lesson One** Deductive proof.
- Lesson Two** The polygon.
- Lesson Three** The parallelogram and its properties.
- Lesson Four** The special cases of the parallelogram.
- Lesson Five** The triangle : - Theorem (1), exterior angle of the triangle.
- Lesson Six** Theorem (2), theorem (3).
- Lesson Seven** Pythagoras' theorem.
- Lesson Eight** Geometric transformations.
- Lesson Nine** Reflection in a straight line.
- Lesson Ten** Reflection in a point.
- Lesson Eleven** Translation.
- Lesson Twelve** Rotation.



Unit Objectives : By the end of this unit, student should be able to :

- use the deductive proof to prove the verity of the theorems.
- recognize the polygon and the difference between the convex polygon and the concave polygon.
- find the sum of the measures of the interior angles and the exterior angles of any polygon.
- recognize the regular polygon and find the measure of its interior angle.
- recognize the parallelogram and its properties.
- deduce when the quadrilateral be a parallelogram.
- recognize the special cases of the parallelogram (Rectangle - Rhombus - Square).
- deduce that the sum of measures of the interior angles of a triangle is 180°
- recognize the exterior angle of a triangle and its measure.
- deduce the relation between the length of the line segment joining the midpoints of two sides of a triangle and the length of the third side.
- recognize the Pythagoras' theorem.
- recognize the properties of the reflection in a straight line, the reflection in a point, the translation and the rotation.
- find an image of a geometric figure by using the reflection, the translation and the rotation.

Revision on the important topics have been studied on geometry in the first term

1 Relations between angles

The two adjacent supplementary angles

The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are supplementary.

For example:

In the opposite figure :

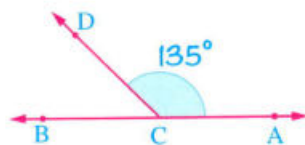
$$\text{If } \overrightarrow{AB} \cap \overrightarrow{CD} = \{C\}$$

, therefore

$$m(\angle ACD) + m(\angle DCB) = 180^\circ \text{ "Straight angle"}$$

$$\text{and if } m(\angle ACD) = 135^\circ$$

$$\text{, then } m(\angle DCB) = 180^\circ - 135^\circ = 45^\circ$$



Vertically opposite angles (V.O.A.)

If two straight lines intersect , then each two vertically opposite angles are equal in measure.

In the opposite figure :

$$m(\angle 1) = m(\angle 2) \text{ , } m(\angle 3) = m(\angle 4)$$

For example:

In the opposite figure :

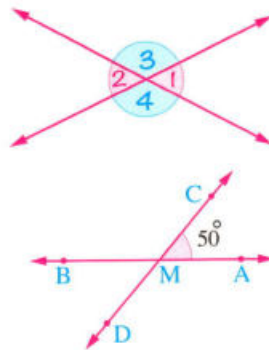
$$\text{If } \overrightarrow{AB} \cap \overrightarrow{CD} = \{M\}$$

$$\text{, } m(\angle AMC) = 50^\circ$$

$$\text{, then } m(\angle DMB) = m(\angle AMC) = 50^\circ \text{ (vertically opposite angles)}$$

$$\text{, } m(\angle CMB) = 180^\circ - m(\angle AMC) = 180^\circ - 50^\circ = 130^\circ$$

$$\text{, then } m(\angle AMD) = m(\angle CMB) = 130^\circ \text{ (vertically opposite angles)}$$





Accumulative angles at a point

The sum of measures of the accumulative angles at a point is 360°

In the opposite figure :

$$m(\angle 1) + m(\angle 2) + m(\angle 3) + m(\angle 4) + m(\angle 5) = 360^\circ$$

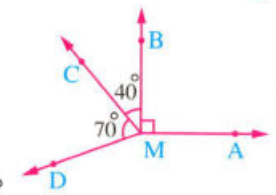
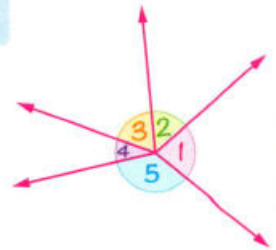
For example:

In the opposite figure :

If \overrightarrow{MA} , \overrightarrow{MB} , \overrightarrow{MC} and \overrightarrow{MD} are rays having the same starting point M

$$\text{, then } m(\angle AMB) + m(\angle BMC) + m(\angle CMD) + m(\angle DMA) = 360^\circ$$

$$\text{So , } m(\angle DMA) = 360^\circ - (90^\circ + 40^\circ + 70^\circ) = 160^\circ$$



2 Parallelism

If a straight line intersects two parallel straight lines , then :

1 Each two alternate angles are equal in measure.

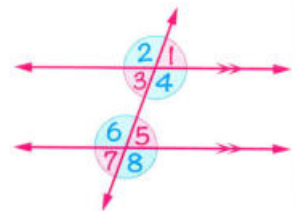
- $m(\angle 3) = m(\angle 5)$ "Alternate angles"
- $m(\angle 4) = m(\angle 6)$ "Alternate angles"

2 Each two corresponding angles are equal in measure.

- $m(\angle 1) = m(\angle 5)$ "Corresponding angles"
- $m(\angle 2) = m(\angle 6)$ "Corresponding angles"
- $m(\angle 3) = m(\angle 7)$ "Corresponding angles"
- $m(\angle 4) = m(\angle 8)$ "Corresponding angles"

3 Each two interior angles in the same side of the transversal are supplementary.

- $m(\angle 3) + m(\angle 6) = 180^\circ$ "Interior angles in the same side of the transversal"
- $m(\angle 4) + m(\angle 5) = 180^\circ$ "Interior angles in the same side of the transversal"

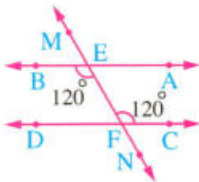


How to prove that two straight lines are parallel ?

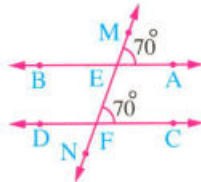
The two straight lines are parallel if a third straight line intersects them (as a transversal) and one of the following cases is satisfied :

- 1 Two alternate angles have the same measure.
- 2 Two corresponding angles have the same measure.
- 3 Two interior angles in the same side of the transversal are supplementary.

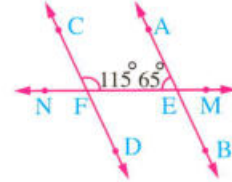
Notice each of the following figures where : \overleftrightarrow{AB} and \overleftrightarrow{CD} are two straight lines and \overleftrightarrow{MN} is a transversal to them :



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ because :
 $m(\angle BEF) = m(\angle EFC)$
 $= 120^\circ$
 and they are two alternate angles.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ because :
 $m(\angle AEM) = m(\angle CFE)$
 $= 70^\circ$
 and they are two corresponding angles.

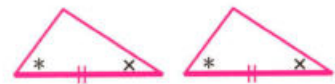
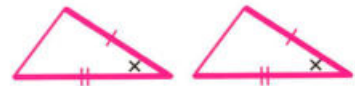


$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ because :
 $m(\angle AEF) + m(\angle CFE)$
 $= 65^\circ + 115^\circ = 180^\circ$
 and they are interior angles in the same side of the transversal.

3 Cases of congruence of two triangles

Two triangles are congruent if one of the following cases is satisfied :

- 1 Congruence of two sides and the included angle of one triangle to the corresponding parts of the other triangle.
- 2 Congruence of two angles and the side drawn between their vertices of one triangle to the corresponding parts of the other triangle.
- 3 Congruence of each side of one triangle to the corresponding side of the other triangle.
- 4 Two right-angled triangles are congruent , if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.



Lesson

1

Deductive proof



- Deductive proof is a theoretical method to prove theorems and reaching to results. In a deductive proof, we do not need to use geometric instruments in measuring, but we use definitions, properties, facts and previous theorems to get results, by writing mathematical statements, which means that when we write any statement, we have to mention the reason for which this statement is true.

For example:

If you know that
ABCD is a rectangle,
you can write the following



Mathematical statement	Reason
• ABCD is a rectangle	Given
• $AB = CD$	opposite sides in the rectangle are equal in length
• $m(\angle B) = 90^\circ$	Angles of the rectangle are right
• $\overline{AD} \parallel \overline{BC}$	opposite sides in the rectangle are parallel

How to write the proof in geometry ?

- Read carefully the problem, in order to determine what is “given”, which are all information given in the problem, and “required”, which is the question we need to answer in the problem.

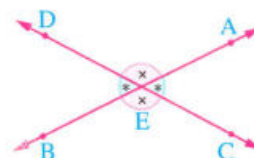
- 2 Use the given information in the problem to draw a clear geometric figure (if the figure is not already given) and put on the figure the given information in the problem as : side lengths , angles measures and so on.
- 3 Write the given in form of points.
- 4 Write the required to be proved.
- 5 Think about a plan for the proof which is the main steps needed in order to arrive to the required.
- 6 Write the proof by writing a mathematical statements mention for each statement a reason that makes this statement is true.
- 7 Be sure that you answered the question of the problem.

In the following, some examples showing how to write the deductive proof :

1 If two straight lines intersect , then the measures of each two vertically opposite angles are equal.

Given

\overleftrightarrow{AB} and \overleftrightarrow{CD} are
two straight lines
intersecting at E



Required to
prove (R.T.P.)

$$m(\angle AED) = m(\angle BEC)$$

Proof

$\therefore \angle AED$ and $\angle AEC$ are two adjacent angles
where $\overrightarrow{EC} \cup \overrightarrow{ED} = \overleftrightarrow{CD}$

$$\therefore m(\angle AED) + m(\angle AEC) = 180^\circ$$

, $\therefore \angle AEC$ and $\angle BEC$ are

two adjacent angles

where $\overrightarrow{EA} \cup \overrightarrow{EB} = \overleftrightarrow{AB}$

$$\therefore m(\angle AEC) + m(\angle BEC) = 180^\circ$$

$$\therefore m(\angle AED) + m(\angle AEC) = m(\angle AEC) + m(\angle BEC)$$

$$\therefore m(\angle AED) = m(\angle BEC)$$

(Q.E.D.)*

Similarly , you can prove that $m(\angle AEC) = m(\angle BED)$

* Q.E.D. is an abbreviation for quod erat demonstrandum.

It is a Latin abbreviation which means to be demonstrated

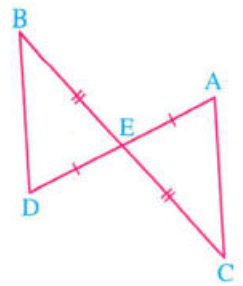


Example 1

In the opposite figure :

$\overline{AD} \cap \overline{BC} = \{E\}$ where $AE = DE$ and $BE = CE$

Prove that : $\triangle AEC \equiv \triangle DEB$



Solution

Given $\overline{AD} \cap \overline{BC} = \{E\}$ where $AE = DE$, $BE = CE$

R.T.P. $\triangle AEC \equiv \triangle DEB$

Proof $\therefore \overline{AD} \cap \overline{BC} = \{E\}$

$\therefore m(\angle AEC) = m(\angle DEB)$ (V.O.A)

\therefore In $\triangle AEC$ and DEB :

$\begin{cases} AE = DE \text{ (given)} \\ CE = BE \text{ (given)} \\ m(\angle AEC) = m(\angle DEB) \text{ (by proof)} \end{cases}$

$\therefore \triangle AEC \equiv \triangle DEB$

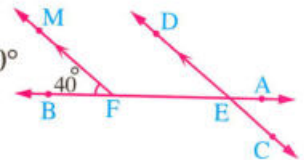
(Q.E.D.)

TRY 1 by yourself

In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{E\}$, $\overline{CD} \parallel \overline{FM}$, $F \in \overline{AB}$ and $m(\angle MFB) = 40^\circ$

Complete the following proof to find : $m(\angle AEC)$



Given

Required to
find (R.T.F.)

Proof

$\therefore \overline{CD} \parallel \dots\dots\dots$ (given) and \overline{AB} is a transversal.

$\therefore m(\angle DEB) = m(\angle \dots\dots\dots) = 40^\circ$ (corresponding angles)

$\therefore \overline{AB} \cap \overline{CD} = \{E\}$

$\therefore m(\angle AEC) = m(\angle \dots\dots\dots)$ (V.O.A)

$\therefore m(\angle AEC) = \dots\dots\dots^\circ$

(The req.)

2 The sum of the measures of the accumulative angles at a point is equal to 360°

Given \overrightarrow{OA} , \overrightarrow{OB} , \overrightarrow{OC} and \overrightarrow{OD} are rays that start at O

R.T.P. The sum of the measures of the accumulative angles at O is 360°

Construction Draw \overrightarrow{DO} and $E \in \overrightarrow{DO}$

Proof

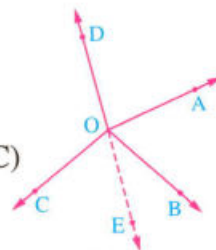
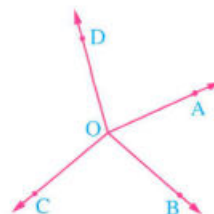
$$\therefore m(\angle EOB) + m(\angle BOA) + m(\angle AOD) = 180^\circ$$

$$, m(\angle EOC) + m(\angle COD) = 180^\circ$$

$$\therefore m(\angle EOB) + m(\angle BOA) + m(\angle AOD) + m(\angle EOC) + m(\angle COD) = 180^\circ + 180^\circ = 360^\circ$$

$$\therefore m(\angle AOB) + m(\angle BOC) + m(\angle COD) + m(\angle DOA) = 360^\circ$$

(Q.E.D.)



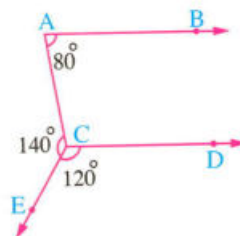
Example 2

In the opposite figure :

$$m(\angle BAC) = 80^\circ, m(\angle DCE) = 120^\circ$$

$$\text{and } m(\angle ACE) = 140^\circ$$

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{CD}$



Solution

Given

$$m(\angle BAC) = 80^\circ, m(\angle DCE) = 120^\circ,$$

$$m(\angle ACE) = 140^\circ$$

R.T.P.

$$\overrightarrow{AB} \parallel \overrightarrow{CD}$$

Proof

$$\therefore m(\angle DCA) + m(\angle DCE) + m(\angle ACE) = 360^\circ$$

(accumulative angles at C)

$$\therefore m(\angle DCA) = 360^\circ - (120^\circ + 140^\circ) = 100^\circ$$

$$\therefore m(\angle BAC) + m(\angle DCA) = 80^\circ + 100^\circ = 180^\circ$$

And they are interior angles in the same side of the transversal \overrightarrow{AC}

$$\therefore \overrightarrow{AB} \parallel \overrightarrow{CD}$$

(Q.E.D.)



TRY by yourself 2

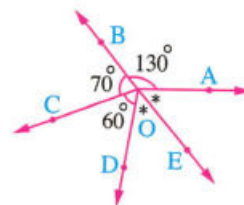
In the opposite figure :

$$m(\angle AOB) = 130^\circ, m(\angle BOC) = 70^\circ,$$

$$m(\angle COD) = 60^\circ \text{ and } \overrightarrow{OE} \text{ bisects } \angle AOD$$

Complete the following proof to prove that :

\overrightarrow{OE} and \overrightarrow{OB} are on one straight line.



Given

R.T.P.

Proof $\therefore m(\angle AOB) + m(\angle BOC) + m(\angle COD) + m(\angle AOD) = \dots\dots\dots^\circ$

(Accumulative angles at O)

$$\therefore m(\angle AOD) = \dots\dots\dots^\circ - \dots\dots\dots^\circ = \dots\dots\dots^\circ$$

$\therefore \overrightarrow{OE}$ bisects $\angle \dots\dots\dots$ (given)

$$\therefore m(\angle AOE) = \frac{1}{2} m(\angle \dots\dots\dots)$$

$$\therefore m(\angle AOE) = \frac{1}{2} \times \dots\dots\dots^\circ = \dots\dots\dots^\circ$$

$$\therefore m(\angle AOE) + m(\angle AOB) = \dots\dots\dots^\circ + \dots\dots\dots^\circ = \dots\dots\dots^\circ$$

$\therefore \overrightarrow{OE}$ and \overrightarrow{OB} are on one straight line.

(Q.E.D.)

Enrich your knowledge

Euclid

He was a Greek mathematician. He lived in Alexandria.

Euclid introduced the system of axioms.

Since this time, geometry of Euclid was considered a model of logical proof.

Euclid's Axioms (notations) :

- Things which are equal to one thing are equal to each other.
- If equals are added to equals, then the sums are equal.
- Things which coincide with one another are equal to each other.
- The whole is greater than the part.



Euclid
(325 B.C. - 265 B.C.)

Lesson 2

The polygon



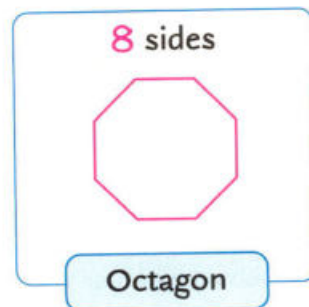
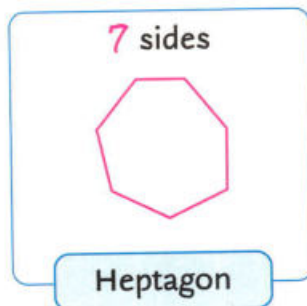
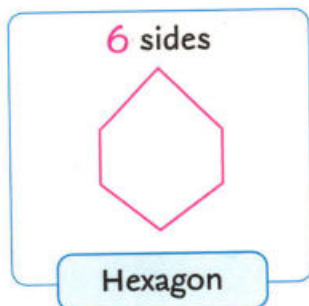
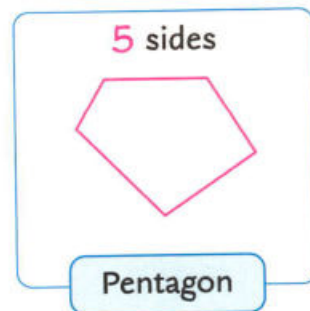
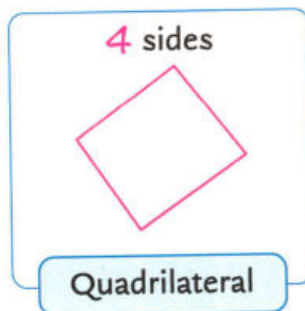
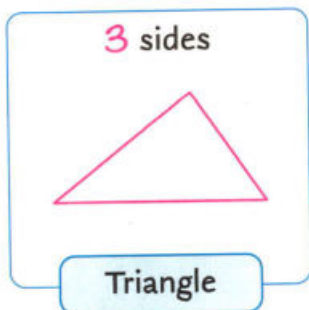
The polygon

It is a simple closed line that consists of three line segments , or more. The polygon is named according to the number of its sides.

Notice that :

The simple line is the line that does not cut itself.

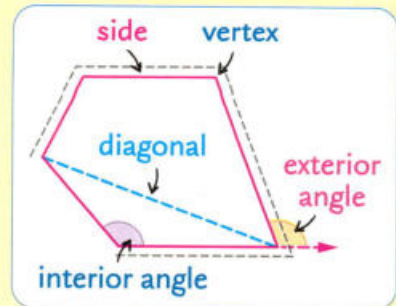
Examples for some polygons :





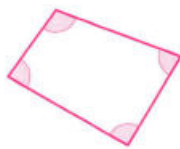
! Remarks

- ① Each line segment of the line segments forming the polygon is called a **side**.
- ② Each point resulted from intersecting of two adjacent sides of the polygon is called a **vertex**.
- ③ The sum of the side lengths of the polygon is called the **perimeter of the polygon**.
- ④ Each line segment joining two non-adjacent vertices of the polygon is called a **diagonal**.
- ⑤ The included angle between two adjacent sides of the polygon is called an **interior angle**.
- ⑥ The included angle between a side of the polygon and the extension of its adjacent side is called an **exterior angle**.
- ⑦ The number of sides of any polygon = the number of its vertices = the number of its interior angles.

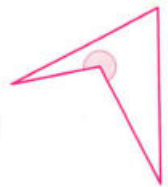


Convex polygon and concave polygon

- The polygon is **convex** if the measure of any of its interior angles is less than 180°



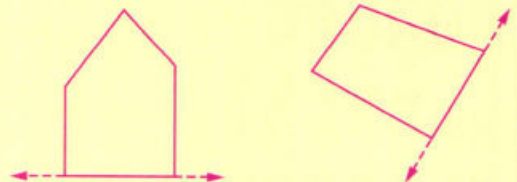
- The polygon is **concave** if the measure of one of its interior angles at least is greater than 180° "reflex angle"



! Remarks

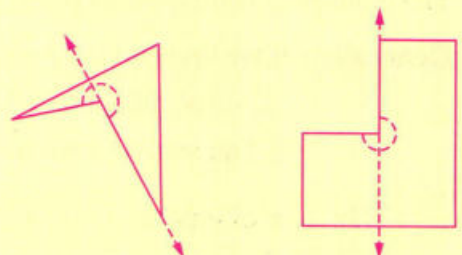
• In the convex polygon :

If a straight line is drawn to pass through any two consecutive vertices, then the remaining vertices lie on one side of this straight line.



• In the concave polygon :

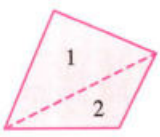
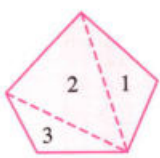
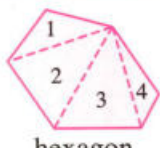
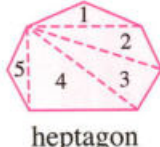
There are straight lines passing through two consecutive vertices and the remaining vertices lie on two different sides of the straight lines.



The sum of measures of the interior angles of the polygon

We knew before that : The sum of measures of the interior angles of the triangle equals 180°
 We can use that to deduce a general rule to find the sum of measures of the interior angles of any polygon whose number of sides is n

If we draw from any vertex of the polygon all diagonals that pass through this vertex , then the surface of this polygon will be divided into a number of triangles as shown in the following table :

The polygon	The number of its sides	The number of the resulted triangles	The sum of measures of the interior angles of the polygon
 quadrilateral	4	2	$2 \times 180^\circ = 360^\circ$
 pentagon	5	3	$3 \times 180^\circ = 540^\circ$
 hexagon	6	4	$4 \times 180^\circ = 720^\circ$
 heptagon	7	5	$5 \times 180^\circ = 900^\circ$

From the previous, notice that :

The number of resulted triangles = the number of the sides of the polygon $- 2$

Generally : If we draw all the possible diagonals that out of a vertex of a polygon having (n) sides, then the surface of this polygon will be divided into $(n - 2)$ triangles.
 \therefore The sum of measures of the interior angles of the triangle = 180°

\therefore The sum of measures of the interior angles of a polygon of n sides equals $(n - 2) \times 180^\circ$



For example:

- The sum of measures of the interior angles of the octagon $= (8 - 2) \times 180^\circ = 1080^\circ$
- The sum of measures of the interior angles of the enneagon (nonagon)
 $= (9 - 2) \times 180^\circ = 1260^\circ$

Example 1

Complete the following table :

Number of sides of the polygon	10	3	12	15
Sum of measures of its interior angles

Solution

Number of sides of the polygon	10	3	12	15
Sum of measures of its interior angles	$8 \times 180^\circ = 1440^\circ$	$1 \times 180^\circ = 180^\circ$	$10 \times 180^\circ = 1800^\circ$	$13 \times 180^\circ = 2340^\circ$

Example 2

The sum of measures of the interior angles of a polygon is 2160° .
Find the number of its sides.

Solution

\therefore The sum of measures of the interior angles of a polygon of n sides equals $(n - 2) \times 180^\circ$

$$\therefore 2160^\circ = (n - 2) \times 180^\circ \quad \therefore n - 2 = \frac{2160}{180} = 12 \quad \therefore n = 14$$

\therefore The number of sides of this polygon is 14 sides.

TRY by yourself 1

Complete the following table :

Number of sides of the polygon	11	16
Sum of measures of its interior angles	900°	540°

Example 3

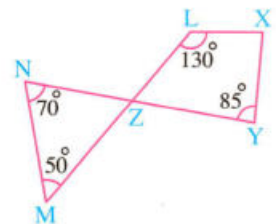
In the opposite figure :

$$\overline{LM} \cap \overline{YN} = \{Z\}, m(\angle M) = 50^\circ,$$

$$m(\angle N) = 70^\circ, m(\angle Y) = 85^\circ \text{ and}$$

$$m(\angle L) = 130^\circ$$

Find : $m(\angle X)$



Solution

Given

R.T.F.

Proof

$$m(\angle M) = 50^\circ, m(\angle N) = 70^\circ, m(\angle Y) = 85^\circ, m(\angle L) = 130^\circ$$

$$m(\angle X)$$

In $\triangle ZMN$:

$$\therefore m(\angle M) = 50^\circ, m(\angle N) = 70^\circ$$

$$\therefore m(\angle NZM) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$$

$$\therefore m(\angle LZY) = m(\angle NZM) \quad (\text{V.O.A.})$$

$$\therefore m(\angle LZY) = 60^\circ$$

\therefore The figure XYZL is a quadrilateral.

$$\begin{aligned} \therefore \text{The sum of measures of its interior angles} &= (4 - 2) \times 180^\circ \\ &= 2 \times 180^\circ = 360^\circ \end{aligned}$$

$$\therefore m(\angle X) = 360^\circ - (130^\circ + 85^\circ + 60^\circ) = 85^\circ \quad (\text{The req.})$$

Example 4

If the ratio among the measures of the interior angles of a quadrilateral is $2 : 3 : 3 : 4$, find the smallest measure of the angles of that quadrilateral.

Solution

\therefore The ratio among the measures of the interior angles of a quadrilateral is $2 : 3 : 3 : 4$

\therefore The measures of the interior angles of this figure are $2x, 3x, 3x$ and $4x$

\therefore The sum of measures of the interior angles of the quadrilateral $= (4 - 2) \times 180^\circ = 2 \times 180^\circ = 360^\circ$

$$\therefore 2x + 3x + 3x + 4x = 360^\circ \quad \therefore 12x = 360^\circ \quad \therefore x = \frac{360^\circ}{12} = 30^\circ$$

\therefore The smallest measure $= 2x$

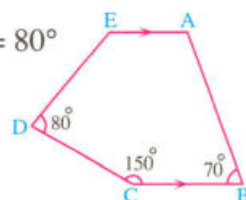
$$\therefore \text{The smallest measure of the angles} = 2 \times 30^\circ = 60^\circ \quad (\text{The req.})$$

TRY by yourself 2

In the opposite figure:

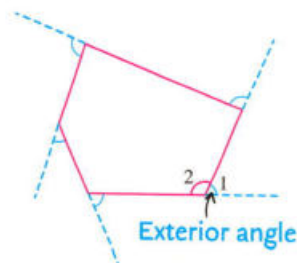
$\overline{AE} \parallel \overline{BC}$, $m(\angle B) = 70^\circ$, $m(\angle C) = 150^\circ$ and $m(\angle D) = 80^\circ$

Find: $m(\angle E)$



The sum of measures of the exterior angles of the convex polygon which has n sides

- We mentioned that the exterior angle of the polygon is the angle included between one side and the extension of its adjacent side and although we can draw two exterior angles equal in measure at each vertex of the polygon, but the rule of the sum of measures of the exterior angles use only one exterior angle at each vertex.



- At any vertex of the polygon, we find that the sum of measures of the interior angle and the exterior angle equals 180°

In the previous figure : $m(\angle 1) + m(\angle 2) = 180^\circ$ as an example.

In the previous pentagon, we find that the sum of measures of the five exterior and five interior angles of the pentagon equals $5 \times 180^\circ$

Since the sum of measures of the interior angles equals $3 \times 180^\circ$

\therefore The sum of measures of the five exterior angles of the pentagon $= 2 \times 180^\circ = 360^\circ$

We can deduce that for any convex polygon of n sides as follows :

The sum of measures of the exterior angles + the sum of measures of the interior angles $= n \times 180^\circ$

\therefore The sum of measures of the exterior angles $+ (n - 2) \times 180^\circ = n \times 180^\circ$

\therefore The sum of measures of the exterior angles $= n \times 180^\circ - (n - 2) \times 180^\circ$
 $= 180^\circ n - 180^\circ n + 360^\circ = 360^\circ$

So we get :

The sum of measures of the exterior angles of a convex polygon of n sides $= 360^\circ$
 (taking into account one exterior angle at each vertex)

The regular polygon The polygon is regular if :

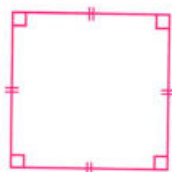
- All its sides are equal in length.
- All its angles are equal in measure.

As examples for the regular polygons :

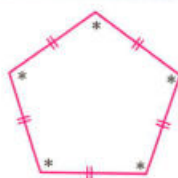
Equilateral triangle



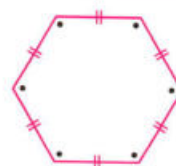
Square



Regular pentagon



Regular hexagon



The measure of the interior angle of a regular polygon

We knew that the sum of measures of the interior angles of a polygon of n sides $= (n - 2) \times 180^\circ$

Then :

If the polygon is regular , then its interior angles (whose number is n) are equal in measure.

$$\therefore \text{The measure of each interior angle of the regular polygon of } n \text{ sides} = \frac{(n-2) \times 180^\circ}{n}$$

For example:

- The measure of each interior angle of the equilateral triangle $= \frac{(3-2) \times 180^\circ}{3} = 60^\circ$
- The measure of each interior angle of the square $= \frac{(4-2) \times 180^\circ}{4} = 90^\circ$

Example 5

Complete the following table :

Number of sides of the regular polygon	5	8	12	6
Measure of one of its interior angles

Solution

Number of sides of the regular polygon	5	8	12	6
Measure of one of its interior angles	$\frac{3 \times 180^\circ}{5}$ $= 108^\circ$	$\frac{6 \times 180^\circ}{8}$ $= 135^\circ$	$\frac{10 \times 180^\circ}{12}$ $= 150^\circ$	$\frac{4 \times 180^\circ}{6}$ $= 120^\circ$

Example 6

The measure of one of the interior angles of a regular polygon is 144°

Find the number of its sides.

Solution

$$\therefore \text{The measure of each interior angle of the regular polygon of } n \text{ sides} = \frac{(n-2) \times 180^\circ}{n}$$

$$\therefore \frac{(n-2) \times 180^\circ}{n} = 144^\circ$$

$$\therefore 180^\circ n - 360^\circ = 144^\circ n$$

$$\therefore 36^\circ n = 360^\circ$$

$$\therefore \text{The number of sides} = 10 \text{ sides.}$$

$$\therefore (n-2) \times 180^\circ = 144^\circ n$$

$$\therefore 180^\circ n - 144^\circ n = 360^\circ$$

$$\therefore n = 10$$



Another solution

- \therefore The measure of the exterior angle of the polygon
 $= 180^\circ - \text{the measure of the interior angle} = 180^\circ - 144^\circ = 36^\circ$
- \therefore The sum of the measures of the exterior angles $= 360^\circ$
- \therefore The number of the exterior angles $= \frac{360^\circ}{36^\circ} = 10$ angles.
- \therefore The number of sides $= 10$ sides.

Notice that :

The number of the polygon sides = The number of its vertices
 = The number of its interior angles = The number of its exterior angles

! Remark

The number of sides of the regular polygon in which the measure of one of its interior angles is $X^\circ = \frac{360^\circ}{180^\circ - X}$

For example:

The number of sides of the regular polygon which the measure of one of its interior angles is $144^\circ = \frac{360^\circ}{180^\circ - 144^\circ} = 10$ sides.

TRY 3 by yourself

Complete the following table :

The number of sides of the regular polygon	3	10
The measure of one of its interior angles $^\circ$ $^\circ$	135°	160°

Lesson

3

The parallelogram and its properties



* In primary stage you have studied the parallelogram and its properties and in this lesson you will remember first what you studied before , then you will study when a quadrilateral will be a parallelogram.

Definition

Parallelogram is a quadrilateral , in which each two opposite sides are parallel.

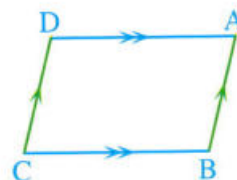
For example:

In the opposite figure :

If ABCD is a quadrilateral in which

$$\overline{AB} \parallel \overline{DC} \quad , \quad \overline{AD} \parallel \overline{BC}$$

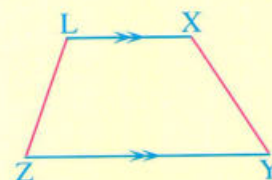
, then ABCD is a parallelogram.



Remark

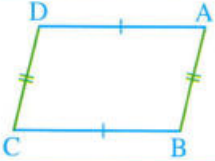
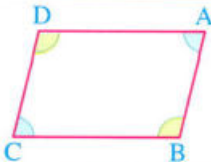
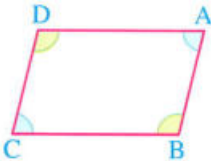
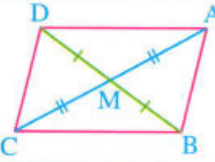
A quadrilateral in which only two sides are parallel is called a trapezium , as shown in the opposite figure in which :

$$\overline{XL} \parallel \overline{YZ}$$





Properties of parallelogram

1	Each two opposite sides are equal in length.		<ul style="list-style-type: none"> • $AB = DC$ • $AD = BC$
2	Each two opposite angles are equal in measure.		<ul style="list-style-type: none"> • $m(\angle A) = m(\angle C)$ • $m(\angle B) = m(\angle D)$
3	The sum of measures of each two consecutive angles is 180°		<ul style="list-style-type: none"> • $m(\angle A) + m(\angle B) = 180^\circ$ • $m(\angle B) + m(\angle C) = 180^\circ$ • $m(\angle C) + m(\angle D) = 180^\circ$ • $m(\angle D) + m(\angle A) = 180^\circ$
4	The two diagonals bisect each other.		<ul style="list-style-type: none"> • $AM = CM$ • $BM = DM$

The perimeter of the parallelogram = The sum of two consecutive sides $\times 2$

Example 1

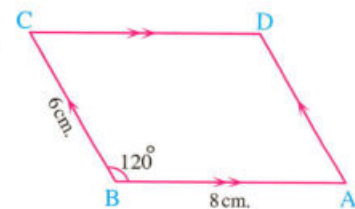
In the opposite figure :

ABCD is a parallelogram in which :

$AB = 8 \text{ cm.}$, $BC = 6 \text{ cm.}$ and $m(\angle B) = 120^\circ$

Find :

- 1 The length of each of \overline{CD} and \overline{DA}
- 2 The measure of each of $\angle D$, $\angle A$ and $\angle C$
- 3 The perimeter of ABCD



Solution

Given

R.T.F.

ABCD is a parallelogram , $AB = 8 \text{ cm.}$, $BC = 6 \text{ cm.}$ and $m(\angle B) = 120^\circ$

- 1 CD and DA
- 2 $m(\angle D)$, $m(\angle A)$ and $m(\angle C)$
- 3 The perimeter of ABCD

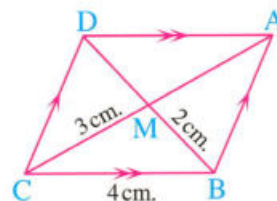
Proof

\therefore ABCD is a parallelogram
 $\therefore CD = AB = 8 \text{ cm.}$ (Properties of a parallelogram)
 and $DA = CB = 6 \text{ cm.}$ (Properties of a parallelogram) (First req.)
 $\therefore m(\angle D) = m(\angle B) = 120^\circ$ (Properties of a parallelogram)
 $\therefore m(\angle A) + m(\angle B) = 180^\circ$ (Properties of a parallelogram)
 $\therefore m(\angle B) = 120^\circ$
 $\therefore m(\angle A) = 180^\circ - 120^\circ = 60^\circ$ and $m(\angle C) = m(\angle A) = 60^\circ$ (Second req.)
 The perimeter of ABCD = $(AB + BC) \times 2$
 $= (8 + 6) \times 2 = 14 \times 2 = 28 \text{ cm.}$ (Third req.)

Example 2

In the opposite figure :

ABCD is a parallelogram whose diagonals intersect at M. If $BC = 4 \text{ cm.}$, $BM = 2 \text{ cm.}$ and $MC = 3 \text{ cm.}$, **then find :** the perimeter of $\triangle AMD$



Solution

Given

ABCD is a parallelogram whose diagonals intersect at M, $BC = 4 \text{ cm.}$, $BM = 2 \text{ cm.}$ and $MC = 3 \text{ cm.}$

R.T.F.

The perimeter of $\triangle AMD$

Proof

\therefore ABCD is a parallelogram
 $\therefore AD = BC = 4 \text{ cm.}$ (Two opposite sides in a parallelogram)
 \therefore The two diagonals bisect each other.
 $\therefore MD = MB = 2 \text{ cm.}$ and $AM = MC = 3 \text{ cm.}$
 \therefore The perimeter of $\triangle AMD = AD + MD + AM = 4 + 2 + 3 = 9 \text{ cm.}$

(The req.)

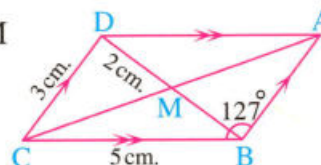
TRY by yourself 1

In the opposite figure :

ABCD is a parallelogram whose diagonals intersect at M
 If $BC = 5 \text{ cm.}$, $DC = 3 \text{ cm.}$, $DM = 2 \text{ cm.}$
 and $m(\angle ABC) = 127^\circ$

Complete the following :

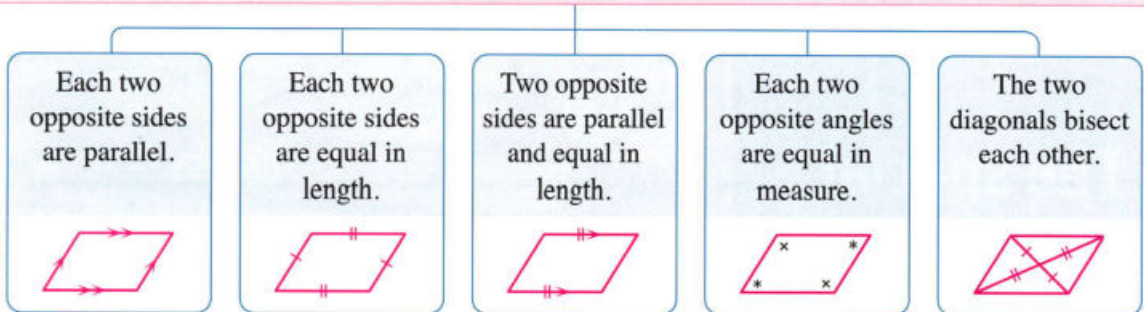
- 1 AB = cm. and AD = cm.
- 2 BD = cm.
- 3 $m(\angle ADC) = \dots\dots\dots^\circ$, $m(\angle BAD) = \dots\dots\dots^\circ$ and $m(\angle BCD) = \dots\dots\dots^\circ$
- 4 The perimeter of $\square ABCD = \dots\dots\dots \text{ cm.}$





When does a quadrilateral represent a parallelogram ?

A quadrilateral represents a parallelogram if one of the following conditions satisfies



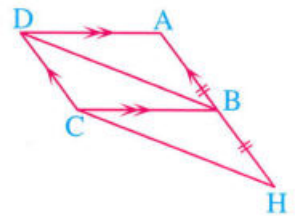
Example 3

In the opposite figure :

ABCD is a parallelogram , $H \in \overrightarrow{AB}$

where : $AB = BH$

Prove that : BHCD is a parallelogram.



Solution

Given

ABCD is a parallelogram and $AB = BH$

R.T.P.

BHCD is a parallelogram.

Proof

\therefore ABCD is a parallelogram

$\therefore AB = CD$

$\therefore AB = BH$ (Given)

$\therefore DC = BH$ (1)

$\therefore \overline{AB} \parallel \overline{DC}$, $H \in \overrightarrow{AB}$

$\therefore \overline{BH} \parallel \overline{DC}$ (2)

From (1) and (2) :

$\therefore DC = BH$ and $\overline{DC} \parallel \overline{BH}$

\therefore BHCD is a parallelogram.

(Q.E.D.)

TRY 2 by yourself

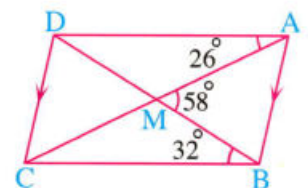
In the opposite figure :

ABCD is a quadrilateral , its diagonals intersect at M ,

$\overline{AB} \parallel \overline{CD}$, $m(\angle AMB) = 58^\circ$, $m(\angle MBC) = 32^\circ$

and $m(\angle MAD) = 26^\circ$

Prove that : ABCD is a parallelogram.



The special cases of the parallelogram



We studied in the previous lesson that the parallelogram is a quadrilateral in which each two opposite sides are parallel, we notice that this condition is verified also in each of **rectangle**, **rhombus** and **square**.

So, we said that each of rectangle, rhombus and square is a special case of the parallelogram with the same properties of it which stated in the previous lesson, as well as some another properties for each figure. In this lesson, we will represent each figure of them and its properties.

1 Rectangle

The rectangle is a parallelogram with a right angle.

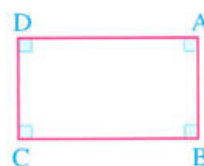


Properties of the rectangle

The rectangle has the same properties of the parallelogram and some additional properties as the following :

- The four angles of the rectangle are all equal in measure and the measure of each is 90°

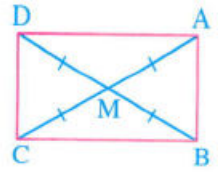
$$m(\angle A) = m(\angle B) = m(\angle C) = m(\angle D) = 90^\circ$$





- 2 The two diagonals of the rectangle are equal in length.

$AC = BD$ and as the two diagonals bisect each other
 , then $AM = BM = CM = DM$



The perimeter of the rectangle = (length + width) \times 2

2 Rhombus

The rhombus is a parallelogram in which two adjacent sides are equal in length.

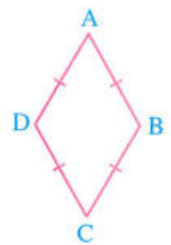


Properties of the rhombus

The rhombus has the same properties of the parallelogram and some additional properties as the following :

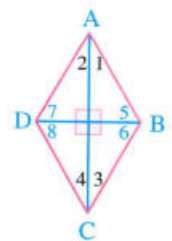
- 1 The four sides of the rhombus are all equal in length.

$$AB = BC = CD = DA$$



- 2 The two diagonals of the rhombus are perpendicular and bisect each of its interior angles.

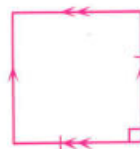
- $\overline{AC} \perp \overline{BD}$
- $m(\angle 1) = m(\angle 2) = m(\angle 3) = m(\angle 4)$
- $m(\angle 5) = m(\angle 6) = m(\angle 7) = m(\angle 8)$



The perimeter of the rhombus = the length of one side \times 4

3 Square

The square is a parallelogram with a right angle and two adjacent sides are equal in length.

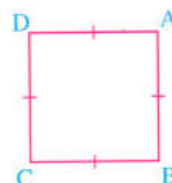


Properties of the square

The square has the same properties of the parallelogram and some additional properties as the following :

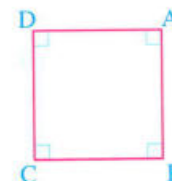
- 1 The four sides of the square are all equal in length.

$$AB = BC = CD = DA$$



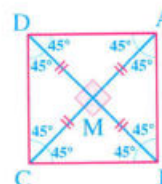
- 2 The four angles of the square are all equal in measure and each of them is of measure 90°

$$m(\angle A) = m(\angle B) = m(\angle C) = m(\angle D) = 90^\circ$$



- 3 The two diagonals of the square are equal in length , perpendicular and each diagonal bisects the two vertices angles which this diagonal joins into two angles each one of measure 45° .

- $AC = BD$ and hence $AM = BM = CM = DM$
- $\overline{AC} \perp \overline{BD}$



The perimeter of the square = the length of one side $\times 4$

! Remarks

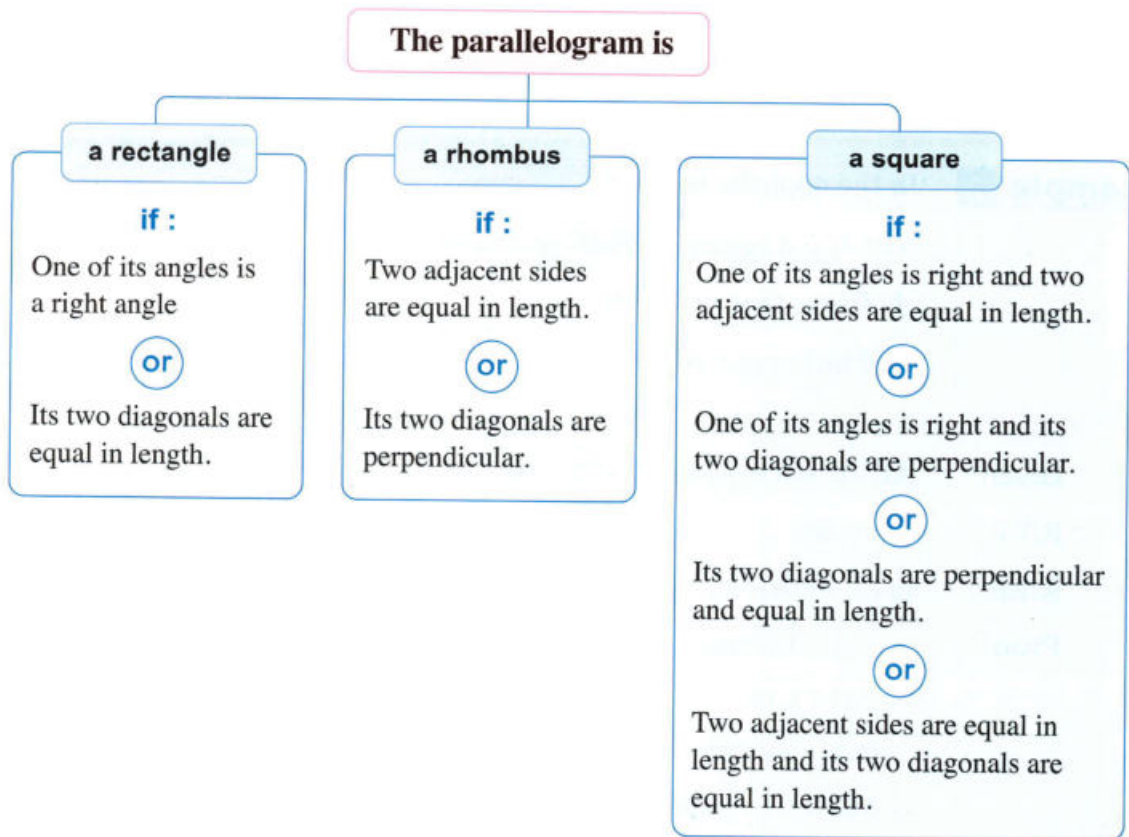
We can also define the square as follows :

- 1 A square is a rectangle with two adjacent sides equal in length.
- 2 A square is a rectangle with two perpendicular diagonals.
- 3 A square is a rhombus with a right angle.
- 4 A square is a rhombus with two diagonals equal in length.



Notice that :

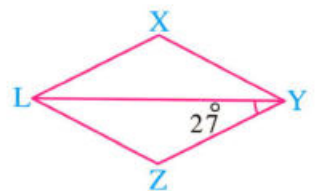
To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram , as we see in the previous lesson , then :



Example 1

In the opposite figure :

XYZL is a rhombus in which $m(\angle LYZ) = 27^\circ$
Calculate the measures of the angles of the rhombus XYZL



Solution

Given

R.T.F.

Proof

XYZL is a rhombus in which $m(\angle LYZ) = 27^\circ$

$m(\angle XYZ)$, $m(\angle XLZ)$, $m(\angle X)$ and $m(\angle Z)$

$\therefore \overline{YL}$ is a diagonal in the rhombus XYZL

$\therefore \overline{YL}$ bisects $\angle XYZ$

$$\therefore m(\angle XYZ) = 2 \times 27^\circ = 54^\circ$$

\therefore each two opposite angles in the rhombus are equal in measure.

$$\therefore m(\angle XLZ) = 54^\circ$$

\therefore The rhombus is a special case of the parallelogram

\therefore Each two consecutive angles are supplementary

$$\therefore m(\angle X) + m(\angle XYZ) = 180^\circ$$

$$\therefore m(\angle X) + 54^\circ = 180^\circ \quad \therefore m(\angle X) = 126^\circ$$

$$\therefore m(\angle Z) = 126^\circ$$

(The req.)

Try to solve this example by another method using the properties of the rhombus

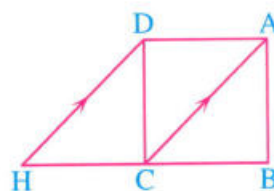
Example 2

In the opposite figure :

ABCD is a square, $H \in \overrightarrow{BC}$ such that $\overrightarrow{DH} \parallel \overrightarrow{AC}$

1 Prove that : $CH = BC$

2 Find : $m(\angle ADH)$



Solution

Given

ABCD is a square and $\overrightarrow{DH} \parallel \overrightarrow{AC}$

R.T.P.

$CH = BC$

R.T.F.

$m(\angle ADH)$

Proof

$\therefore \overrightarrow{AD} \parallel \overrightarrow{BC}$ (two opposite sides in the square) and $H \in \overrightarrow{BC}$

$\therefore \overrightarrow{AD} \parallel \overrightarrow{CH}$

$\therefore \overrightarrow{DH} \parallel \overrightarrow{AC}$ (given)

\therefore ACHD is a parallelogram.

$$\therefore CH = AD$$

But $AD = BC$ (two opposite sides in the square)

$$\therefore CH = BC$$

(First req.)

$\therefore \overrightarrow{AC}$ is a diagonal in the square.

$\therefore \overrightarrow{CA}$ bisects $\angle BCD$

$$\therefore m(\angle BCD) = 90^\circ$$

$$\therefore m(\angle ACD) = 45^\circ$$

$\therefore \overrightarrow{DH} \parallel \overrightarrow{AC}$ and \overrightarrow{CD} is their transversal.

$$\therefore m(\angle CDH) = m(\angle ACD) = 45^\circ \text{ (two alternate angles)}$$

$$\therefore m(\angle ADC) = 90^\circ \text{ (property of the square)}$$

$$\therefore m(\angle ADH) = m(\angle ADC) + m(\angle CDH)$$

$$= 90^\circ + 45^\circ = 135^\circ$$

(Second req.)

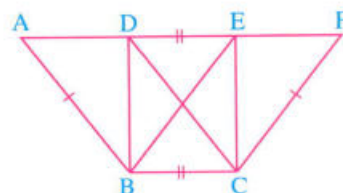
Example 3

In the opposite figure :

ABCD, EBCF are two parallelograms,

D and E belong to \overrightarrow{AF} , $AB = FC$, $BC = DE$

Prove that : The figure DBCE is a rectangle.





Solution

Given

ABCD and EBCF are two parallelograms, $AB = FC$, $BC = DE$

R.T.P.

The figure DBCE is a rectangle.

Proof

\therefore ABCD is a parallelogram.

$\therefore \overline{AD} \parallel \overline{BC}$

\therefore D and E belong to \overleftrightarrow{AF}

$\therefore \overline{DE} \parallel \overline{BC}$

$\therefore DE = BC$

\therefore DBCE is a parallelogram.

\therefore ABCD is a parallelogram.

$\therefore AB = DC$

\therefore EBCF is a parallelogram.

$\therefore FC = EB$ but $AB = FC$

$\therefore DC = EB$

\therefore DBCE is a parallelogram and its diagonals are equal in length.

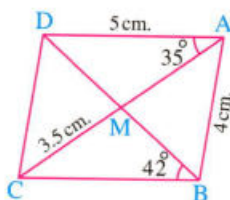
\therefore DBCE is a rectangle.

(Q.E.D.)

TRY
by yourself

Using the given in each figure, complete where M is the intersection point of the diagonals :

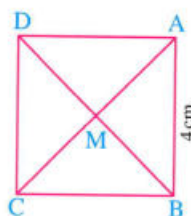
1



ABCD is a parallelogram :

- The perimeter of $\triangle ABC = \dots\dots\dots$ cm.
- $m(\angle AMB) = \dots\dots\dots^\circ$

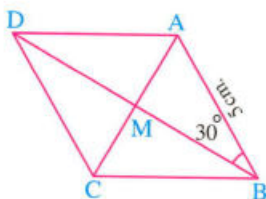
2



ABCD is a square :

- The perimeter of the square = $\dots\dots\dots$ cm.
- $m(\angle BAC) = \dots\dots\dots^\circ$

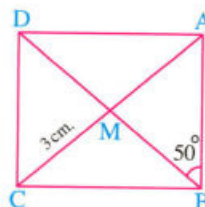
3



ABCD is a rhombus :

- $AD = \dots\dots\dots$ cm.
- $m(\angle BAM) = \dots\dots\dots^\circ$

4



ABCD is a rectangle :

- $BD = \dots\dots\dots$ cm.
- $m(\angle MCD) = \dots\dots\dots^\circ$

Lesson 5

The triangle



Theorem 1

The sum of the measures of the interior angles of a triangle is 180°

Given

ABC is a triangle

R.T.P.

$$m(\angle A) + m(\angle B) + m(\angle ACB) = 180^\circ$$

Construction

Draw $\overleftrightarrow{CX} \parallel \overleftrightarrow{AB}$

Proof

$\therefore \angle XCY$ is a straight angle

$$\therefore m(\angle XCA) + m(\angle ACB) + m(\angle YCB) = 180^\circ$$

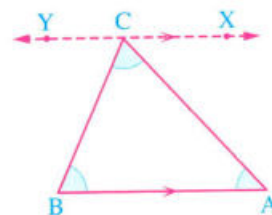
$$\therefore \overleftrightarrow{XY} \parallel \overleftrightarrow{AB}$$

$$\therefore m(\angle XCA) = m(\angle A) \quad (\text{alternate angles})$$

$$, m(\angle YCB) = m(\angle B) \quad (\text{alternate angles})$$

$$\therefore m(\angle A) + m(\angle ACB) + m(\angle B) = 180^\circ$$

(Q.E.D.)



Example 1

In the opposite figure :

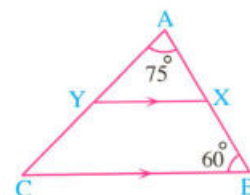
ABC is a triangle in which $m(\angle A) = 75^\circ$,

$m(\angle B) = 60^\circ$,

$X \in \overleftrightarrow{AB}$ and $Y \in \overleftrightarrow{AC}$

Such that : $\overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$

Find : $m(\angle AYX)$





Solution

Given

$$\overline{XY} \parallel \overline{BC}, m(\angle A) = 75^\circ \text{ and } m(\angle B) = 60^\circ$$

R.T.F.

$$m(\angle AYX)$$

Proof

$$\therefore m(\angle A) = 75^\circ \text{ and } m(\angle B) = 60^\circ \text{ (given)}$$

, the sum of measures of the interior angles

$$\text{of } \triangle ABC = 180^\circ$$

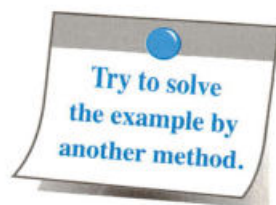
$$\therefore m(\angle C) = 180^\circ - (75^\circ + 60^\circ)$$

$$= 180^\circ - 135^\circ = 45^\circ$$

$\therefore \overline{XY} \parallel \overline{BC}$ and \overleftrightarrow{AC} is a transversal.

$$\therefore m(\angle AYX) = m(\angle C) = 45^\circ \text{ (corresponding angles)}$$

(The req.)



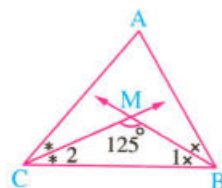
Example 2

In the opposite figure :

\overrightarrow{BM} bisects $\angle ABC$, \overrightarrow{CM} bisects $\angle ACB$

$$\text{and } m(\angle BMC) = 125^\circ$$

Find : $m(\angle A)$



Solution

Given

\overrightarrow{BM} bisects $\angle ABC$, \overrightarrow{CM} bisects $\angle ACB$ and $m(\angle BMC) = 125^\circ$

R.T.F.

$$m(\angle A)$$

Proof

\therefore The sum of measures of the interior angles of $\triangle MBC = 180^\circ$

$$\text{and } m(\angle BMC) = 125^\circ$$

$$\therefore m(\angle 1) + m(\angle 2) = 180^\circ - 125^\circ = 55^\circ$$

$$\text{But } m(\angle ABC) = 2m(\angle 1) \text{ and } m(\angle ACB) = 2m(\angle 2)$$

$$\therefore m(\angle ABC) + m(\angle ACB) = 2 \times 55^\circ = 110^\circ$$

\therefore The sum of measures of the interior angles of $\triangle ABC = 180^\circ$

$$\therefore m(\angle A) = 180^\circ - 110^\circ = 70^\circ$$

(The req.)

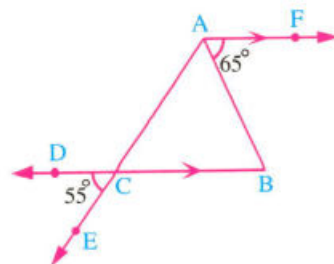
TRY
by yourself **1**

In the opposite figure :

$$\overrightarrow{BD} \cap \overrightarrow{AE} = \{C\}, \overrightarrow{AF} \parallel \overrightarrow{BC}$$

$$, m(\angle BAF) = 65^\circ \text{ and } m(\angle DCE) = 55^\circ$$

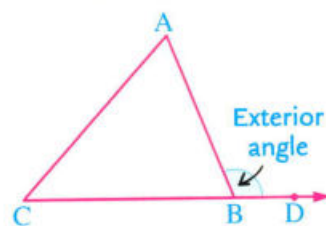
Find : The measures of the interior angles of $\triangle ABC$



The exterior angle of the triangle

In the opposite figure :

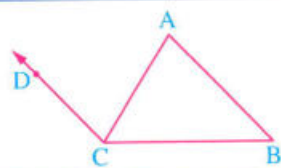
If ABC is a triangle, $D \in \overrightarrow{CB}$ and $D \notin \overline{CB}$, then $\angle ABD$ is called an exterior angle of $\triangle ABC$



Notice that :

In the opposite figure :

$\angle ACD$ is not an exterior angle of $\triangle ABC$ because $D \notin \overrightarrow{BC}$



The measure of an exterior angle of a triangle

The measure of an exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.

In the opposite figure :

If ABC is a triangle, $D \in \overrightarrow{CB}$ and $D \notin \overline{CB}$,

$$\text{then } m(\angle ABD) = m(\angle A) + m(\angle C)$$

We can prove that as follows :

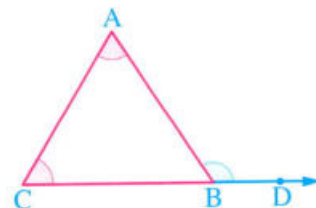
$$\therefore m(\angle A) + m(\angle C) + m(\angle ABC) = 180^\circ$$

$$, m(\angle ABD) + m(\angle ABC) = 180^\circ$$

$$\therefore m(\angle ABD) + m(\angle ABC) = m(\angle A) + m(\angle C) + m(\angle ABC)$$

$$\therefore m(\angle ABD) = m(\angle A) + m(\angle C)$$

(Q.E.D.)



Notice that :

The measure of the exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

i.e. In the previous figure : $m(\angle ABD) > m(\angle A)$ and $m(\angle ABD) > m(\angle C)$



Example 3

In the opposite figure :

ABC is a triangle , $D \in \overrightarrow{BC}$ and $E \in \overrightarrow{AC}$

where \overrightarrow{BE} bisects $\angle ABC$,

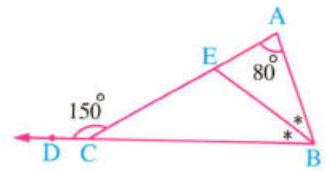
$m(\angle A) = 80^\circ$ and

$m(\angle ACD) = 150^\circ$

Find :

1 $m(\angle ABC)$

2 $m(\angle BEC)$



Solution

Given

\overrightarrow{BE} bisects $\angle ABC$, $m(\angle A) = 80^\circ$ and $m(\angle ACD) = 150^\circ$

R.T.F.

1 $m(\angle ABC)$

2 $m(\angle BEC)$

Proof

$\therefore \angle ACD$ is an exterior angle of $\triangle ABC$

$\therefore m(\angle ACD) = m(\angle A) + m(\angle ABC)$

$\therefore 150^\circ = 80^\circ + m(\angle ABC)$

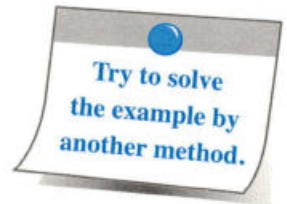
$\therefore m(\angle ABC) = 150^\circ - 80^\circ = 70^\circ$ (First req.)

$\therefore \overrightarrow{BE}$ bisects $\angle ABC$ (given)

$\therefore m(\angle ABE) = \frac{1}{2} m(\angle ABC) = \frac{70^\circ}{2} = 35^\circ$

$\therefore \angle BEC$ is an exterior angle of $\triangle ABE$

$\therefore m(\angle BEC) = m(\angle A) + m(\angle ABE) = 80^\circ + 35^\circ = 115^\circ$ (Second req.)



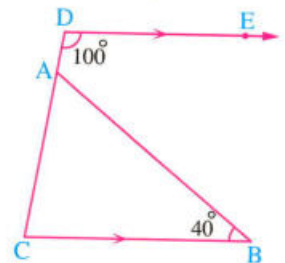
TRY 2 by yourself

In the opposite figure :

$A \in \overrightarrow{DC}$, $\overrightarrow{DE} \parallel \overrightarrow{CB}$,

$m(\angle D) = 100^\circ$ and $m(\angle B) = 40^\circ$

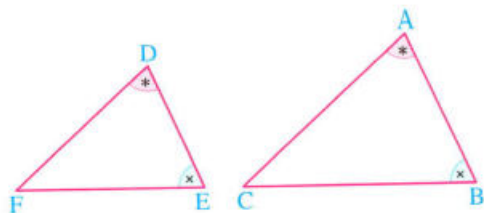
Find : $m(\angle BAD)$



Remark 1

If two angles of one triangle equal two angles of another triangle in measure, then the third angle of the first triangle is equal in measure to the third angle of the other triangle.

In $\triangle ABC$ and DEF ,
if $m(\angle A) = m(\angle D)$ and $m(\angle B) = m(\angle E)$,
then $m(\angle C) = m(\angle F)$

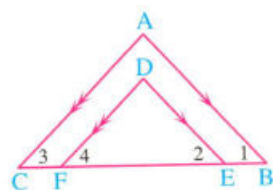


Example 4

In the opposite figure :

ABC and DEF are two triangles, $E \in \overline{BC}$,
 $F \in \overline{BC}$, $\overline{DE} \parallel \overline{AB}$ and $\overline{DF} \parallel \overline{AC}$

Prove that : $m(\angle A) = m(\angle D)$



Solution

Given

$\overline{DE} \parallel \overline{AB}$ and $\overline{DF} \parallel \overline{AC}$

R.T.P.

$m(\angle A) = m(\angle D)$

Proof

$\therefore \overline{DE} \parallel \overline{AB}$ and \overline{BC} is a transversal to them.

$\therefore m(\angle 1) = m(\angle 2)$ (corresponding angles)

$\therefore \overline{DF} \parallel \overline{AC}$ and \overline{BC} is a transversal to them.

$\therefore m(\angle 3) = m(\angle 4)$ (corresponding angles)

In $\triangle ABC$ and DEF :

$\therefore m(\angle 1) = m(\angle 2)$ and $m(\angle 3) = m(\angle 4)$

$\therefore m(\angle A) = m(\angle D)$

(Q.E.D.)

Remark 2

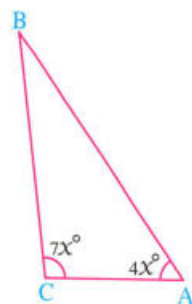
- If the sum of measures of two angles in a triangle equals 90° , then the third angle is **right**.
- If the sum of measures of two angles in a triangle is **less than 90°** , then the third angle is **obtuse**.
- If the sum of measures of two angles in a triangle is **more than 90°** , then the third angle is **acute**.

**Example 5**

In the opposite figure :

ABC is a triangle in which $m(\angle A) = 2m(\angle B) = 4x^\circ$
and $m(\angle C) = 7x^\circ$

Prove that : $\angle C$ is an obtuse angle.

**Solution**

Given

R.T.P.

Proof

$$m(\angle A) = 2m(\angle B) = 4x^\circ \text{ and } m(\angle C) = 7x^\circ$$

$\angle C$ is an obtuse angle.

$$\therefore 2m(\angle B) = 4x^\circ$$

$$\therefore m(\angle B) = 2x^\circ$$

$$\therefore m(\angle A) + m(\angle B) = 4x^\circ + 2x^\circ = 6x^\circ$$

$$\therefore m(\angle C) = 7x^\circ$$

$$\therefore m(\angle A) + m(\angle B) < m(\angle C)$$

$\therefore \angle C$ is an obtuse angle.

(Q.E.D.)

! Remark 3

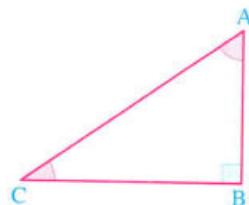
If the measure of an angle in a triangle equals the sum of measures of the other two angles, then the triangle is right-angled.

In the opposite figure :

If ABC is a triangle in which : $m(\angle A) + m(\angle C) = m(\angle B)$

$$\text{, then } m(\angle B) = \frac{180^\circ}{2} = 90^\circ$$

i.e. ΔABC is right-angled at B

**Example 6**

ABC is a triangle in which $m(\angle A) : m(\angle B) : m(\angle C) = 2 : 3 : 5$

Prove that the triangle is right-angled, then mention the right angle.

Solution

Given

R.T.P.

Proof

ABC is a triangle in which $m(\angle A) : m(\angle B) : m(\angle C) = 2 : 3 : 5$

ΔABC is right-angled and mention the right angle.

$\therefore m(\angle A) + m(\angle B)$ is equivalent to 5 parts and $m(\angle C)$
is equivalent to 5 parts

$$\therefore m(\angle A) + m(\angle B) = m(\angle C)$$

\therefore The sum of measures of the interior angles of a triangle = 180°

$$\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$$

$$\therefore m(\angle A) + m(\angle B) = m(\angle C) = 90^\circ$$

$\therefore \Delta ABC$ is right-angled at C

(Q.E.D.)

Lesson 6

Theorem 2 and its corollary, and theorem 3



Theorem 2

The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.

Given

D is the midpoint of \overline{AB} , $\overrightarrow{DE} \parallel \overline{BC}$

R.T.P.

E is the midpoint of \overline{AC}

Construction

Draw $\overrightarrow{AX} \parallel \overline{BC}$

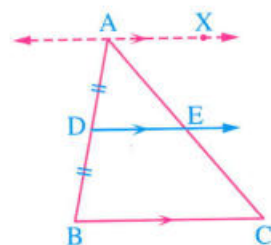
Proof

$\therefore \overrightarrow{AX} \parallel \overrightarrow{DE} \parallel \overline{BC}$

, \overline{AB} and \overline{AC} are two transversals to them at D and E respectively.

$\therefore AD = DB$ $\therefore AE = EC$

$\therefore E$ is the midpoint of \overline{AC}



(Q.E.D.)

Example 1

In the opposite figure :

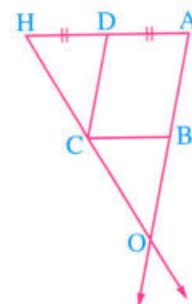
ABCD is a parallelogram ,

$H \in \overline{AD}$ such that : $AD = DH$, $\overline{HC} \cap \overline{AB} = \{O\}$

Prove that :

1 $HC = CO$

2 $AB = BO$





Solution

Given

ABCD is a parallelogram, $AD = DH$ and $\overleftrightarrow{HC} \cap \overleftrightarrow{AB} = \{O\}$

R.T.P.

1 $HC = CO$

2 $AB = BO$

Proof

In $\triangle HAO$:

$\therefore D$ is the midpoint of \overline{HA} (given) ,

$\overline{DC} \parallel \overline{AO}$ (definition of the parallelogram)

$\therefore C$ is the midpoint of \overline{HO}

i.e. $HC = CO$ (theorem)

(Q.E.D. 1)

$\therefore C$ is the midpoint of \overline{OH} (proved)

, $\overline{CB} \parallel \overline{HA}$ (definition of the parallelogram)

$\therefore B$ is the midpoint of \overline{AO}

i.e. $AB = BO$ (theorem)

(Q.E.D. 2)

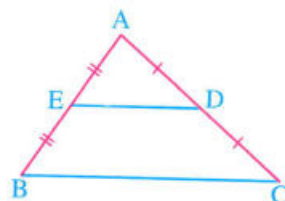
Corollary The line segment joining the midpoints of two sides of a triangle is parallel to the third side.

In the opposite figure :

If ABC is a triangle in which D

is the midpoint of \overline{AC} ,

E is the midpoint of \overline{AB} , then : $\overline{ED} \parallel \overline{BC}$



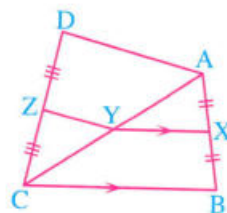
Example 2

In the opposite figure :

X is the midpoint of \overline{AB} ,

$\overline{XY} \parallel \overline{BC}$ and Z is the midpoint of \overline{DC}

Prove that : $\overline{YZ} \parallel \overline{AD}$



Solution

Given

X is the midpoint of \overline{AB} , Z is the midpoint of \overline{CD} and $\overline{XY} \parallel \overline{BC}$

R.T.P.

$\overline{YZ} \parallel \overline{AD}$

Proof

In $\triangle ABC$:

$\therefore AX = XB$, $\overline{XY} \parallel \overline{BC}$

$\therefore AY = YC$ (theorem)

In $\triangle ACD$: $\therefore AY = YC$ (proved)

, $DZ = ZC$ (given)

$\therefore \overline{YZ} \parallel \overline{AD}$ (corollary)

(Q.E.D.)

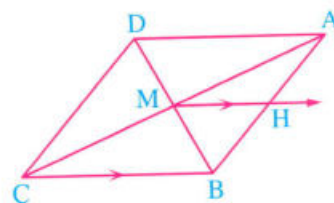
TRY 1
by yourself

In the opposite figure :

ABCD is a parallelogram and M is the point of intersection of its two diagonals.

Draw $\overrightarrow{MH} \parallel \overrightarrow{BC}$ to cut \overline{AB} at H

Prove that : $AH = HB$



Theorem 3

The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.

Given

ABC is a triangle , D is the midpoint of \overline{AB} , H is the midpoint of \overline{AC}

R.T.P.

$$DH = \frac{1}{2} BC$$

Construction

Draw $\overrightarrow{HO} \parallel \overline{AB}$ to cut \overline{BC} at O

Proof

\therefore D is the midpoint of \overline{AB} , H is the midpoint of \overline{AC}

$\therefore \overline{DH} \parallel \overline{BC}$ (corollary)

$\therefore \overline{HO} \parallel \overline{AB}$ (construction) , H is the midpoint of \overline{AC}

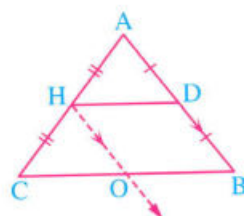
\therefore O is the midpoint of \overline{BC}

$$\therefore BO = \frac{1}{2} BC$$

\therefore the figure DHOB is a parallelogram.

$$\therefore DH = BO = \frac{1}{2} BC$$

(Q.E.D.)



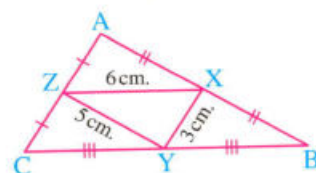
Example 3

In the opposite figure :

ABC is a triangle in which X , Y and Z are the midpoints of \overline{AB} , \overline{BC} and \overline{CA} respectively.

If $XY = 3$ cm. , $YZ = 5$ cm. and $ZX = 6$ cm. ,

then find : the perimeter of $\triangle ABC$



Solution

Given

ABC is a triangle in which X , Y and Z are the midpoints of \overline{AB} , \overline{BC} and \overline{CA} respectively , $XY = 3$ cm. , $YZ = 5$ cm. and $ZX = 6$ cm.

R.T.F.

The perimeter of $\triangle ABC$

Proof

In $\triangle ABC$:

\therefore X is the midpoint of \overline{AB} and Z is the midpoint of \overline{AC}

$$\therefore XZ = \frac{1}{2} BC \text{ (theorem)}$$

$$\therefore BC = 6 \times 2 = 12 \text{ cm.}$$



Similarly : \because X is the midpoint of \overline{AB} and Y is the midpoint of \overline{BC}

$$\therefore XY = \frac{1}{2} AC$$

$$\therefore AC = 3 \times 2 = 6 \text{ cm.}$$

Similarly : \because Y is the midpoint of \overline{BC} and Z is the midpoint of \overline{AC}

$$\therefore YZ = \frac{1}{2} AB$$

$$\therefore AB = 5 \times 2 = 10 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle ABC = AB + BC + CA = 10 + 12 + 6 = 28 \text{ cm.}$$

(The req.)

Example 4

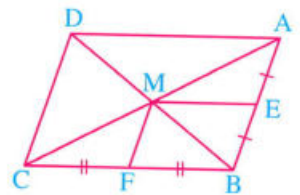
In the opposite figure :

ABCD is a parallelogram in which :

$\overline{AC} \cap \overline{BD} = \{M\}$, E is the midpoint of \overline{AB}

, F is the midpoint of \overline{BC}

Prove that : The figure EBFM is a parallelogram.



Solution

Given

ABCD is a parallelogram , E is the midpoint of \overline{AB}

, F is the midpoint of \overline{BC}

R.T.P.

The figure EBFM is a parallelogram.

Proof

\because ABCD is a parallelogram whose diagonals intersect at M

\therefore M is the midpoint of each of \overline{AC} and \overline{BD}

\therefore In $\triangle ABC$:

\because E is the midpoint of \overline{AB} , M is the midpoint of \overline{AC}

$$\therefore \overline{EM} \parallel \overline{BC}$$

$$\therefore \overline{EM} \parallel \overline{BF}$$

$$\therefore EM = \frac{1}{2} BC \text{ (theorem)}$$

$$\therefore EM = BF$$

\therefore The figure EBFM is a parallelogram.

(Q.E.D.)

TRY 2

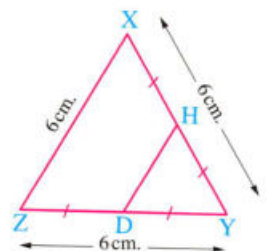
by yourself

In the opposite figure :

XYZ is an equilateral triangle whose side is of length 6 cm. ,

D is the midpoint of \overline{YZ} and H is the midpoint of \overline{XY}

Prove that : $\triangle HYD$ is an equilateral triangle and find its perimeter.



Pythagoras' theorem



In the opposite figure :

- If ABC is a right-angled triangle at A in which :

AB = 4 length units , **AC** = 3 length units

, **BC** = 5 length units , then :

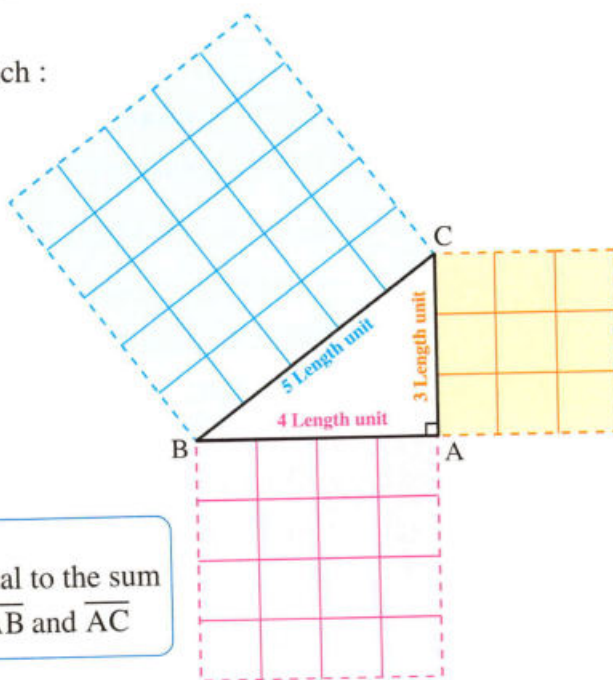
- The area of the square drawn on \overline{AB}
= $(AB)^2 = 16$ square unit.
- The area of the square drawn on \overline{AC}
= $(AC)^2 = 9$ square unit.
- The area of the square drawn on \overline{BC}
= $(BC)^2 = 25$ square unit.

i.e.

The area of the square drawn on \overline{BC} is equal to the sum of the areas of the two squares drawn on \overline{AB} and \overline{AC}

In other words

$$(BC)^2 = (AB)^2 + (AC)^2$$



The verbal formula of this relation is defined by Pythagoras' theorem.

Pythagoras' theorem

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.





We can also write the previous theorem as follows :

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

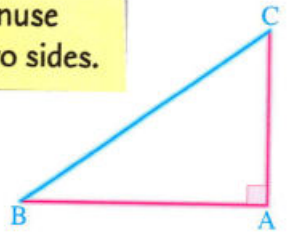
i.e. If ABC is a right-angled triangle at A, then :

$$(BC)^2 = (AB)^2 + (AC)^2$$

• From the previous relation, we can deduce the following two relations :

$$(AB)^2 = (BC)^2 - (AC)^2$$

$$(AC)^2 = (BC)^2 - (AB)^2$$



Example 1 In each of the following figures :

Find the side length which is denoted by sign (?) :

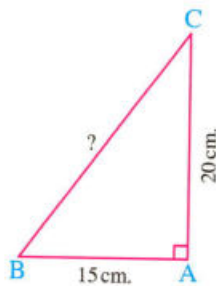


Fig. (1)

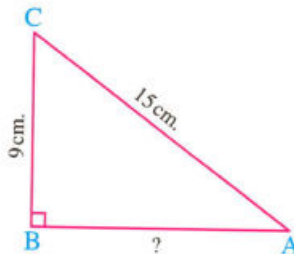


Fig. (2)

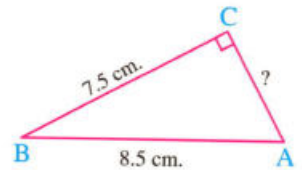


Fig. (3)

Solution

In fig. (1) :

$\because \Delta ABC$ is right-angled at A

$$\therefore (BC)^2 = (AB)^2 + (AC)^2 = (15)^2 + (20)^2 = 225 + 400 = 625$$

$$\therefore BC = \sqrt{625} = 25 \text{ cm.}$$

In fig. (2) :

$\because \Delta ABC$ is right-angled at B

$$\therefore (AB)^2 = (AC)^2 - (BC)^2 = (15)^2 - (9)^2 = 225 - 81 = 144$$

$$\therefore AB = \sqrt{144} = 12 \text{ cm.}$$

In fig. (3) :

$\because \Delta ABC$ is right-angled at C

$$\therefore (AC)^2 = (AB)^2 - (BC)^2 = (8.5)^2 - (7.5)^2 = 72.25 - 56.25 = 16$$

$$\therefore AC = \sqrt{16} = 4 \text{ cm.}$$

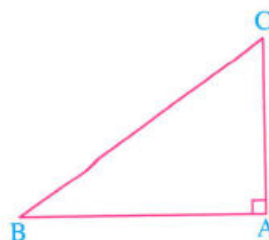
TRY by yourself 1

In the opposite figure :

ΔABC is right-angled at A

Complete the following table :

AB	8 cm.	12 cm.	12 cm.	20 cm.
AC	6 cm.	9 cm.	12 cm.	4.5 cm.
BC	13 cm.	25 cm.	20 cm.	7.5 cm.



Example 2

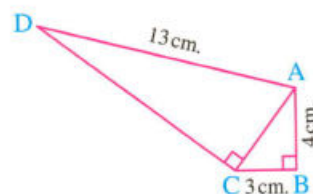
In the opposite figure :

ABCD is a quadrilateral in which :

$m(\angle B) = m(\angle ACD) = 90^\circ$

, $AB = 4$ cm. , $BC = 3$ cm. and $AD = 13$ cm.

Find : The length of each of \overline{AC} and \overline{CD}



Solution

Given

$m(\angle B) = m(\angle ACD) = 90^\circ$, $AB = 4$ cm. , $BC = 3$ cm. and $AD = 13$ cm.

R.T.F.

The length of each of \overline{AC} and \overline{CD}

Proof

$\therefore \Delta ABC$ is a right-angled triangle at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = (4)^2 + (3)^2 = 16 + 9 = 25$

$\therefore AC = \sqrt{25} = 5$ cm.

(First req.)

$\therefore \Delta ACD$ is a right-angled triangle at C

$\therefore (CD)^2 = (AD)^2 - (AC)^2$ (Pythagoras' theorem)

$\therefore (CD)^2 = (13)^2 - (5)^2 = 169 - 25 = 144$

$\therefore CD = \sqrt{144} = 12$ cm.

(Second req.)



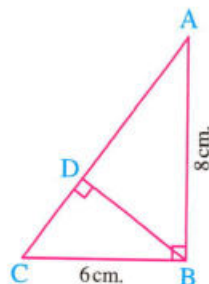
Example 3

In the opposite figure :

$\triangle ABC$ is a right-angled triangle at B , $D \in \overline{AC}$

such that $\overline{BD} \perp \overline{AC}$, $AB = 8$ cm. and $CB = 6$ cm.

Find : The length of \overline{BD}



Solution

Given

$\triangle ABC$ is right-angled at B , $\overline{BD} \perp \overline{AC}$, $AB = 8$ cm. and $CB = 6$ cm.

R.T.F.

The length of \overline{BD}

Proof

$\therefore \triangle ABC$ is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = 64 + 36 = 100 \qquad \therefore AC = \sqrt{100} = 10$ cm.

, \therefore the area of $\triangle ABC = \frac{1}{2} BC \times AB = \frac{1}{2} \times 6 \times 8 = 24$ cm²

, \therefore the area of $\triangle ABC = \frac{1}{2} AC \times BD$

$\therefore 24 = \frac{1}{2} \times 10 \times BD \qquad \therefore 24 = 5 BD$

$\therefore BD = \frac{24}{5} = 4.8$ cm.

(The req.)

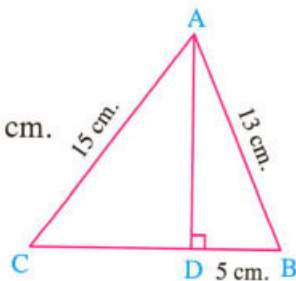
TRY 2 by yourself

In the opposite figure :

$\triangle ABC$ is a triangle in which $AB = 13$ cm.

, $AC = 15$ cm., $D \in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$ and $BD = 5$ cm.

Find : The length of \overline{DC}



Enrichment information (for reading only)

You can get three numbers representing the lengths of sides of a right-angled triangle as follows :

- 1 If M is an even number bigger than 2, then the numbers $M, \left(\frac{M}{2}\right)^2 - 1, \left(\frac{M}{2}\right)^2 + 1$ represent three lengths of sides of a right-angled triangle as shown in the following table :

M	$\left(\frac{M}{2}\right)^2 - 1$	$\left(\frac{M}{2}\right)^2 + 1$	Side lengths of the right-angled triangle
4	$\frac{16}{4} - 1 = 3$	$\frac{16}{4} + 1 = 5$	4, 3, 5
6	$\frac{36}{4} - 1 = 8$	$\frac{36}{4} + 1 = 10$	6, 8, 10
8	$\frac{64}{4} - 1 = 15$	$\frac{64}{4} + 1 = 17$	8, 15, 17
10	$\frac{100}{4} - 1 = 24$	$\frac{100}{4} + 1 = 26$	10, 24, 26

- 2 If M is an odd number bigger than 2, then the numbers $M, \frac{M^2 - 1}{2}, \frac{M^2 + 1}{2}$ represent three lengths of sides of a right-angled triangle as shown in the following table :

M	$\frac{M^2 - 1}{2}$	$\frac{M^2 + 1}{2}$	Side length of the right angled triangle
3	$\frac{9 - 1}{2} = 4$	$\frac{9 + 1}{2} = 5$	3, 4, 5
5	$\frac{25 - 1}{2} = 12$	$\frac{25 + 1}{2} = 13$	5, 12, 13
7	$\frac{49 - 1}{2} = 24$	$\frac{49 + 1}{2} = 25$	7, 24, 25
9	$\frac{81 - 1}{2} = 40$	$\frac{81 + 1}{2} = 41$	9, 40, 41

Lesson

8

Geometric transformations

In this lesson, you will recognize the meaning of the geometric transformation, also recognize quickly three types of them :

1 Reflection.

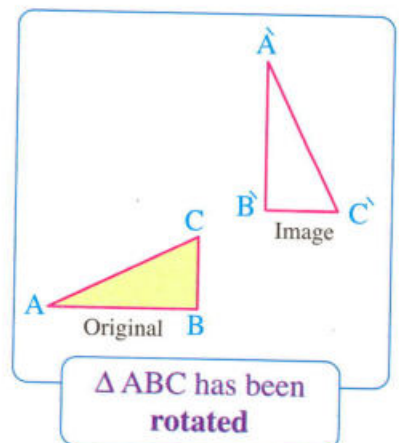
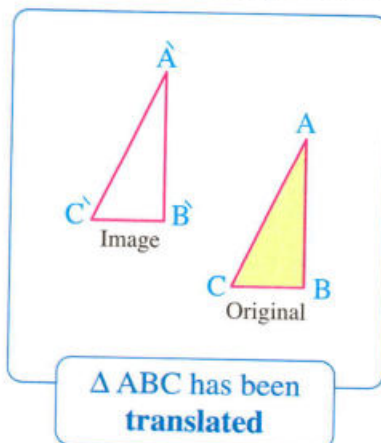
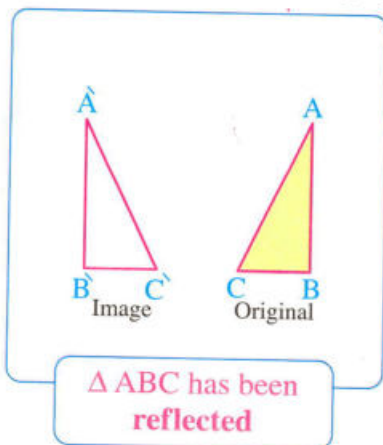
2 Translation.

3 Rotation.

And you will study each of them in details next lessons.

The concept of the geometric transformation

In each of the following figures , notice the image of ΔABC :



In each of the previous figures , notice that :

- The point A has been transferred to \hat{A}
- The point B has been transferred to \hat{B}

- The point C has been transferred to \hat{C}

Thus, all the points of ΔABC have been transferred to another position, then we say that ΔABC has been transformed from a position to another position.

From the previous, we deduce that

If all the points of a geometrical figure have moved according to a certain system, then we will obtain an image in a new position to the same figure, then we say that this figure has been under the effect of a geometric transformation.

i.e. The geometric transformation maps each point P in the plane onto an image point \hat{P} in the same plane.

Example

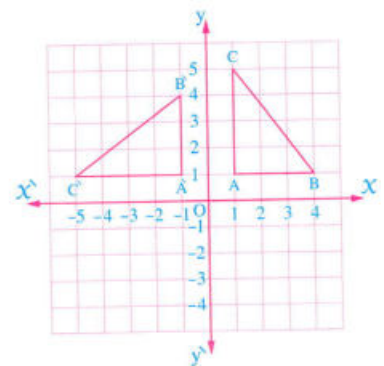
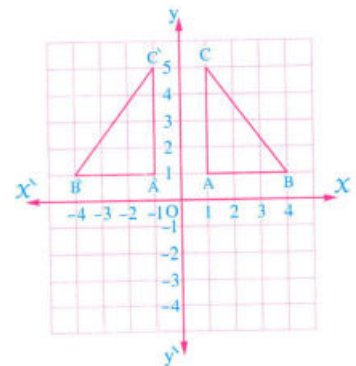
Draw the image of ΔABC where A (1, 1), B (4, 1), C (1, 5) according to each of the following transformations, then describe its type :

- 1 $(X, y) \longrightarrow (-X, y)$
- 2 $(X, y) \longrightarrow (-y, X)$
- 3 $(X, y) \longrightarrow (X, y - 5)$

Solution

- 1 $A(1, 1) \xrightarrow{(-X, y)} \hat{A}(-1, 1)$
 $B(4, 1) \xrightarrow{(-X, y)} \hat{B}(-4, 1)$
 $C(1, 5) \xrightarrow{(-X, y)} \hat{C}(-1, 5)$
 The transformation is reflection.

- 2 $A(1, 1) \xrightarrow{(-y, X)} \hat{A}(-1, 1)$
 $B(4, 1) \xrightarrow{(-y, X)} \hat{B}(-1, 4)$
 $C(1, 5) \xrightarrow{(-y, X)} \hat{C}(-5, 1)$
 The transformation is rotation.



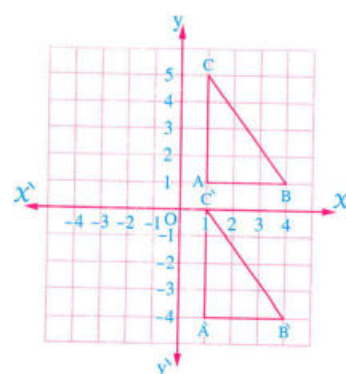


$$3 \quad A(1, 1) \xrightarrow{(X, y-5)} \hat{A}(1, -4)$$

$$B(4, 1) \xrightarrow{(X, y-5)} \hat{B}(4, -4)$$

$$C(1, 5) \xrightarrow{(X, y-5)} \hat{C}(1, 0)$$

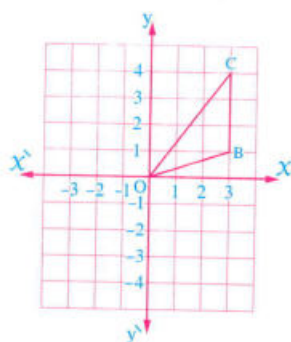
The transformation is translation.



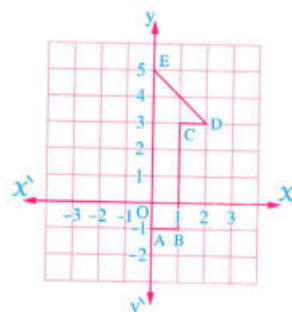
TRY
by yourself

Map each of the following shapes due to the geometric transformation above it, then describe its type :

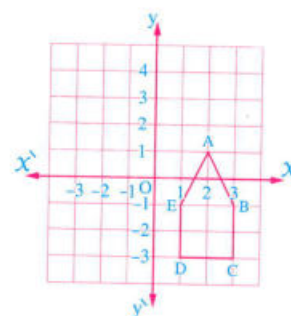
$$1 \quad (X, y) \longrightarrow (-X, -y)$$



$$2 \quad (X, y) \longrightarrow (-X, y)$$



$$3 \quad (X, y) \longrightarrow (X, y+2)$$



Lesson 9

Reflection in a straight line



Prelude

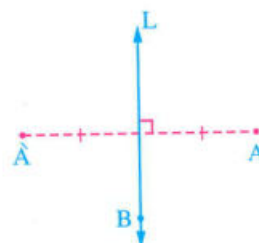
If you stand in front of a plane mirror, then you will see your picture (image) reflected in the mirror in the same size and details and you will notice also that the distance between the image and the mirror equals the real distance between you and the mirror. If you approach the mirror, then you will find that your image approaches also the mirror.



Definition of reflection in a straight line

Reflection in the straight line L maps each point A to the point \hat{A} in the same plane such that :

- 1 If $A \notin L$, then the straight line L is the perpendicular bisector to the line segment $\overline{A\hat{A}}$
- 2 If $B \in L$, then B is reflected onto itself
i.e. \hat{B} coincides B





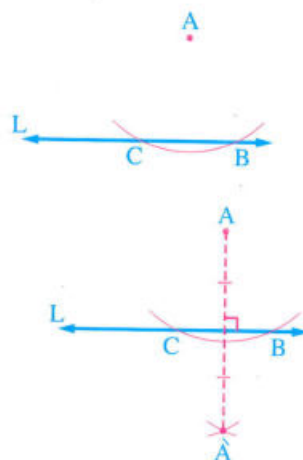
Finding the image of a point by reflection in a given straight line

- To find \hat{A} which is the image of A by reflection in the straight line L, we do as follows :

- 1 Draw an arc of a circle with centre A to cut L at B and C
- 2 With the same radius length taking B and C as centres, draw two arcs in the other side of the straight line L to intersect at \hat{A} , then \hat{A} is the image of A by reflection in L

Check by measuring that :

$\overline{AA} \perp L$ and L bisects \overline{AA}



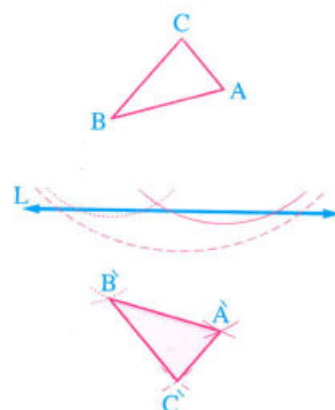
Finding the image of a polygon by reflection in a given straight line

- To find the image of a polygon as $\triangle ABC$ by reflection in the straight line L, we do as follows :

- 1 Find the image of each vertex of $\triangle ABC$ by reflection in the straight line L as we did before (\hat{A} is the image of A, \hat{B} is the image of B and \hat{C} is the image of C)
- 2 Draw $\overline{\hat{A}\hat{B}}$, $\overline{\hat{B}\hat{C}}$ and $\overline{\hat{C}\hat{A}}$, then $\triangle \hat{A}\hat{B}\hat{C}$ is the image of $\triangle ABC$ by reflection in the straight line L

Check by measuring that :

- $AB = \hat{A}\hat{B}$, $BC = \hat{B}\hat{C}$ and $CA = \hat{C}\hat{A}$
- $m(\angle A) = m(\angle \hat{A})$, $m(\angle B) = m(\angle \hat{B})$ and $m(\angle C) = m(\angle \hat{C})$





$\triangle ABC \equiv \triangle \hat{A}\hat{B}\hat{C}$

From the previous, we deduce that :

Reflection is a geometrical transformation which transforms the geometrical shape into another one congruent to it (equal to it in its side lengths and angle measures), but the direction of reading the shape is the opposite direction of reading the image.

Notice that :

The reading of $\triangle ABC$ is clockwise  while the reading of $\triangle \hat{A}\hat{B}\hat{C}$ is anticlockwise .

Properties of reflection in a straight line

Illustrated example

Draw the image of the rectangle ABCD in which $AB = 4$ cm., $BC = 2$ cm. by reflection in \overleftrightarrow{AB}

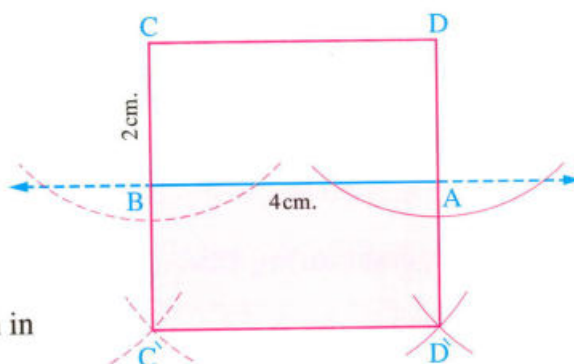
Solution

First

We draw the rectangle ABCD in which :
 $AB = 4$ cm. and $BC = 2$ cm.

Second

To find the image of the rectangle ABCD by reflection in \overleftrightarrow{AB} , we do as follows :



- 1 The images of A and B by reflection in \overleftrightarrow{AB} are the same because they belong to \overleftrightarrow{AB}
- 2 We find the image of D by reflection in \overleftrightarrow{AB} , let it be D' , the image of C by reflection in \overleftrightarrow{AB} , let it be C' , then we get the rectangle $AB'C'D'$ to be the image of the rectangle ABCD by reflection in \overleftrightarrow{AB}

Notice that :

- 1 $AD = A'D'$, $DC = D'C'$, $CB = C'B$ and \overleftrightarrow{AB} is a common side.

i.e.

Reflection in a straight line reserves the lengths of the line segments.

- 2 $m(\angle BAD) = m(\angle B'A'D')$
 $m(\angle ABC) = m(\angle A'B'C')$
 $m(\angle C) = m(\angle C')$ and $m(\angle D) = m(\angle D')$

i.e.

Reflection in a straight line reserves the measures of the angles.

- 3 From the rectangle ABCD : $\overline{AD} \parallel \overline{BC}$
from the rectangle $AB'C'D'$: $\overline{A'D'} \parallel \overline{B'C'}$
 \therefore The images of the two parallel line segments are also two parallel line segments.

i.e.

Reflection in a straight line reserves the parallelism.

- 4 The reading of the rectangle ABCD is in the clockwise direction while the reading of the rectangle $AB'C'D'$ is in anticlockwise direction.

i.e.

Reflection in a straight line doesn't reserve the orientation of the vertices of the figure.

- 5 If a point lies on \overline{DC} and we find its image by reflection in \overleftrightarrow{AB} , we find its image lie on $\overline{D'C'}$

i.e.

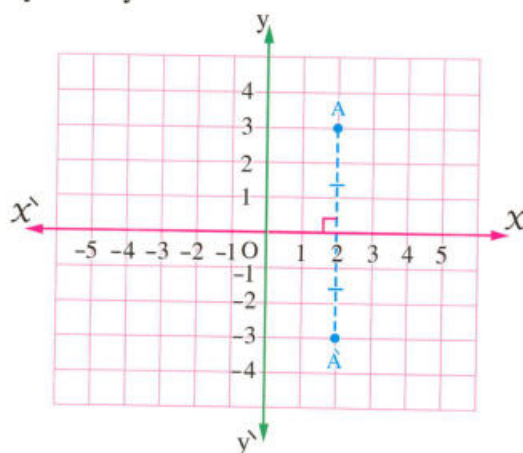
Reflection in a straight line reserves the betweenness.



Reflection in the Cartesian coordinates plane

Reflection in the x -axis

- To find the image of the point $A(2, 3)$ by reflection in \overleftrightarrow{XX} (the x -axis): draw $\overline{AA'}$ such that \overleftrightarrow{XX} is the line of symmetry of it.



Then we find that :

$$A(2, 3) \longrightarrow \hat{A}(2, -3)$$

- i.e. The reflection in the x -axis changes the sign of the 2nd projection (y-coordinates)

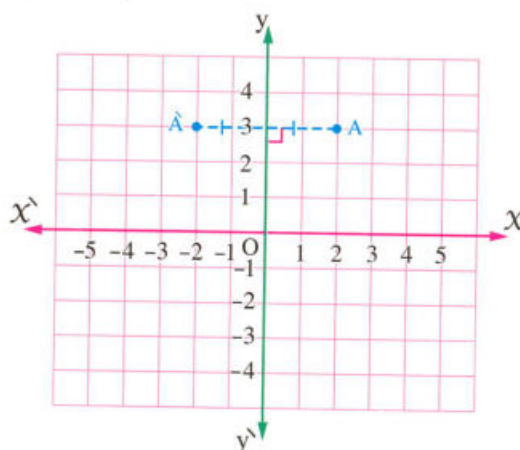
$$A(X, y) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} \hat{A}(X, -y)$$

For example:

- $(2, 4) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (2, -4)$
- $(-2, -1) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (-2, 1)$
- $(-5, 3) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (-5, -3)$
- $(7, -2) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (7, 2)$

Reflection in the y -axis

- To find the image of the point $A(2, 3)$ by reflection in \overleftrightarrow{YY} (the y -axis): draw $\overline{AA'}$ such that \overleftrightarrow{YY} is the line of symmetry of it.



Then we find that :

$$A(2, 3) \longrightarrow \hat{A}(-2, 3)$$

- i.e. The reflection in the y -axis changes the sign of the 1st projection (X-coordinates)

$$A(X, y) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} \hat{A}(-X, y)$$

For example:

- $(2, 4) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (-2, 4)$
- $(-2, -1) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (2, -1)$
- $(-5, 3) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (5, 3)$
- $(7, -2) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (-7, -2)$

Remarks

- The image of the point $(x, 0)$ by reflection in the x -axis is itself because it lies on the x -axis

For example: $(5, 0) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (5, 0)$

- The image of the point $(0, y)$ by reflection in the y -axis is itself because it lies on the y -axis

For example: $(0, -3) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (0, -3)$

- The image of the point $(0, 0)$ by reflection in the x -axis or the y -axis is itself because it lies on both two axes.

TRY by yourself 1 Complete the following table :

The point	$(5, 1)$	$(2, -3)$	$(-1, 4)$	$(-2, -6)$	$(0, -1)$	$(3, 0)$	$(0, 0)$
Its image by reflection in the x -axis							
Its image by reflection in the y -axis							

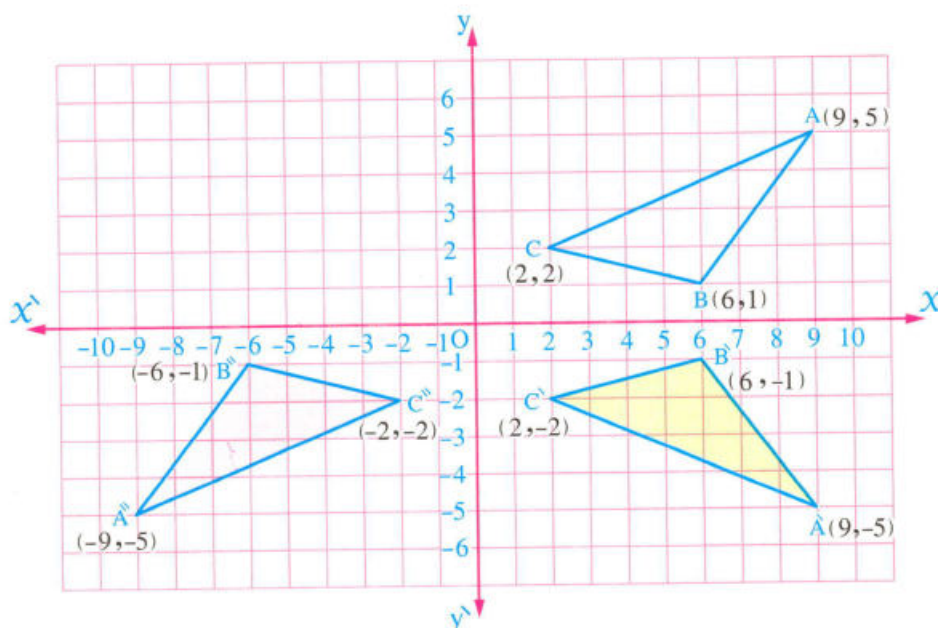
Example

Draw on a square lattice the triangle ABC where $A(9, 5)$, $B(6, 1)$ and $C(2, 2)$:

- Draw $\triangle A'B'C'$ which is the image of $\triangle ABC$ by reflection in the x -axis

- Draw $\triangle A''B''C''$ which is the image of $\triangle A'B'C'$ by reflection in the y -axis

Solution

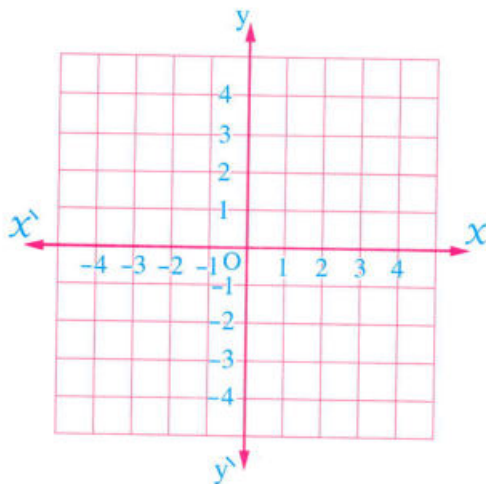




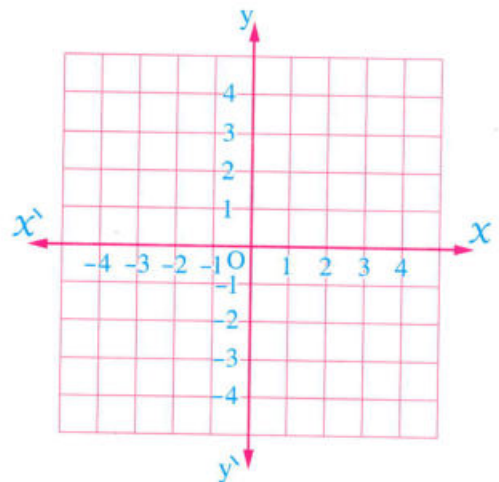
TRY 2 by yourself

Draw $\triangle ABC$ where $A(1, 1)$, $B(4, 1)$ and $C(3, 3)$, then draw its image by reflection in :

1 The x -axis



2 The y -axis



Symmetry

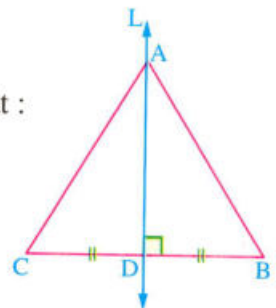
In the opposite figure :

ABC is a triangle, $\overline{AD} \perp \overline{BC}$, D is the midpoint of \overline{BC} , we find that :

- The image of A by reflection in L is itself (A)
- The image of B by reflection in L is C
- The image of C by reflection in L is B

i.e. The image of $\triangle ABC$ by reflection in L is $\triangle ACB$

We can say that $\triangle ABC$ is transformed to itself by reflection in the straight line L , therefore the straight line L is called the axis of symmetry of $\triangle ABC$



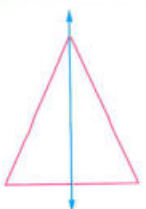
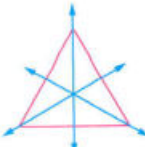


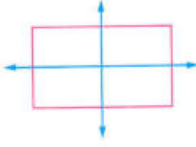
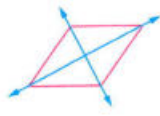
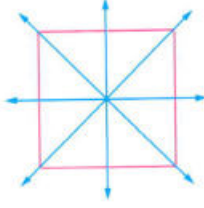

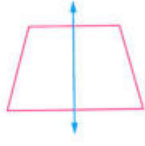
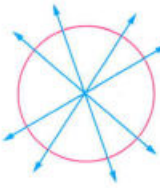
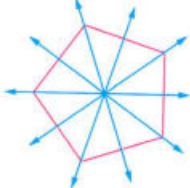
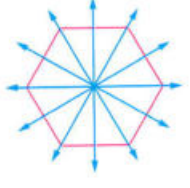
From the previous, we can deduce the definition of the axis of symmetry as follows :

If the reflection in a line transforms the figure to itself, then this line is called an axis of symmetry of the figure.

Remark

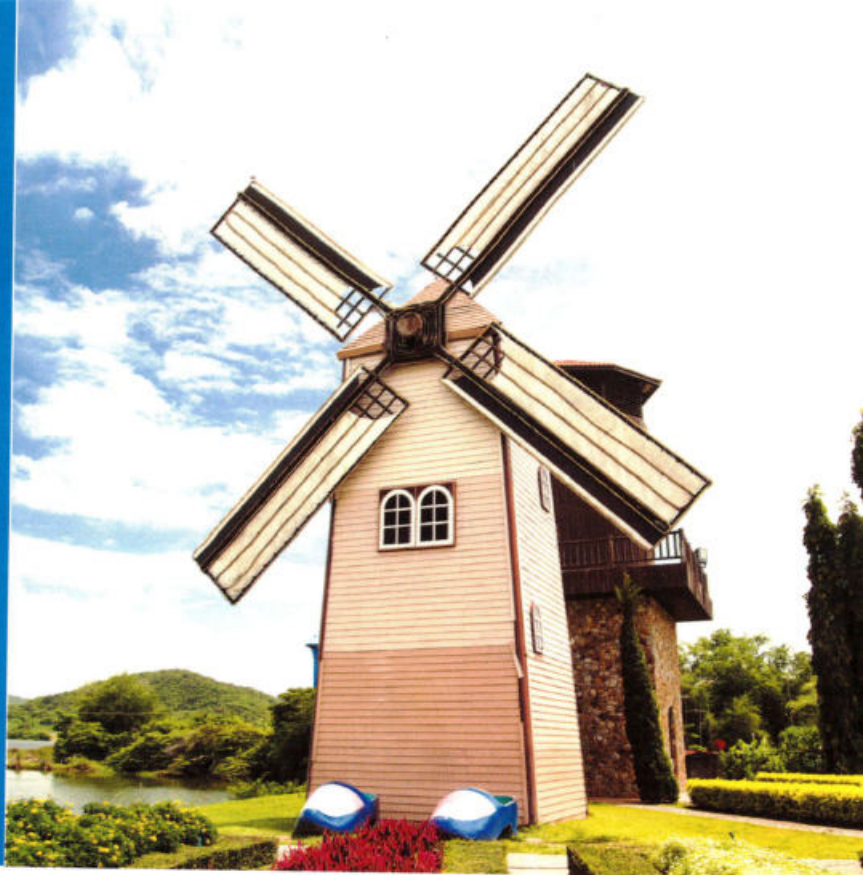
The axis of symmetry divides the figure into two congruent figures.

The axes of symmetry of some geometric figures

The figure			
Number of axes of symmetry	1	3	Zero (does not exist)
The figure			
Number of axes of symmetry	Zero (does not exist)	2	2
The figure			
Number of axes of symmetry	4	Zero (does not exist)	1
The figure			
Number of axes of symmetry	An infinite number	5	6

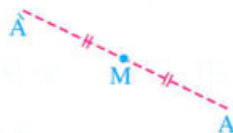
Lesson 10

Reflection in a point



Definition of reflection in a point

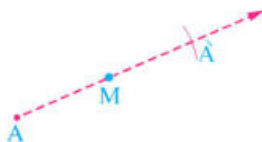
Reflection in a point M maps each point A in the plane to the point \hat{A} in the same plane where M is the midpoint of the line segment $\overline{A\hat{A}}$,
the point M is called the centre of reflection and the image of M by reflection in M is itself.



Finding the image of a point by reflection in a given point

- To find the image of a point as A by reflection in M , we do as follows :

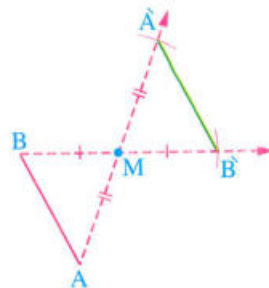
- 1 Draw \overline{AM}
- 2 Using the compasses with a radius length equals MA , then use M as a centre and draw an arc to intersect \overline{AM} at a point as \hat{A} ,
then \hat{A} is the image of the point A by reflection in the point M
- 3 From the previous, we found that : $MA = M\hat{A}$



Finding the image of a line segment by reflection in a given point

• To find the image of a line segment as \overline{AB} by reflection in M , we do as follows :

- 1 Find the image of A by reflection in M to be \hat{A} as we mentioned before.
- 2 Similarly find the image of B by reflection in M to be \hat{B}
- 3 Draw $\overline{\hat{A}\hat{B}}$ to be the image of \overline{AB} by reflection in the point M



Notice that :

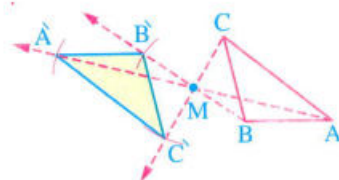
$$\hat{A}\hat{B} = AB \text{ and } \overline{\hat{A}\hat{B}} \parallel \overline{AB}$$

i.e. The image of a line segment by reflection in a point is a line segment parallel to the original one and its length equals the length of the original line segment.

Finding the image of a polygon by reflection in a given point

• To find the image of a polygon as the triangle ABC by reflection in M , we do as follows :

- 1 Find the image of each vertex of the vertices of the triangle ABC by reflection in the point M as we mentioned before to be :



\hat{A} is the image of A , \hat{B} is the image of B and \hat{C} is the image of C

- 2 Draw $\overline{\hat{A}\hat{B}}$, $\overline{\hat{B}\hat{C}}$ and $\overline{\hat{C}\hat{A}}$ to get $\triangle \hat{A}\hat{B}\hat{C}$ which is the image of $\triangle ABC$ by reflection in the point M

Notice that :

$\triangle ABC \equiv \triangle \hat{A}\hat{B}\hat{C}$, therefore it is said that the reflection in a point is **isometric**.

From the previous, we deduce that :

Reflection in a point is a geometric transformation that maps the geometric figure to another geometric figure congruent to it and has the same orientation of its vertices.



Properties of reflection in a point

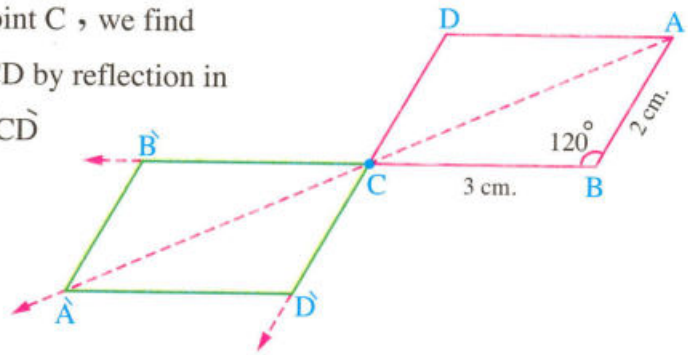
Illustrated example

Draw the parallelogram ABCD in which $AB = 2 \text{ cm}$, $BC = 3 \text{ cm}$, and $m(\angle B) = 120^\circ$, then draw its image by reflection in the point C and show what you observe.

Solution

Finding the image of each vertex of the vertices of $\square ABCD$

by reflection in the point C, we find the image of $\square ABCD$ by reflection in the point C is $\square \tilde{A}\tilde{B}\tilde{C}\tilde{D}$

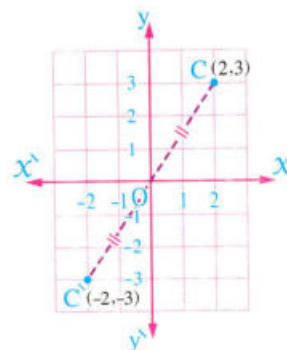


Notice that :

1 $\tilde{A}\tilde{B} = AB$, $\tilde{B}\tilde{C} = BC$, $\tilde{C}\tilde{D} = CD$ and $\tilde{D}\tilde{A} = DA$	i.e.	Reflection in a point reserves the lengths of the line segments.
2 $m(\angle \tilde{A}) = m(\angle A)$, $m(\angle \tilde{B}) = m(\angle B)$, $m(\angle \tilde{B}\tilde{C}\tilde{D}) = m(\angle BCD)$ and $m(\angle \tilde{D}) = m(\angle D)$	i.e.	Reflection in a point reserves the measures of the angles.
3 From the parallelogram ABCD : $\overline{AB} \parallel \overline{DC}$, From the parallelogram $\tilde{A}\tilde{B}\tilde{C}\tilde{D}$: $\overline{\tilde{A}\tilde{B}} \parallel \overline{\tilde{D}\tilde{C}}$ \therefore The images of the two parallel line segments are also two parallel line segments.	i.e.	Reflection in a point reserves the parallelism.
4 The reading of the parallelogram ABCD is in the clockwise direction and the reading of the parallelogram $\tilde{A}\tilde{B}\tilde{C}\tilde{D}$ is in the clockwise direction also.	i.e.	Reflection in a point reserves the orientation of the vertices of the figure.
5 Putting a point belongs to \overline{AB} , we find its image by reflection in C belongs to $\overline{\tilde{A}\tilde{B}}$	i.e.	Reflection in a point reserves the betweenness.

Reflection in the origin point

- If the point C is a point in the Cartesian coordinates plane where C (2, 3)
- To find the image of the point C by reflection in the origin point O using the same previous method, we will find it \hat{C} (-2, -3)



Notice that :

The signs of the two projections of the ordered pair (2, 3) have been changed, hence we can define the reflection in the origin point as follows :

Definition

If A (X, y) is a point in the Cartesian coordinates plane, then the image of the point A by reflection in the origin point O is \hat{A} (-X, -y)

i.e. Reflection in the origin point converts the sign of each of the coordinates of the point.

\therefore The image of the point (X, y) $\xrightarrow[\text{the origin point}]{\text{by reflection in}}$ (-X, -y)

For example:

- The image of the point (2, 3) $\xrightarrow[\text{the origin point}]{\text{by reflection in}}$ (-2, -3)
- The image of the point (-4, 1) $\xrightarrow[\text{the origin point}]{\text{by reflection in}}$ (4, -1)
- The image of the point (5, -2) $\xrightarrow[\text{the origin point}]{\text{by reflection in}}$ (-5, 2)
- The image of the point (-3, -6) $\xrightarrow[\text{the origin point}]{\text{by reflection in}}$ (3, 6)

! Remark

The image of the point (0, 0) by reflection in the origin point is itself.



Example

Draw $\triangle ABC$ where $A(4, 1)$, $B(2, 4)$ and $C(-1, 3)$, then map draw image by reflection in the origin point.

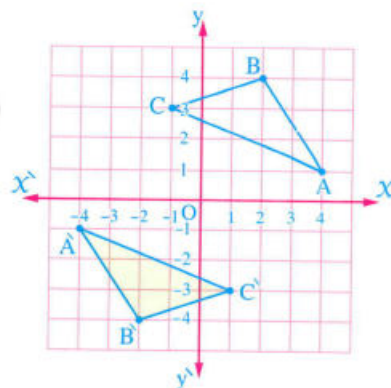
Solution

$$\therefore (x, y) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-x, -y)$$

$$\therefore A(4, 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \hat{A}(-4, -1)$$

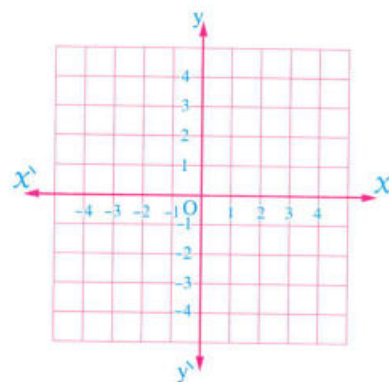
$$, B(2, 4) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \hat{B}(-2, -4)$$

$$, C(-1, 3) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \hat{C}(1, -3)$$



TRY by yourself

Draw on a square lattice $\triangle ABC$, where $A(-2, 1)$, $B(4, -2)$ and $C(2, 3)$, then draw its image by reflection in the origin point.

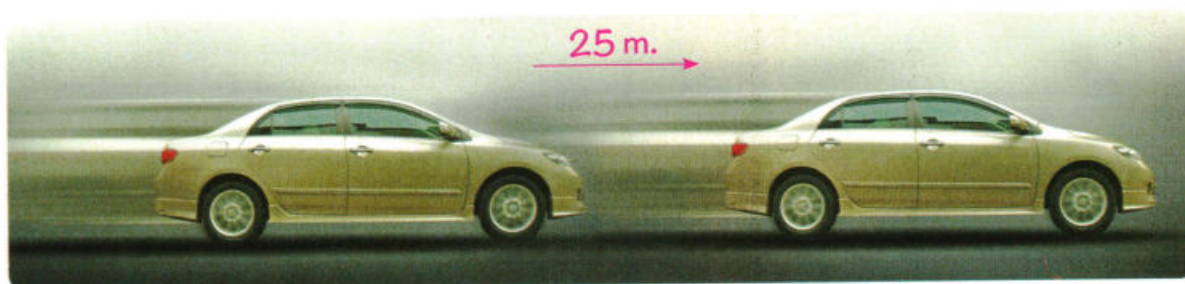


Translation



Prelude

If a car moved a distance 25 metres in a straight line forward, then we say that :
The car translated for a distance 25 metres forward.



i.e. To determine the new position of the car after its movement, we should know two important elements which are :

- 1 The magnitude of the translation (25 metres).
- 2 The direction of the translation (forward in a straight line).

According to this, we can say that :

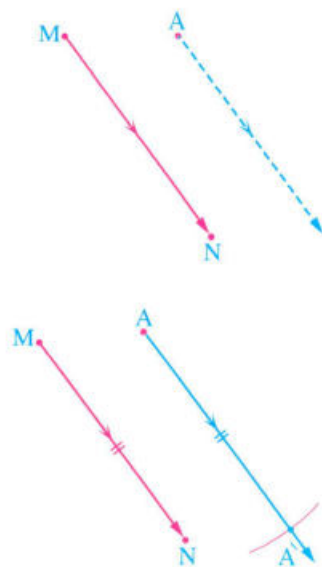
Translation is a geometrical transformation which maps each point A in the plane to another point \hat{A} in the same plane with a constant distance in a certain direction.



Translation in the plane

Finding the image of a point by a given translation

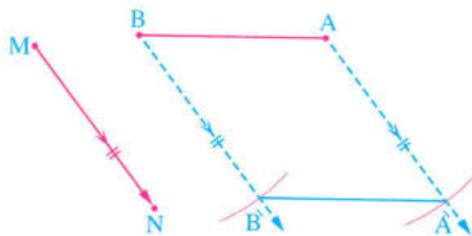
- To find \hat{A} which is the image of A by translation MN in the direction of \overrightarrow{MN} , we do as follows :
 - 1 Draw from A a ray parallel to \overrightarrow{MN} and in the same direction.
 - 2 By the compasses in A as a centre with radius = MN , draw an arc to intersect the ray drawn from A at the point \hat{A} ($A\hat{A} = MN$ and $A\hat{A} \parallel \overrightarrow{MN}$)
- Then \hat{A} is the image of A by translation of magnitude MN in the direction of \overrightarrow{MN}



Finding the image of a line segment by a given translation

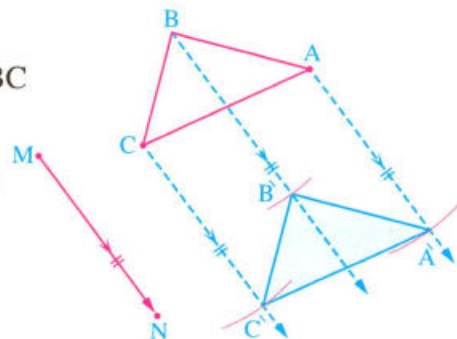
- To find the image of \overline{AB} by translation MN in the direction of \overrightarrow{MN} , we do as follows :
 - 1 Find the image of the point A by translation MN in the direction of \overrightarrow{MN} as we mentioned before, say \hat{A}
 - 2 Similarly, we find the image of the point B by translation MN in the direction of \overrightarrow{MN} , say \hat{B}
 - 3 Draw $\overline{\hat{A}\hat{B}}$ to be the image of \overline{AB} by translation MN in the direction of \overrightarrow{MN}

Check that : $AB = \hat{A}\hat{B}$ and $\overline{AB} \parallel \overline{\hat{A}\hat{B}}$



Finding the image of a polygon by a given translation

- To find the image of a polygon as $\triangle ABC$ by translation MN in the direction of \overrightarrow{MN} , we do as follows :
 - 1 Find the image of each vertex of the vertices of $\triangle ABC$ by translation MN in the direction of \overrightarrow{MN} as we mentioned before (say \hat{A} for A , \hat{B} for B and \hat{C} for C)
 - 2 Draw $\overline{\hat{A}\hat{B}}$, $\overline{\hat{B}\hat{C}}$ and $\overline{\hat{C}\hat{A}}$ then $\triangle \hat{A}\hat{B}\hat{C}$ is the image of $\triangle ABC$ by translation MN in the direction of \overrightarrow{MN}



Check that :

- $AB = \hat{A}\hat{B}$, $BC = \hat{B}\hat{C}$ and $CA = \hat{C}\hat{A}$
- $m(\angle A) = m(\angle \hat{A})$, $m(\angle B) = m(\angle \hat{B})$, $m(\angle C) = m(\angle \hat{C})$

From the previous , we deduce that translation is a geometrical transformation which maps the geometrical figure to another geometrical figure congruent to it.

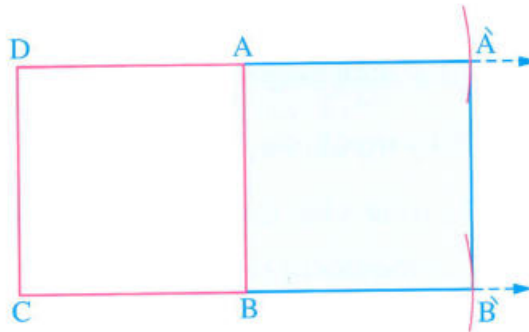
Properties of translation

Illustrated example

Draw the square ABCD whose side length is 3 cm., then draw its image by translation AB in the direction \overrightarrow{DA}

Solution

The square $\hat{A}\hat{B}\hat{B}\hat{A}$ is the image of the square ABCD by translation AB in the direction \overrightarrow{DA}



Notice that :

1 $\hat{A}\hat{B} = AB$, $AB = DC$

i.e.

Translation reserves the lengths of the line segments.

2 $m(\angle \hat{A}) = m(\angle BAD)$, $m(\angle \hat{B}) = m(\angle CBA)$

i.e.

Translation reserves the measures of the angles.

3 From the square ABCD : $\overline{AB} \parallel \overline{DC}$, from the square $\hat{A}\hat{B}\hat{B}\hat{A}$: $\overline{\hat{A}\hat{B}} \parallel \overline{\hat{A}\hat{B}}$
 \therefore The images of the two parallel line segments are also two parallel line segment.

i.e.

Translation reserves the parallelism.

4 The reading of the square ABCD is in the clockwise direction and the reading of the square $\hat{A}\hat{B}\hat{B}\hat{A}$ is in the clockwise direction also.

i.e.

Translation reserves the orientation of the vertices of the figure.



- 5** If you take a point lies on \overline{AB} and find its image by the previous translation, you will find its image lies on $\overline{A'B'}$

i.e.

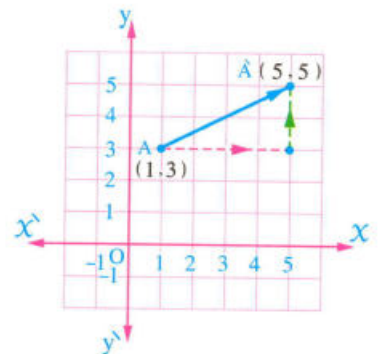
Translation reserves the betweenness.

Translation in the Cartesian plane

If $A(1, 3)$ is a point in the orthogonal coordinates plane and to find its image \hat{A} by translation with magnitude 4 length units in the direction of \overrightarrow{OX} followed by a translation with magnitude 2 length units in the direction of \overrightarrow{OY}

From the graph, we get \hat{A} to be the point $(5, 5)$

i.e. $\hat{A}(1 + 4, 3 + 2)$



According to this :

Translation in the orthogonal Cartesian coordinates plane transforms each point by a displacement a in the direction of the X -axis followed by a displacement b in the direction of the y -axis

i.e. The image of the point $A(X, y) \longrightarrow$ the point $\hat{A}(X + a, y + b)$

Example 1

Find the images of the points $A(2, 5)$, $B(-4, 3)$ and $C(2, 0)$ by translation $(X, y) \longrightarrow (X + 2, y - 3)$

Solution

$\therefore (X, y) \longrightarrow (X + 2, y - 3)$, then :

- The image of $A(2, 5)$ is $\hat{A}(2 + 2, 5 - 3)$

i.e. $\hat{A}(4, 2)$

- The image of $B(-4, 3)$ is

$\hat{B}(-4 + 2, 3 - 3)$

i.e. $\hat{B}(-2, 0)$

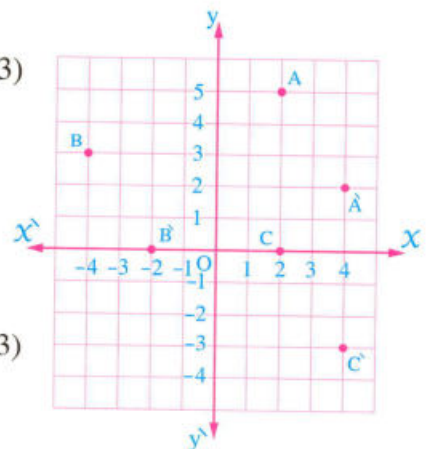
- The image of $C(2, 0)$ is $\hat{C}(2 + 2, 0 - 3)$

i.e. $\hat{C}(4, -3)$

Notice that :

The translation $(X, y) \longrightarrow (X + 2, y - 3)$

transforms each point to another point by a right horizontal displacement of 2 units and a vertical displacement of 3 units downwards.



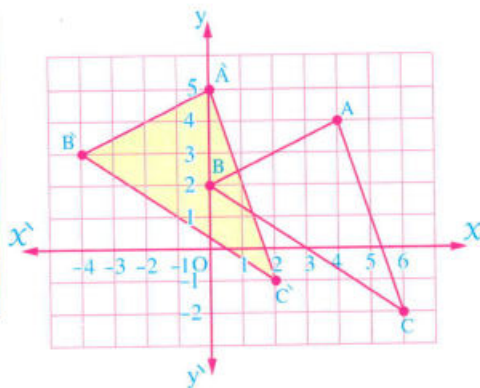
Example 2

Draw on a square lattice $\triangle ABC$ where $A(4, 4)$, $B(0, 2)$, $C(6, -2)$, then draw its image by translation $(x, y) \longrightarrow (x - 4, y + 1)$

Solution

The point	Its image by the translation
(x, y)	$(x - 4, y + 1)$
$A(4, 4)$	$\hat{A}(0, 5)$
$B(0, 2)$	$\hat{B}(-4, 3)$
$C(6, -2)$	$\hat{C}(2, -1)$

$\therefore \triangle \hat{A}\hat{B}\hat{C}$ is the image of $\triangle ABC$ by translation $(x, y) \longrightarrow (x - 4, y + 1)$

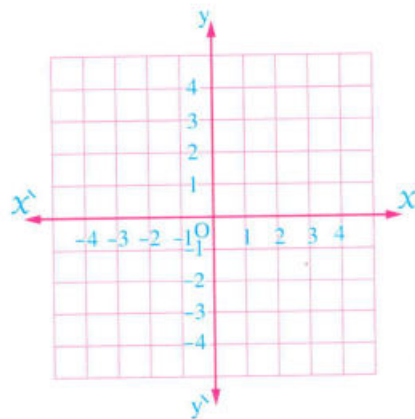


Remark

The translation $(x, y) \longrightarrow (x + a, y + b)$ can be written as the translation (a, b) for example :
The translation $(x, y) \longrightarrow (x + 2, y - 1)$ can be written as the translation $(2, -1)$

TRY 1 by yourself

On a square lattice, draw $\triangle ABC$ where $A(-3, 2)$, $B(-1, 1)$, $C(-2, 0)$, then draw its image by translation :
 $(x, y) \longrightarrow (x + 2, y + 1)$





Example 3

Find the image of each of the two points A (4, -1) and B (0, -3) by translation with magnitude \overline{MN} in the direction of \overrightarrow{MN} where M (4, 2) and N (1, 4)

Solution

- By noticing the following graph, we find that the translation with magnitude \overline{MN} in the direction of \overrightarrow{MN} where M (4, 2) and N (1, 4) is equivalent to :

- Horizontal displacement from 4 to 1

i.e. A displacement = 3 units to the left (-3)

- Vertical displacement from 2 to 4

i.e. A displacement = 2 units upwards (2)

i.e. $(X, y) \longrightarrow (X - 3, y + 2)$,

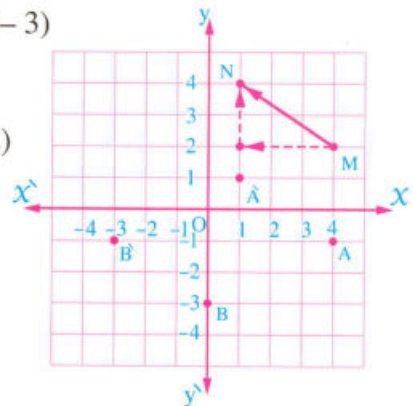
thus we get :

$$A(4, -1) \longrightarrow \hat{A}(4 - 3, -1 + 2)$$

i.e. $\hat{A}(1, 1)$

$$B(0, -3) \longrightarrow \hat{B}(0 - 3, -3 + 2)$$

i.e. $\hat{B}(-3, -1)$



Notice that :

The translation with magnitude \overline{MN} in the direction of \overrightarrow{MN} where M (4, 2) and N (1, 4) is equivalent to :

- A horizontal displacement (in the X-axis direction) from 4 to 1 = $1 - 4 = -3$
- A vertical displacement (in the y-axis direction) from 2 to 4 = $4 - 2 = 2$

i.e. The rule of translation is $(X, y) \longrightarrow (X - 3, y + 2)$

Example 4

Draw the image of $\triangle ABC$ where $A(5, 2)$, $B(4, 5)$ and $C(2, 2)$ by translation BC in the direction of \overrightarrow{BC} and write the rule of the translation.

Solution

$$\because B(4, 5), C(2, 2)$$

\therefore The translation BC in the direction of \overrightarrow{BC} is equivalent to :

- Horizontal displacement $= 2 - 4 = -2$
- Vertical displacement $= 2 - 5 = -3$

Thus the rule of translation

$$\text{is } (x, y) \longrightarrow (x - 2, y - 3)$$

Thus :

$$A(5, 2) \longrightarrow \hat{A}(5 - 2, 2 - 3)$$

$$\text{i.e. } \hat{A}(3, -1)$$

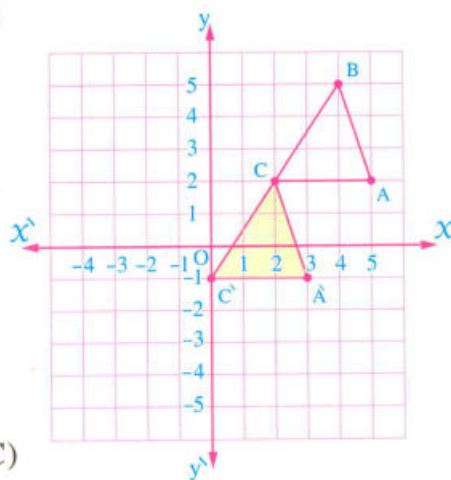
$$B(4, 5) \longrightarrow \hat{B}(4 - 2, 5 - 3)$$

$$\text{i.e. } \hat{B}(2, 2) \quad (\text{Notice : } \hat{B} \text{ coincides } C)$$

$$C(2, 2) \longrightarrow \hat{C}(2 - 2, 2 - 3)$$

$$\text{i.e. } \hat{C}(0, -1)$$

$$\text{i.e. } \triangle \hat{A}\hat{C}\hat{C} \text{ is the image of } \triangle ABC \text{ by translation } BC \text{ in the direction of } \overrightarrow{BC}$$

**TRY**
by yourself**2**

Draw the square $ABCD$ where $A(4, -2)$, $B(4, -5)$, $C(1, -5)$ and $D(1, -2)$, then draw its image by translation CA in the direction of \overrightarrow{CA}



Example 5

If the image of the point A $(-3, 2)$ by translation in the Cartesian coordinates plane is $\hat{A}(2, -2)$:

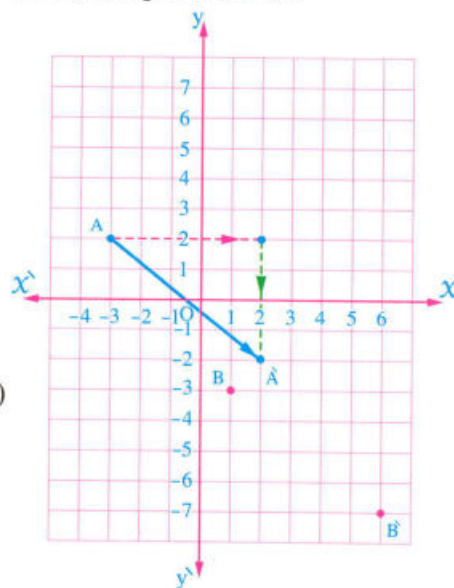
- 1 Find the rule of translation.
- 2 Find the image of B $(1, -3)$ by the same translation.

Solution

- 1 By noticing the following graph, we find that the translation which makes $\hat{A}(2, -2)$ the image of A $(-3, 2)$ is equivalent to :

- Horizontal displacement of 5 units to the right side (5)
- Vertical displacement of 4 units downwards (-4)
 \therefore The rule of translation is
 $(X, y) \longrightarrow (X + 5, y - 4)$

- 2 B $(1, -3) \longrightarrow \hat{B}(1 + 5, -3 - 4)$
 i.e. $\hat{B}(6, -7)$



Example 6

If $\hat{A}(7, -2)$ is the image of A by the translation whose rule is $(X, y) \longrightarrow (X - 3, y + 1)$, find A

Solution

Let A be (X, y)

$$\therefore A(X, y) \longrightarrow \hat{A}(X - 3, y + 1)$$

$$\therefore \hat{A}(7, -2)$$

$$\therefore (X - 3, y + 1) = (7, -2)$$

$$\therefore X - 3 = 7$$

$$\therefore y + 1 = -2$$

$$\therefore A(10, -3)$$

$$\therefore X = 10$$

$$\therefore y = -3$$

Notice that :



If $(X, y) = (a, b)$,
 then $X = a, y = b$

Lesson 12

Rotation



Prelude

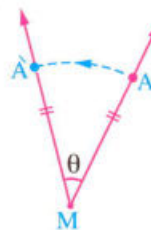
If you stand in front of Ferris wheel in the funfair, you will find that the carriage moves in a circular motion around a fixed point in the direction of clockwise  or in the direction of anticlockwise , this motion is called “rotation”



Definition of rotation

If M is a fixed point in the plane, then the rotation around M with an angle of measure θ is a geometric transformation transforming each point A in the plane to another point \hat{A} in the same plane such that $m(\angle AMA\hat{A}) = \theta$, $MA = M\hat{A}$ this rotation is denoted by $R(M, \theta)$ where :

- M is the centre of rotation.
- θ is the measure of the angle of rotation.



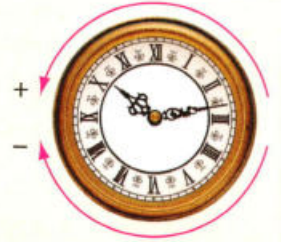
According to this definition, the rotation is determined completely if we know the following elements

- 1 The centre of the rotation.
- 2 The measure of the angle of the rotation (θ)
- 3 The direction of rotation.



! Remark

The measure of rotation angle is positive if the rotation is anticlockwise and it is negative if the rotation is clockwise.

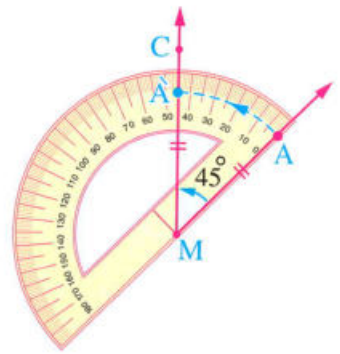


Rotation in the plane

Finding the image of a point by a given rotation

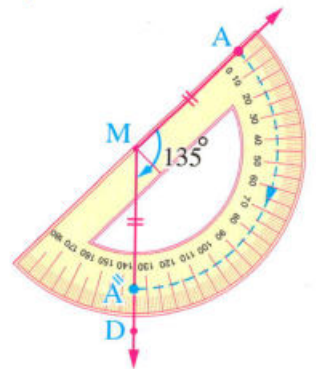
Firstly : Finding the image of the point A by rotation around the point M with an angle of measure 45° i.e. $R(M, 45^\circ)$:

- Draw the ray \overrightarrow{MA}
- Put the protractor with its straight edge on \overrightarrow{MA} and in the anticlockwise direction, then draw \overrightarrow{MC} such that $m(\angle AMC) = 45^\circ$
- Use the compasses at the point M as a centre with radius = MA , draw an arc to cut \overrightarrow{MC} at \hat{A} , then \hat{A} is the image of the point A by rotation around M with an angle of measure 45°



Secondly : Finding the image of the point A by rotation around the point M with an angle of measure (-135°) i.e. $R(M, -135^\circ)$:

- Repeat the same previous steps, then draw \overrightarrow{MD} in the clockwise direction such that $m(\angle AMD) = 135^\circ$, then determine on it the point \hat{A} such that $M\hat{A} = MA$, then \hat{A} is the image of A by rotation around M with an angle of measure (-135°)



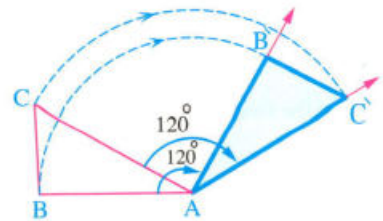
! Remark

If \hat{A} is the image of A by rotation around M with an angle θ , then A is the image of \hat{A} by rotation around M with an angle of measure $(-\theta)$

Finding the image of a polygon by a given rotation

The opposite figure shows how to find the image of $\triangle ABC$ by the rotation $R(A, -120^\circ)$ by finding the image of each vertex of its vertices, then $\triangle AB'C'$ is the image of $\triangle ABC$ by rotation $R(A, -120^\circ)$

Notice that : $\triangle AB'C' \equiv \triangle ABC$



! Remark

From the previous figure, the image of the point A by rotation $R(A, -120^\circ)$ is itself because it is the centre of rotation.

Properties of rotation

Through our study of rotation, we found that the rotation is a geometric transformation that maps the figure to another congruent figure to it.

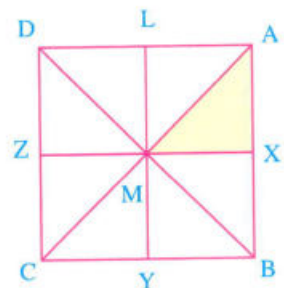
Therefore it is said that the rotation in the plane is **isometric**

, thus we can deduce some of properties and add other properties through our study of the following illustrated example.

Illustrated example

In the opposite figure :

ABCD is a square whose diagonals intersect at M, X, Y, Z and L are the midpoints of its sides \overline{AB} , \overline{BC} , \overline{CD} and \overline{DA} respectively. **Find :**



- 1 The image of $\triangle AXM$ by rotation $R(M, 90^\circ)$, then mention what you observe.
- 2 The image of each of \overline{AB} and \overline{DC} by rotation $R(M, -90^\circ)$, then mention what you observe.
- 3 The image of each of B, Y and C by rotation $R(M, 180^\circ)$, then mention what you observe.

Solution

- 1 \therefore D is the image of A by rotation $R(M, 90^\circ)$, L is the image of X by rotation $R(M, 90^\circ)$ and M is the image of itself (The centre of rotation).
 $\therefore \triangle DLM$ is the image of $\triangle AXM$ by rotation $R(M, 90^\circ)$


We notice that :

- $DL = AX$, $LM = XM$ and $DM = AM$

i.e.

Rotation in the plane reserves the lengths of the line segments.

- $m(\angle DLM) = m(\angle AXM)$,
 $m(\angle LDM) = m(\angle XAM)$ and
 $m(\angle DML) = m(\angle AMX)$

i.e.

Rotation in the plane reserves the measures of the angles.

- Reading $\triangle AXM$ is in the clockwise direction and reading its image $\triangle DLM$ is in the clockwise direction also.

i.e.

Rotation in the plane reserves the orientation of the vertices of the figure.

- 2 $\therefore B$ is the image of A by rotation $R(M, -90^\circ)$, C is the image of B by rotation $R(M, -90^\circ)$

$\therefore \overline{BC}$ is the image of \overline{AB} by rotation $R(M, -90^\circ)$

$\therefore A$ is the image of D by rotation $R(M, -90^\circ)$, D is the image of C by rotation $R(M, -90^\circ)$

$\therefore \overline{AD}$ is the image of \overline{DC} by rotation $R(M, -90^\circ)$

We notice that :

- $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$

i.e.

Rotation in the plane reserves the parallelism.

- 3 D is the image of B , L is the image of Y and A is the image of C by rotation $(M, 180^\circ)$

We notice that :

- $Y \in \overline{BC}$
 and L (The image of Y) $\in \overline{AD}$

i.e.

Rotation in the plane reserves the betweenness.

- B, Y, C are collinear , D, L, A are also collinear.

i.e.

Rotation in the plane reserves the collinearity.

Rotation in the Cartesian plane

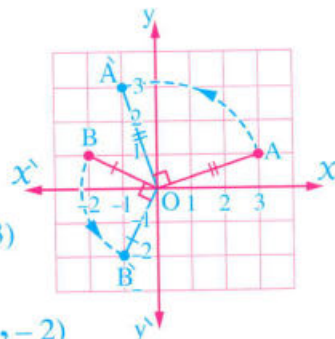
First Rotation about the origin point (O) with an angle of measure 90°

The opposite figure shows the two images of the two points

A (3, 1) and B (-2, 1) by rotation $R(O, 90^\circ)$

By noticing the figure, we find that :

- The image of the point A (3, 1) $\xrightarrow[\text{R}(O, 90^\circ)]{\text{by rotation}}$ the point $\hat{A}(-1, 3)$
- The image of the point B (-2, 1) $\xrightarrow[\text{R}(O, 90^\circ)]{\text{by rotation}}$ the point $\hat{B}(-1, -2)$



From the previous, we deduce the following rule :

The image of the point (X, y) $\xrightarrow[\text{R}(O, 90^\circ)]{\text{by rotation}}$ the point (-y, X)

Remarks

- The image of the point (X, y) $\xrightarrow[\text{R}(O, -90^\circ)]{\text{by rotation}}$ the point (y, -X)

For example: The image of the point (2, -3) $\xrightarrow[\text{R}(O, -90^\circ)]{\text{by rotation}}$ the point (-3, -2)

- Rotation about the origin point with an angle of measure 270° is equivalent to rotation about the origin point with an angle of measure (-90°)

For example: The image of the point (2, -3) $\xrightarrow[\text{R}(O, 270^\circ)]{\text{by rotation}}$ the point (-3, -2)

Second Rotation about the origin point (O) with an angle of measure 180°

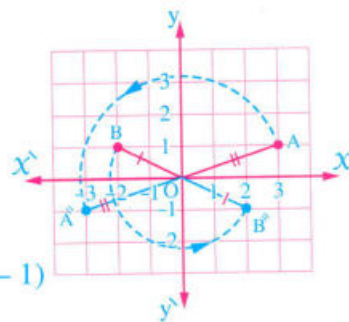
The opposite figure shows the two images of the

two points A (3, 1) and B (-2, 1)

by rotation $R(O, 180^\circ)$

By noticing the figure, we find that :

- The image of the point A (3, 1) $\xrightarrow[\text{R}(O, 180^\circ)]{\text{by rotation}}$ the point $\hat{A}(-3, -1)$
- The image of the point B (-2, 1) $\xrightarrow[\text{R}(O, 180^\circ)]{\text{by rotation}}$ the point $\hat{B}(2, -1)$





From the previous, we deduce the following rule :

The image of the point (X, y) $\xrightarrow[\text{R}(O, 180^\circ)]{\text{by rotation}}$ the point $(-X, -y)$

Remarks

- 1 The image of the point A (X, y) by rotation $R(O, 180^\circ)$ is the same image of the point A by rotation $R(O, -180^\circ)$
- 2 The image of the point A (X, y) about the origin point with an angle of measure $\pm 360^\circ$ is the same point A (X, y)
- 3 Rotation with an angle of measure 90° is called a $\frac{1}{4}$ turn.
- 4 Rotation with an angle of measure 180° is called a $\frac{1}{2}$ turn.
- 5 Rotation with an angle of measure 360° is called the identity rotation because it returns the figure to its original position.

Example 1

Complete the following table :

	The point	Its image by rotation $R(O, \pm 180^\circ)$	Its image by rotation $R(O, 90^\circ)$
1	$(3, 2)$
2	$(-3, 4)$
3	$(-2, -1)$
4	$(5, -2)$
5	$(6, 0)$

Solution

- 1 $(-3, -2), (-2, 3)$
- 2 $(3, -4), (-4, -3)$
- 3 $(2, 1), (1, -2)$
- 4 $(-5, 2), (-2, -5)$
- 5 $(0, -6), (0, 6)$

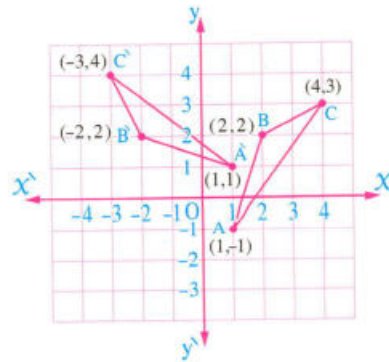
Example 2

On a square lattice, draw $\triangle ABC$ where A $(1, -1)$, B $(2, 2)$ and C $(4, 3)$:

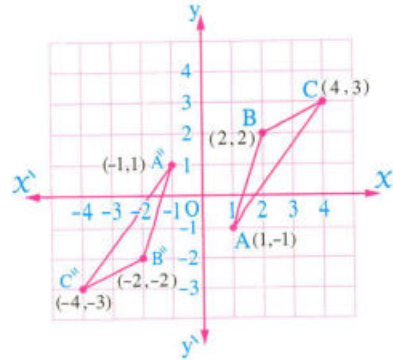
- 1 Draw $\triangle \hat{A}\hat{B}\hat{C}$ which is the image of $\triangle ABC$ by rotation $R(O, 90^\circ)$
- 2 Draw $\triangle \hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}$ which is the image of $\triangle ABC$ by rotation $R(O, 180^\circ)$

Solution

1



2

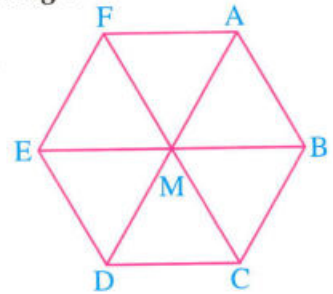


TRY
by yourself

1. In the opposite figure :

ABCDEF is a regular hexagon. Complete the following :

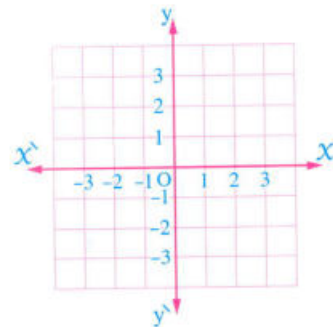
- 1 The image of the point A by rotation around M with an angle of measure 180° is
- 2 The image of \overline{AB} by rotation around M with an angle of measure (-60°) is
- 3 The image of $\triangle CMD$ by rotation around M with an angle of measure 120° is



2. In the opposite figure :

On the square lattice ,
draw \overline{AB} where A (2 , 1) and B (1 , 3) ,
then draw its image by rotation :

- 1 $R(O, 90^\circ)$
- 2 $R(O, 180^\circ)$



Optical illusion :

Look at the picture.
Turn the book with an angle of
measure 180° and look at it again.
What do you notice ?





By a group of supervisors

EXERCISES

1st PREP.
2024
SECOND TERM



Maths

Contents

First

Algebra and Statistics

UNIT **1** Numbers and Algebra.

UNIT **2** Statistics and Probability.



Second

Geometry and Measurement

UNIT **3** Geometry and Measurement.

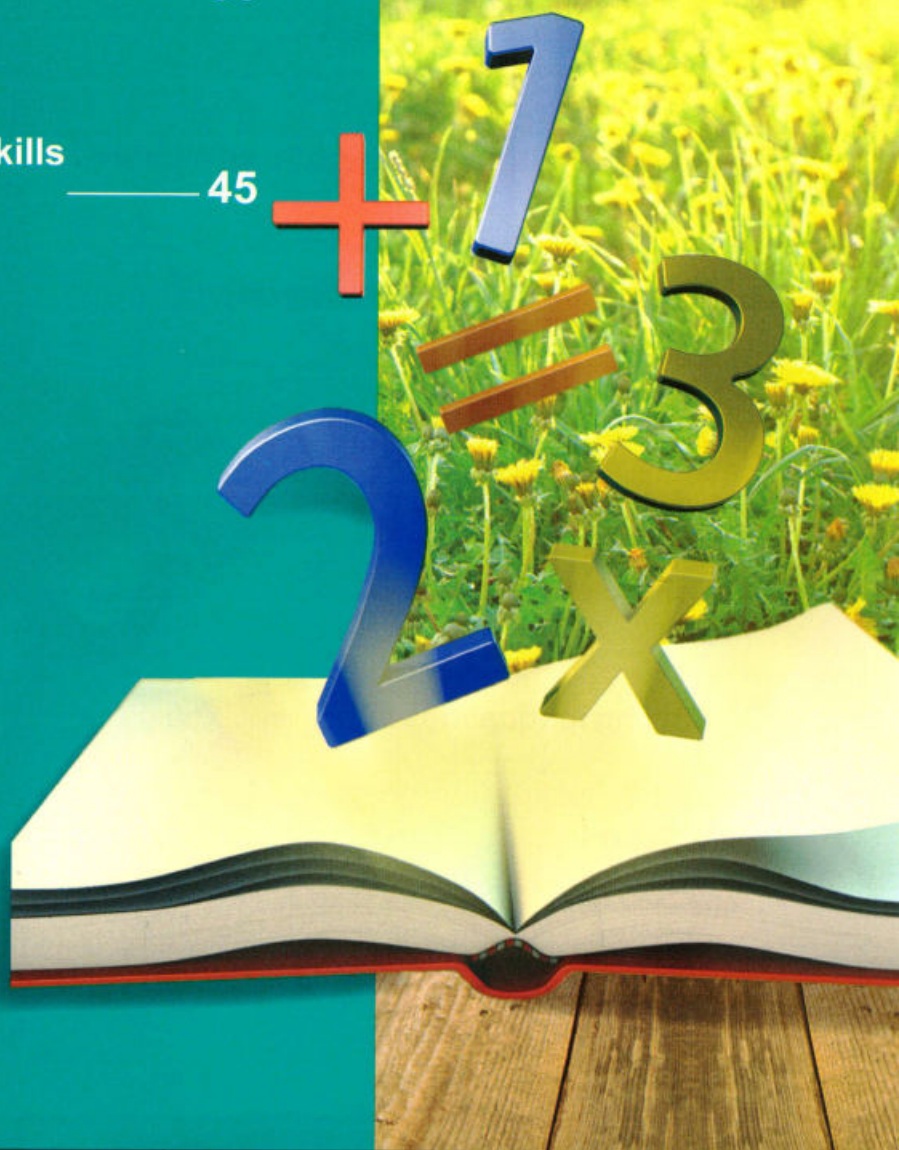


First | Algebra and Statistics

UNIT **1** Numbers and Algebra — 6

UNIT **2** Statistics and Probability — 36

Accumulative Basic skills — 45
"TIMSS Problems"

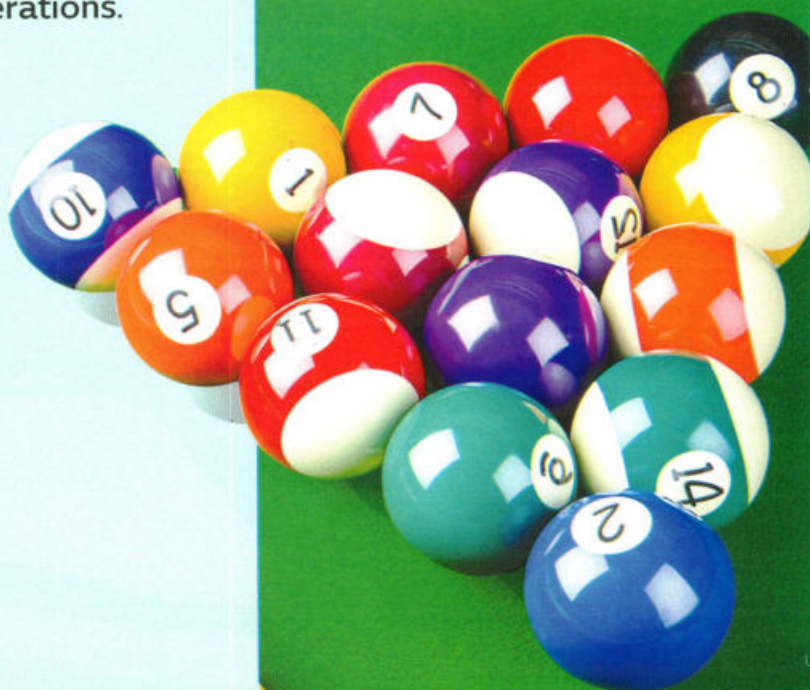


1 Numbers and Algebra

Exercises of the unit :

1. Repeated multiplication.
2. Non-negative integer powers.
3. Negative integer powers.
4. Scientific notation of the rational number.
5. Order of mathematical operations.
6. The square root of a perfect square rational number.
7. Solving equations in \mathbb{Q} .
8. Solving inequalities in \mathbb{Q} .

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test on each
lesson





Exercise

1

Repeated
multiplication

From the school book



Remember

Understand

Apply

Problem Solving



Interactive test

1 Calculate each of the following, then put the result in the simplest form :

1 $(\frac{1}{3})^4$

2 $(\frac{3}{5})^2$

3 $(-\frac{1}{7})^3$

4 $(-\frac{3}{4})^4$

5 $(\frac{5}{9})^0$

6 $(-2\frac{1}{2})^3$

7 $(0.04)^2$

8 $(1.5)^3$

9 $(-3.2)^2$

2 Calculate each of the following, then put the result in the simplest form :

1 $8 \times (\frac{1}{2})^3$

2 $(-\frac{3}{4})^2 \times \frac{8}{27}$

3 $(-\frac{3}{5})^3 \times (-\frac{25}{27})$

4 $(\frac{3}{5})^2 \div (-\frac{9}{125})$

5 $(\frac{4}{3})^2 \times (\frac{3}{2})^3$

6 $(-\frac{5}{6})^2 \div 3\frac{3}{4}$

7 $(2\frac{1}{2})^2 \times \frac{4}{25}$

8 $2\frac{7}{9} \div (-1\frac{2}{3})^2$

3 Calculate each of the following, then put the result in the simplest form :

1 $(\frac{4}{5})^2 \times \frac{5}{16} \times (\frac{2}{3})^0$

2 $\frac{3}{4} \times (-\frac{2}{3})^3 \times (\frac{3}{2})^2$

3 $(-\frac{5}{3})^4 \times (-\frac{3}{5})^3 \times (-1)^7$

4 $(-\frac{2}{3})^3 \times (\frac{1}{3})^3 \div (-\frac{2}{9})^2$

5 $[(\frac{5}{2})^3 \div (\frac{3}{2})^4] \times (\frac{3}{5})^3$

6 $(-\frac{1}{2})^3 \div [8 \times (-\frac{1}{2}) \times \frac{3}{4}]$

4 Choose the correct answer from those given :

- 1 The multiplicative inverse of the number $(\frac{2}{5})^0$ is
 (a) $\frac{5}{2}$ (b) $-\frac{2}{5}$ (c) 1 (d) 0
- 2 The additive inverse of the number $(-3)^0$ is
 (a) 1 (b) -3 (c) 3 (d) $-(3)^0$
- 3 The multiplicative inverse of the number $(-1)^3$ is
 (a) $(-1)^3$ (b) $(-1)^2$ (c) 1^3 (d) 1^2
- 4 The additive inverse of the number $(-\frac{2}{5})^2$ is
 (a) $\frac{4}{25}$ (b) $-\frac{4}{25}$ (c) $\frac{25}{4}$ (d) $-\frac{25}{4}$
- 5 $(\frac{1}{4})^0 + \frac{1}{4} = \dots\dots\dots$
 (a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{2}{4}$
- 6 $(\frac{5}{3})^2 \times (\frac{3}{5})^0 = \dots\dots\dots$
 (a) $\frac{5}{3}$ (b) $\frac{25}{9}$ (c) 0 (d) 1
- 7 If $x = y$, then $(\frac{3}{5})^{x-y} = \dots\dots\dots$
 (a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) 1 (d) 0
- 8 $(\frac{a}{b})^2 \times \frac{b^2}{a^2} = \dots\dots\dots$ (where $ab \neq 0$)
 (a) ab (b) $(\frac{a}{b})^4$ (c) $(ab)^0$ (d) $\frac{a}{b}$
- 9 If $x = -\frac{1}{2}$ and $y = 3$, then $x^y = \dots\dots\dots$
 (a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$
- 10 If $y^{26} + y^{27} = 0$, then $y = \dots\dots\dots$
 (a) 1 (b) -1 (c) 2 (d) -2

5 Complete the following :

1 $\frac{8}{27} = (\frac{2}{3})^{\dots\dots\dots}$

3 $-\frac{64}{125} = (-\frac{4}{5})^{\dots\dots\dots}$

5 $0.027 = (\frac{3}{10})^{\dots\dots\dots}$


7 If $\frac{x}{y} = -\frac{2}{5}$, then $(\frac{x}{y})^3 = \dots\dots\dots$

2 $\frac{9}{16} = (\frac{3}{4})^{\dots\dots\dots}$

4 $2\frac{1}{4} = (\frac{3}{2})^{\dots\dots\dots}$

6 $64\% = (\frac{4}{5})^{\dots\dots\dots}$

8 If $x = \frac{1}{2}$ and $y = \frac{2}{3}$, then $x^2 y^2 = \dots\dots\dots$

9  $\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 = \dots\dots\dots$

10 $2^2 + 2^2 = 2^{\dots\dots\dots}$


11 $\frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \dots\dots\dots$ (in the same pattern)


12 The greater number of the two numbers $\left(\frac{1}{4}\right)^2$ and $\left(-\frac{8}{3}\right)^5$ is $\dots\dots\dots$

6 If $x = -\frac{2}{3}$ and $y = -\frac{1}{3}$, find the value of : $x^2 + y^3$ « $\frac{11}{27}$ »

7 If $a = \frac{2}{3}$ and $b = -\frac{4}{3}$, find the value of : $|a^3 \div b^3|$ « $\frac{1}{8}$ »

8 If $x = 0.5$, $y = -\frac{2}{3}$ and $z = -3$, find the value of : $9xy^2 - z^3$ « 29 »

9  If $a = -\frac{1}{2}$, $b = 2$ and $c = \frac{3}{4}$, find the numerical value of : $a^3b^2 + b^2c - 8abc$ « $8\frac{1}{2}$ »

10  If $x = -\frac{3}{2}$, $y = \frac{1}{2}$ and $z = -\frac{4}{3}$, find the numerical value of each of the following in its simplest form :

1 $x^2y^2z^2$

« 1 »

2 $x^2 \div z^2$

« $\frac{81}{64}$ »

3 $x^2 - yz^2$

« $\frac{49}{36}$ »

4 $\frac{x^2y^2z^2}{x+y}$

« -1 »

Geometric Application

11 If $V = l^3$ where V is the volume of a cube and l is its edge length, then calculate the volume of the cube whose edge length is $1\frac{1}{2}$ cm. « $\frac{27}{8} \text{ cm}^3$ »

For excellent pupils

12 Choose the correct answer from those given :

1 If $y = \left(\frac{1}{2}\right)^x$ where $x \in \{0, 1, 2, 3\}$, then y takes its maximum value when $x = \dots\dots\dots$

(a) 0

(b) 1

(c) 2

(d) 3

2 If $y = \left(-\frac{2}{5}\right)^x$ where $x \in \{0, 1, 3, 4\}$, then y takes its minimum value when $x = \dots\dots\dots$

(a) 0

(b) 1

(c) 3

(d) 4

13 Arrange the following numbers ascendingly without expanding :

$\left(\frac{2}{3}\right)^2, \left(-\frac{2}{3}\right)^3, \left(-\frac{1}{3}\right)^2, \left(-\frac{1}{3}\right)^3$

Exercise

2

Non-negative integer powers

From the school book

Remember

Understand

Apply

Problem Solving



Interactive test

1 Calculate each of the following, then put the result in the simplest form :

1 $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$

2 $\left(-\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$

3 $\frac{1}{5} \times \left(-\frac{1}{5}\right)^4$

4 $\left(\frac{1}{6}\right)^9 \div \left(\frac{1}{6}\right)^8$

5 $\left(\frac{2}{7}\right)^5 \div \left(\frac{2}{7}\right)^3$

6 $\left(-\frac{3}{5}\right)^7 \div \left(\frac{3}{5}\right)^5$

7 $\left(-\frac{5}{2}\right)^2 \div 2\frac{1}{2}$

8 $\left(\frac{1}{2}\right)^2 \times \frac{1}{2} \times \left(\frac{1}{2}\right)^3$

9 $\left(\frac{4}{5}\right)^8 \div \left(\frac{4}{5}\right)^6 \times \frac{4}{5}$

2 Calculate each of the following, then put the result in the simplest form :

1 $\frac{3^7 \times 3^3}{3^6}$

2 $\frac{2^6 \times 2}{2^3 \times 2^4}$

3 $\frac{(-5)^4 \times 5^2}{5^3}$

4 $\frac{(-2)^5 \times 2^4}{(-2)^3 \times 2^2}$

5 $\frac{(-3)^5 \times (-2)^7}{(-3)^3 \times (-2)^5}$

6 $\frac{x^4 \times y^3 \times x^5}{x^6 \times y^2}$

3 Find each of the following in the simplest form :

1 $\left(\frac{a b}{c}\right)^5$

2 $\left(\frac{5 x}{3 y}\right)^2$

3 $\left(-\frac{2 a b}{3 c}\right)^4$

4 $\left(\frac{x^2}{y^3}\right)^2$

5 $\left(\frac{a^3 b^2}{c^5}\right)^3$

6 $\left(-\frac{c^2}{d}\right)^3$

7 $\left(-\frac{x^3}{y^2}\right)^2$

8 $\frac{(4 x^3 y^2)^7}{(2 x^2 y)^7}$

9 $\frac{(2 a)^3 \times (2 a)^4}{(-2 a)^6 \times a}$

4 Calculate each of the following, then put the result in the simplest form :

1 $\left[\left(\frac{1}{2}\right)^2\right]^2$

2 $\left[(-\frac{3}{2})^2\right]^5$

3 $\left[(2\frac{1}{2})^3\right]^2$

4 $(\frac{3}{5})^{10} \times (\frac{5}{3})^{10}$

5 $\left((\frac{2}{7})^2\right)^3 \times (\frac{7}{2})^6$

6 $(2\frac{1}{2})^2 \times (-\frac{2}{5})^2$

5 Choose the correct answer from those given :

1 $3^2 \times 3^5 = \dots\dots\dots$

(a) 3^7

(b) 3^3

(c) 3^{10}

(d) 3^{25}

2 $5^2 + 5^2 = \dots\dots\dots$

(a) 10^2

(b) 10^4

(c) 5^4

(d) 50

3 $3^5 \times 2^5 = \dots\dots\dots$

(a) 5^{10}

(b) 6^{10}

(c) 6^5

(d) 6^{25}

4 $(5a)^0 = \dots\dots\dots$, $a \neq 0$

(a) 5

(b) a

(c) 5 a

(d) 1

5 $3^{(2^3)} = \dots\dots\dots$

(a) 3^6

(b) 3^5

(c) 3^8

(d) 3^{23}

6 $(5^2)^3 = \dots\dots\dots$

(a) 5^6

(b) 5^5

(c) 5^{23}

(d) 5

7 $3^{10} + 3^{10} + 3^{10} = \dots\dots\dots$

(a) 3^{10}

(b) 3^{30}

(c) 9^{10}

(d) 3^{11}

8 $4^x + 4^x + 4^x + 4^x = \dots\dots\dots$

(a) 4^{x+4}

(b) $4^4 x$

(c) 4^{x+1}

(d) $4 x^4$

9 $\frac{(3^2)^5}{(3^5)^2} = \dots\dots\dots$

(a) 3^{10}

(b) 3^{52}

(c) 3^{25}

(d) 1

10 $\frac{(x^2)^3}{x^3} = \dots\dots\dots$, $x \neq 0$

(a) x^6

(b) x^2

(c) x^3

(d) x

11 $(2y)^3 = \dots\dots\dots$

(a) $2y^3$

(b) $8y$

(c) $8y^3$

(d) $23y$

12 $(b^3)^4 = \dots\dots\dots$

(a) b^{34}

(b) b^7

(c) $b^3 \times b^3 \times b^3$

(d) $b^4 \times b^4 \times b^4$

13 The quarter of the number $4^{20} = \dots\dots\dots$

(a) 4^5

(b) 4^{10}

(c) 4^{19}

(d) 2^{10}

6 Simplify to the simplest form :

$\frac{(2y)^4 \times (3y)^2}{12y^5}$, then find the value of the result at $y = -\frac{1}{6}$

« -2 »

7 If $a = \frac{5}{3}$, $b = -\frac{3}{2}$ and $c = \frac{2}{5}$, find the numerical value of each of :

1 $\frac{(a^2 c^2)^2}{b}$

2 $\left(\frac{2ab}{5c}\right)^3$

« $-\frac{32}{243}$, $-\frac{125}{8}$ »

8 If $x = -\frac{1}{2}$, $y = \frac{3}{4}$ and $z = -\frac{3}{2}$,

find the numerical value of each of the following in the simplest form :

1 $x^3 y^2$

2 $y^3 x^2$

3 $\frac{x^3}{y^2 z^2}$

« $-\frac{9}{128}$, $\frac{27}{256}$, $-\frac{8}{81}$ »

9 Complete the following :

1 $\left(\left(\frac{7}{9}\right)^3\right)^4 = \frac{7^{12}}{3^{\dots\dots\dots}}$

2 If $\left(\frac{3}{4}\right)^5 \times x = \left(\frac{3}{4}\right)^7$, then $x = \dots\dots\dots$

3 The greater number of the two numbers $((-3)^5)^3$ and $((-3)^2)^4$ is $\dots\dots\dots$

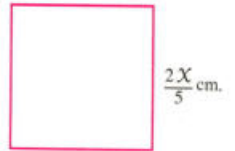
4 $((-1)^5)^2 - ((-1)^3)^2 = \dots\dots\dots$

5 $\frac{4^4}{4^3} + \frac{4^3}{4^2} + \frac{4^2}{4} + 4 = 2 \dots\dots\dots$

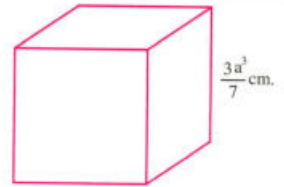
6 $2^{2x} \times 4^x = 4 \dots\dots\dots$

Geometric Applications

- 10 Find the area of the square whose side length is $\frac{2x}{5}$ cm.



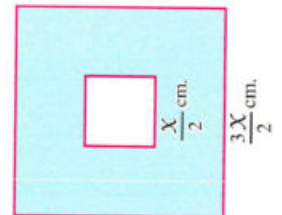
- 11 Find the volume of the cube whose edge length is $\frac{3a^3}{7}$ cm.



- 12 In the opposite figure :

A square is drawn inside another square.

Find the area of the shaded part.



For excellent pupils

- 13 If four times a number is 4^3 , find $\frac{3}{4}$ this number.

« 12 »

- 14 If $x = \frac{1}{5}$ and $y = 5$, find the value of : $x^{15} y^{14}$

« $\frac{1}{5}$ »

- 15 Prove that :

- 1 $5^{x+2} - 5^{x+1} = 20 \times 5^x$
- 2 $3^{15} + 3^{14}$ is divisible by 4

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each exercise



Exercise

3

Negative
integer
powers

From the school book



Remember

Understand

Apply

Problem Solving



Interactive test

1 Evaluate each of the following :

1 4^{-1}

2 5^{-2}

3 $\left(\frac{1}{2}\right)^{-1}$

4 $\left(-\frac{2}{3}\right)^{-2}$

5 $(0.2)^{-2}$

6 $(1.2)^{-1}$

2 Evaluate each of the following :

1 $3^7 \times 3^{-3}$

2 $2^{-2} \times 2^{-3}$

3 $\frac{3}{3^{-2}}$

4 $\frac{6^{-2}}{6^{-3}}$

3 Evaluate each of the following :

1 $(5^{-1})^{-3}$

2 $(3^{-2})^2$

3 $(0.25)^{-2}$

4 $(2^{-1} \times 2^{-2})^3$

5 $\left(\frac{3^{-1}}{3}\right)^2$

6 $\left(\frac{8^4}{8^{-4}}\right)^0$

4 Evaluate each of the following :

1 $\frac{8 \times 8^{-2}}{8^{-3}}$

2 $\frac{7^{-2} \times 7^5}{7^3}$

3 $\frac{2^5 \times 2^{-2}}{2^{-4} \times 2^3}$

4 $\frac{2^3 \times 2^{-3}}{(2^2)^2}$

5 $\frac{(3^{-2})^3}{3^{-2} \times 3^{-6}}$

6 $\left(\frac{9^3 \times 9}{9^5}\right)^{-3}$

7 $\left(\frac{2^5 \times 3^2}{3^4 \times 2^3}\right)^{-1}$

8 $(3^0 \times 2^{-2})^{-2}$

9 $\frac{(10)^2 \times (0.01)^3}{(10)^{-3}}$

- 5** Simplify each of the following and write the result in terms of positive exponents, where the denominator does not equal zero :

1 $7x^{-1}$

2 $x^{-1}y^2$

3 $a^{-2}b^{-3}$

4 $x^3 \times x^{-5}$

5 $x^3 \times x^{-2} \times x^{-1}$

6 $\frac{c^{-5}}{c^2}$

7 $(a^{-2})^3$

8 $(b^{-1})^{-3}$

9 $(a^2 \times a^{-5})^2$

10 $(x^2)^{-3} \times (x^{-3})^{-2}$

11 $\left(\frac{y^5}{y^{-2}}\right)^{-3}$

12 $\frac{x^2 \times x^{-3}}{x^{-4} \times x}$

13 $\frac{(x^2)^{-3} \times (x^{-1})^2}{x^{-3} \times x^{-4}}$

14 $\frac{a^{-1}}{b^2} \left(\frac{a^{-1}}{2b^2}\right)^{-2}$

15 $(x + x^{-1})^2$

- 6** Complete the following :

1 $2^{-3} \times c^0 = \dots\dots\dots$

2 $(b^{-1})^{-3} = b^{\dots\dots\dots}$

3 $2x^{-3} = \frac{2}{\dots\dots\dots}$

4 $(3x^{-1})^2 = 9x^{\dots\dots\dots} = \frac{9}{\dots\dots\dots}$

5 $(3y^{-2})^{-2} = \dots\dots\dots$

6 $(3a^2)^{-1} = \frac{1}{\dots\dots\dots}$

7 $2x^{-2}y^{-3} = \frac{2}{\dots\dots\dots}$

8 $\frac{x^{-5}}{y^{-5}} = (\dots\dots\dots)^5$

9 $\left(\frac{1}{2}\right)^2 + 2^0 - (2)^{-2} = \dots\dots\dots$

10 $(x^2)^{\dots\dots\dots} = \frac{1}{x^4}$

11 $2^{10} \times 2^{-10} = 3^{\dots\dots\dots}$

12 $a^{-5} + 1 = a^{-5} (\dots\dots\dots + \dots\dots\dots)$, where $a \neq 0$

13 If $x = \frac{1}{2}$, $y = \frac{1}{4}$, then $(x - y)^{-1} = \dots\dots\dots$

- 7** Choose the correct answer from those given :

1 If $a^{-1} = \frac{2}{3}$, then $a = \dots\dots\dots$

(a) $-\frac{2}{3}$

(b) $\frac{3}{2}$

(c) $-\frac{3}{2}$

(d) 1

2 If $a = 7^x$ and $b = 7^{-x}$, then $a \times b = \dots\dots\dots$

(a) 7^{2x}

(b) 49^{2x}

(c) 1

(d) 0

3 $\frac{5^x}{5^{-y}} = \dots\dots\dots$

(a) $5^{x \div y}$

(b) 5^{x-y}

(c) 5^{x+y}

(d) $-\frac{x}{y}$

4 $\frac{6a^2x^4}{2a^3x^3} = \dots\dots\dots$

(a) $3ax$

(b) $3a^5x^7$

(c) $\frac{3x}{a}$

(d) $\frac{3}{ax}$


5 $\frac{(-2st^2)^3}{(-4st^2)^2} = \dots\dots\dots$

(a) $-\frac{s^3}{2t}$

(b) $-\frac{s^4}{2t}$

(c) $\frac{s^5}{2t^2}$

(d) $\frac{s^4}{t}$


6  $\left(\frac{m^2}{n^{-3}}\right)^{-1} \left(\frac{3m^{-2}}{n^{-2}}\right)^{-2} = \dots\dots\dots$

(a) $\frac{9m^2}{n^7}$

(b) $\frac{m^2}{9n^7}$

(c) $\frac{m^2}{9n}$

(d) $\frac{9m^6}{n}$

7  $\frac{(2ab^{-2})^0}{3^0 a^{-2} b} = \dots\dots\dots$

(a) $\frac{a^3}{3b^3}$

(b) a^2

(c) 1

(d) $\frac{a^2}{b}$

8 If $a^x = 2$ and $a^{-y} = 3$, then $a^{x-y} = \dots\dots\dots$

(a) 1

(b) -1

(c) $\frac{2}{3}$

(d) 6

9 If $xy^{-1} = \frac{1}{2}$, then $\frac{y}{x} = \dots\dots\dots$

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) 1

(d) 2

10 $3^{-1} + 3^{-1} + 3^{-1} = \dots\dots\dots$

(a) 3^{-3}

(b) 3^3

(c) 9^{-3}

(d) 1

11 The multiplicative inverse of 5^{-1} is $\dots\dots\dots$

(a) $\frac{1}{5}$

(b) 5

(c) -5

(d) $-\frac{1}{5}$

12 $\left(\frac{3}{5}\right)^2 \times \left(\frac{5}{3}\right)^{-2} = \dots\dots\dots$

(a) $\left(\frac{3}{5}\right)^4$

(b) 1

(c) $\left(\frac{3}{5}\right)^{-4}$

(d) 0

8 Complete each of the following by the suitable sign of ($>$), ($<$) or ($=$):

1 $2^{10} \dots\dots\dots 2^{-10}$


2 $3^{-20} \dots\dots\dots 3^2$

3 $5^{-15} \dots\dots\dots 2^{-15}$

4 $(-7)^{-2} \dots\dots\dots (-7)^{19}$

5 $(-1)^{-6} \dots\dots\dots (-1)^{-9}$

6 $(-1)^{-20} \dots\dots\dots (1)^{-10}$

9  Why b^{-3} is not defined when $b = 0$?

10 Calculate the value of $\left(-\frac{3}{5}\right)^x \times \left(\frac{3}{5}\right)^y$ in each of the following cases:

1 $x = -2$ and $y = 2$

« 1 »

2 $x = -1$ and $y = 2$

« $-\frac{3}{5}$ »

11 If $x = -\frac{1}{3}$, $y = \frac{2}{3}$, then find in the simplest form the numerical value of the

expression: $\left(\frac{y}{x^2}\right)^{-2}$

« $\frac{1}{36}$ »

12 Simplify to the simplest form : $\frac{2^{10} \times 3^4}{(12)^5}$

« $\frac{1}{3}$ »

13 Simplify to the simplest form :

$\frac{6^{2n+1} \times 4^{-n}}{2^n \times 3^{2n+1}}$, then find the value of the result when $n = 3$

« $\frac{1}{4}$ »

Life Applications

14 The flea can jump at a height of 200 times of its length.

If a flea of length 2^{-4} inches can jump at a height of 2^3 inches

What does this height represent according to the length of the flea ?



15 The population of a city has been growing exponentially. It is estimated that in (t) years the population (p) will be : $p = 2 (1.03)^t$ million.

1 What will the population be in 2 years ?

2 What is the population now ?

3 What was the population last year ?



For excellent pupils

16 If $2^n = 3$, find the value of :

1 2^{n+1}

2 4^n

3 4^{-n}

4 2^{n-1}

« 6 , 9 , $\frac{1}{9}$, $\frac{3}{2}$ »

17 If $a = 5$ and $b = 5^{-1}$, find the value of : $a^{51} b^{50}$

« 5 »

18 Without expanding , arrange the following ascendingly by inspection :

$(-2)^{-15}$, $(-5)^{20}$, $(-2)^{15}$, 2^{-20} , $(-5)^{15}$, $(-2)^{20}$

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Interactive test

● Remember ● Understand ● Apply ● Problem Solving

9 -0.0003×10^3

6 58

6 – 300.501

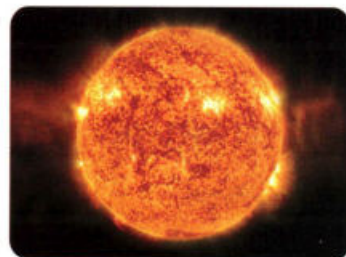


Hydrogen Atom

- 6 The light velocity is about 300 000 km./sec.

Express this velocity by m./sec.

in the standard form.



- 7 Dr. Ahmed Zewail discovered the femto second

which is a millionth of a miliardth of a second.

Express it by the standard form.



- 8 At witting the number 2.74×10^{20} as a whole number, find the number of zeroes that are on the right of the digit 4

- 9 Write the following numbers in the standard form :

1 68×10^5

3 720×10^6

5 -32.4×10^4

7 0.4×10^{-10}

9 0.0036×10^{-4}

2 68×10^{-5}

4 750×10^{-9}

6 -702.5×10^{-8}

8 0.0005×10^{15}

10 0.0020205×10^{12}

- 10 Put the suitable sign (<) or (>) :

1 6.4×10^3 4.6×10^3

3 0.0041 3.2×10^{-2}

5 2.10×10^{-5} 1.82×10^{-5}

7 6.920×10^5 96230

2 6.2×10^4 4.1×10^5

4 4370 3.41×10^4

6 9.1×10^{-4} 1.2×10^{-5}

8 3.69×10^{-4} 0.0000623

- 11 Arrange the following numbers in a descending order :

3.6×10^{-3} , 5.2×10^{-5} , 1×10^{-2} , 8.35×10^{-2} , 6.08×10^{-8}

12 Choose the correct answer from those given :

- 1 $3.04 \times 10^7 = \dots\dots\dots$
 (a) 340 000 (b) 304 000 (c) 3 400 000 (d) 30 400 000
- 2 $2.37 \times 10^{-4} = \dots\dots\dots$
 (a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237
- 3 If $0.00079 = 7.9 a$, then $a = \dots\dots\dots$
 (a) 10^3 (b) 10^{-3} (c) 10^{-4} (d) 10^4
- 4 If $0.0000503 = m \times 10^{-5}$, then $m = \dots\dots\dots$
 (a) 503 (b) 5.03 (c) 50.3 (d) 0.503
- 5 If the thickness of a sheet of paper is 0.012 cm., then a ream of 400 sheets is of height $\dots\dots\dots$
 (a) 48×10^{-3} cm. (b) 48×10^{-2} cm. (c) 4.8×10^0 cm. (d) 48 cm.
- 6 Which of the following equals $\frac{1}{2}$ milliard ?
 (a) 50×10^8 (b) 5×10^8 (c) 0.5×10^8 (d) 500×10^7
- 7 Which of the following is the greatest ?
 (a) 6.3×10^5 (b) 9.8×10^4 (c) 5.2×10^5 (d) 7.3×10^4
- 8 Which of the following is the smallest ?
 (a) 0.6×10^5 (b) 0.25×10^5 (c) 7×10^4 (d) 17.5×10^4
- 9 $6\,000 \times 50 = \dots\dots\dots$
 (a) 300×10^2 (b) 30×10^5 (c) 3×10^5 (d) 30×10^4
- 10 $45 \times 900 = \dots\dots\dots$
 (a) 4.05×10^2 (b) 4.05×10^3 (c) 4.05×10^4 (d) 45×10^2
- 11 $0.7 \times 0.005 = \dots\dots\dots$
 (a) 3.5×10^3 (b) 3.5×10^{-2} (c) 3.5×10^2 (d) 3.5×10^{-3}

13 Write the result of each of the following in the standard form :

1 $(6.4 \times 10^8) \times (1.5 \times 10^5)$

2 $(8.2 \times 10^7) \times (2.1 \times 10^{-4})$

3 $(5.02 \times 10^{-4}) \times (0.1 \times 10^{-3})$

4 $(4.4 \times 10^3) \times (2 \times 10)^5$

5 $(3.8 \times 10^8) \div (1.9 \times 10^6)$

6 $(125.5 \times 10^{-3}) \div (5 \times 10^4)$

7 $(8.8 \times 10^{25}) \div (8.8 \times 10^{22})$

8 $(5 \times 10)^4 \div (2.5 \times 10^{-3})$

14 Write the result of each of the following in the standard form :

1 $(3.8 \times 10^5) + (4.6 \times 10^4)$

2 $(4.54 \times 10^4) + (3.76 \times 10^3)$

3 $(5.3 \times 10^8) - (0.8 \times 10^7)$

4 $(2.65 \times 10^{-2}) - (6.34 \times 10^{-3})$

15 Write the result of each of the following in the standard form :

1 5000×3000

2 400×0.00007

3 $8000 \div 0.004$

4 $0.000033 \div 500$

5 $(20\,000)^3$

6 $(0.002)^2$

7 $(0.1)^{-8}$

16 Find the value of n in each of the following :

1 $800\,000 = 8 \times 10^n$

2 $0.000000006 = 6 \times 10^n$

3 $0.00052 = 5.2 \times 10^n$

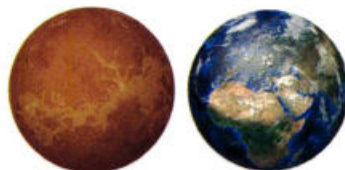
4 $0.000357 = 3.57 \times 10^n$

5 $(0.004)^2 = 1.6 \times 10^n$

6 $76293 = n \times 10^4$

Life Applications

- 17 If the diameter of the Earth is about 1.27×10^4 km. long and the length of the diameter of Mars is about 6.79×10^3 km. Which of the two planets is the greater and what is the difference between the two diameter lengths in the standard form ?



18 If light travels at a speed of 3×10^8 m/s. :

- 1 Calculate the distance from the Sun to the Earth if you know that the light of the Sun takes 8 minutes to reach the Earth.
- 2 If the distance between planet Venus and the Sun is 108 million kilometres , calculate the elapsed time (in minutes) that light takes to reach Venus from the Sun.



For excellent pupils

19 Find the result of the following in the standard form : $\frac{9.02 \times 10^3 + 4.98 \times 10^4}{2.5 \times 10^{-5}}$

20 Without using the calculator , write each of the following numbers in the standard form :

1 $10^{29} - 10^{28}$

2 $2^{19} \times 5^{15}$

21 If $X = 5 + (3 \times 10) + (4 \times 10^2) + (6 \times 10^3) + (9 \times 10^4) + (4 \times 10^5) + (2 \times 10^6)$

Write X in the standard form without using the calculator.



Exercise

5

Order of mathematical operations

From the school book



Interactive test

● Remember

● Understand

● Apply

● Problem Solving

1 Calculate the value of each of the following :

1 $3 + 12 \div 6$

2 $2 \times 6 - 4 \div 2$

3 $4 \times 7 - 3^2$

4 $4 \times 2^3 - 20$

5 $9 + 4 \times 3^2$

6 $144 - 8 \div 2^3$

2 Calculate the value of each of the following :

1 $196 \div (7 - 5)^2$

2 $18 \div (9 - 6) \times (1 + 2)$

3 $20 \div 5 + 8 - (4 - 1)$

4 $10 \times 4 - (2 \times 6 - 8)$

5 $(7 - 4) \times 2 \div (5 - 3)$

6 $(30 - 6) \div 6 \times 30 \div 3$

7 $7 (6^2 \div 2 \times 3)$

8 $12 (2^2) \div 24 + 3^2$

9 $9 (4)^2 \div 2^2 \times 3$

10 $9 \times 10 + 20 \div 2 - 3$

11 $6 \div \frac{1}{3} \times 3 - \frac{1}{3} \times 9$

12 $8 \times \frac{1}{2} \div 2 - 3 \div \frac{1}{5}$

3 Calculate the value of each of the following :

1 $2 - [(7 - 3) - 2]$

2 $[4 - (5 - 2)] - 1$

3 $3 + [5 + 2 (8 \div 4)]$

4 $2^3 + [4 + (2 - 1)]$

5 $[(2 + 23 - 7) \times 2] \div 4$

6 $10 \times 3 \div [4 - (9 - 8)]$

7 $2 + 3 [4 + (6 \times 3 - 8)] \times 2$

8 $2 [(5^2 + 1) - (4^2 - 1)]$

9 $7 - [10 - (-8)] - 3$

10 $[(11 - (-10)) \times 2 \div (-6)]$

4 Calculate the value of each of the following :

1 $\frac{15 + 7}{15 - 4}$

2 $\frac{8 + 20 - 4}{8 - 4}$

3 $\frac{-4 \times (-10)}{-9 + 7}$

4 $\frac{1+15}{8-(2-2)}$

5 $\frac{11-(5-4)}{1+4}$

6 $(3-1)^3 + \frac{7 \times 3}{-1-6} - \frac{2 \times 15}{6}$

7 $\frac{5^2 - 5 \times 2}{(15+3) \div 6}$

8 $\frac{5+2 \times 5}{2^2+1} + 5^2 - 5$

9 $\frac{3^2 \times 6 \div 3}{2 \times 1 + (3+1)^2}$

5 If $X = 3$, what is the numerical value of the expression : $2 \left(\frac{5X+3}{4X-3} \right)$? « 4 »

6 Evaluate the expressions when $t = 2$ and $s = 5$:

1 $(t+s)^2$

2 $(s-t)^3$

3 $\left(\frac{s}{t} \right)^3$

4 $\frac{6^2}{s-1}$

5 $\frac{s-t}{s^3}$

6 $\frac{12}{4s^2}$

7 Evaluate : $16t \div (4s) + 3st$, for $t = 9$ and $s = 6$ « 168 »

8 If $X = 3(5+7) - 4$ and $y = 4(8+2) \div 5$,
find the numerical value of the expression : $X - 4y$ « zero »

9 If $X = 18 - 4 \times 2 \div 2 + 1$ and $y = 8 + 9 \times 3 - 4^2 + 11$,
find the numerical value of the expression : $\left(\frac{y}{X} \right)^{-3}$ « $\frac{1}{8}$ »

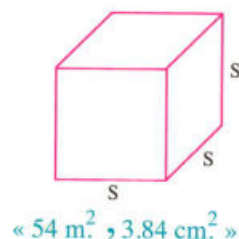
Geometric Applications

10 In the opposite figure :

The total area of a cube is $T = 6s^2$, find T when :

1 $s = 3$ m.

2 $s = 0.8$ cm.

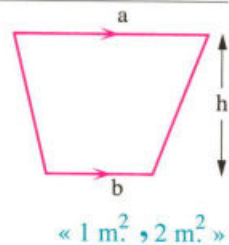


11 In the opposite figure :

The area of a trapezium is $A = \frac{1}{2}h(a+b)$, find A when :

1 $h = 2$ metres, $a = \frac{3}{4}$ metre and $b = \frac{1}{4}$ metre.

2 $h = 4$ metres, $a = \frac{1}{2}$ metre and $b = \frac{1}{2}$ metre.



For excellent pupils

12 Put the parentheses in the place to make each of the following equalities true :

1 $3 + 96 \div 12 \times 4 = 5$

2 $3 + 96 \div 12 \times 4 = 35$

3 $3 + 96 \div 12 \times 4 = 33$

Exercise

6

The square root of a perfect square rational number

From the school book



Interactive test

Remember Understand Apply Problem Solving

1 Find each of the following :

1 $\sqrt{16}$

2 $-\sqrt{25}$

3 $\pm\sqrt{2500}$

4 $\pm\sqrt{40000}$

5 $\sqrt{\frac{9}{49}}$

6 $-\sqrt{\frac{64}{25}}$

7 $\sqrt{0.81}$

8 $\pm\sqrt{1.44}$

9 $\sqrt{6\frac{1}{4}}$

10 $-\sqrt{1\frac{11}{25}}$

11 $-\sqrt{4^2}$

12 $\pm\sqrt{8^2}$

13 $\sqrt{(\frac{81}{100})^2}$

14 $\sqrt{(-\frac{3}{4})^2}$

15 $\pm\sqrt{\frac{576}{1225}}$

16 $-\sqrt{\frac{2.5}{40}}$

17 $-\sqrt{\frac{49a^4}{25b^6}}$

18 $\pm\sqrt{\frac{16b^8}{121h^2}}$

19 $\sqrt{\frac{49a^4b^2}{9}}$

20 $\sqrt{\frac{25x^2y^2}{36}}$

2 Find the two square roots of each of the following numbers :

1 64

2 144

3 $6\frac{1}{4}$

4 0.25

3 Find each of the following :

1 $\sqrt{9} + \sqrt{16}$

2 $\sqrt{36 + 64}$

3 $-\sqrt{225 - 81}$

4 $\sqrt{3^2 + 4^2}$

5 $-\sqrt{(10)^2 - 8^2}$

6 $\sqrt{\frac{9}{16} + 1}$

7 $\sqrt{\frac{5^4 \times 5^3}{5^5}}$

8 $\sqrt{(\frac{1}{2})^4 \times (\frac{1}{3})^4}$

9 $\sqrt{(\frac{1}{4})^2 \times (\frac{1}{4})^3}$

4 Complete the following :

1 $\frac{3}{4} \times \sqrt{\frac{16}{9}} = \dots\dots\dots$


2 $\sqrt{\frac{81}{49}} \times \frac{14}{27} = \dots\dots\dots$

3 $\sqrt{\frac{9}{4}} - \frac{3}{2} + \left(\frac{3}{2}\right)^{\text{zero}} = \dots\dots\dots$

4 $\sqrt{36} + \sqrt{16} = \sqrt{\dots\dots\dots}$

5 The multiplicative inverse of $\sqrt{\frac{4}{25}}$ in the simplest form equals $\dots\dots\dots$

6 The multiplicative inverse of $\sqrt{0.49}$ in the simplest form equals $\dots\dots\dots$

7  The multiplicative inverse of the rational number $\sqrt{\frac{10}{2.5}}$ equals $\dots\dots\dots$

8 The additive inverse of the number $-\sqrt{\frac{9}{16}}$ in the simplest form equals $\dots\dots\dots$

9 The rational number $6\frac{1}{4}$ in the form $\left(\frac{a}{b}\right)^2$ is $\dots\dots\dots$

10 $\sqrt{(-3)^2} = \dots\dots\dots$

11 $\sqrt{a^4b^8} = \dots\dots\dots$

12 If $a = -\frac{1}{2}$ and $b = -\frac{9}{8}$, then $\sqrt{ab} = \dots\dots\dots$

13 If $2x = \sqrt{36}$, then $x = \dots\dots\dots$

14 If $a = 0.000625$, then $\sqrt{a} = 2.5 \times 10^{\dots\dots\dots}$

5 Choose the correct answer from those given :


1 $\sqrt{1\frac{9}{16}} = \dots\dots\dots$

(a) $1\frac{3}{4}$

(b) $-1\frac{3}{4}$

(c) $1\frac{1}{4}$

(d) $-1\frac{1}{4}$

2  $\sqrt{10^2 - 6^2} = \dots\dots\dots$

(a) 4

(b) 8

(c) ± 4

(d) ± 8

3 $\sqrt{18 \times 10 \times 10 \times 18} = \dots\dots\dots$

(a) 18

(b) 180

(c) 10

(d) 100

4 $\sqrt{\sqrt{81}} = \dots\dots\dots$

(a) 81

(b) 27

(c) 9

(d) 3

5 $\sqrt{2^2 + \sqrt{25}} = \dots\dots\dots$

(a) 3

(b) -3

(c) 9

(d) -9

- 6 If $\frac{x}{2} = \frac{8}{x}$, then $x = \dots\dots\dots$
 (a) 4 (b) -4 (c) ± 4 (d) 16
- 7 If $x = \sqrt{\frac{1}{4}}$, then $x^3 = \dots\dots\dots$
 (a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{64}$
- 8 $\sqrt{(a+b)^3(a+b)} = \dots\dots\dots$
 (a) $(a+b)^2$ (b) $a^4 + b^4$ (c) $-(a+b)^2$ (d) $\pm(a+b)^2$
- 9 $\sqrt{1} + \sqrt{4} + \sqrt{9} + \sqrt{16} + \sqrt{25} + \sqrt{36} + \sqrt{49} + \sqrt{64} = \dots\dots\dots$
 (a) 6 (b) $\sqrt{204}$ (c) $\sqrt{81}$ (d) 6^2
- 10 The side length of the square whose area is $16x^2 \text{ cm}^2$ equals $\dots\dots\dots$ cm.
 (a) $8x$ (b) $|4x|$ (c) $2x$ (d) $8x^2$

6 Simplify each of the following to the simplest form :

1 $\sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^{\text{zero}} \times \left(-\frac{2}{7}\right)^2$

2 $\frac{2}{5} \times \sqrt{\frac{9}{16}} \div \left(-\frac{1}{2}\right)^3$

3 $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{4}\right)^{\text{zero}}$

4 $\frac{3}{4} \times \left(-\frac{2}{3}\right)^3 \times \left(\frac{3}{\sqrt{4}}\right)^2$

7 Simplify each of the following to the simplest form :

1 $\sqrt{16} + \sqrt{25}$

2 $\sqrt{\sqrt{16} + \sqrt{25}}$

3 $\sqrt{(\sqrt{16} + \sqrt{25})^2}$

8 Find two rational numbers lying between : $\sqrt{\frac{4}{9}}$ and $\frac{3}{4}$

9 Find each of the following :

1 $\sqrt{5^2 - 2 \times 5 + 1}$

2 $\sqrt{\left(\frac{1}{4}\right)^2 - 2 \times \frac{1}{4} + 1}$

3 $\sqrt{20 \div 5 + 8 - (4 - 1)}$

4 $\sqrt{8 \times (5 + 11) \div (2 + 6)}$

Geometric Applications

- 10 1 \overline{XY} is a line segment where $(XY)^2 = 25 \text{ cm}^2$, E is the midpoint of \overline{XY}

Find the length of : \overline{XE}

« 2.5 cm. »


- 2  If $(AB)^2 = 144 \text{ cm}^2$, $(BC)^2 = 625 \text{ cm}^2$ and $B \in \overline{AC}$

Find the length of : \overline{AC}

« 37 cm. »

- 3 The area of a square is 0.49 cm^2 . Find its perimeter.

« 2.8 cm. »

- 4  The area of a square is equal to the area of a triangle with base = 9 cm. long and its height = 8 cm. Find the side length of the square.


« 6 cm. »

- 5 The area of a circle is 154 cm^2 . Calculate its radius length ($\pi = \frac{22}{7}$)

« 7 cm. »

- 6 The area of a circle is 616 cm^2 . Calculate its circumference ($\pi = \frac{22}{7}$)

« 88 cm. »

- 7  If three quarters of the area of a square is $1\frac{11}{64} \text{ m}^2$

Calculate the side length of the square.

« $1\frac{1}{4} \text{ m.}$ »

- 8  The length of a rectangle is twice its width and its area is 24.5 cm^2

Calculate each of its width and length.

« 3.5 cm. , 7 cm. »




For excellent pupils

- 11 If a and b are the two square roots of c where $c \neq 0$, complete the following :

1 $a + b = \dots\dots\dots$

2 $\frac{a}{b} = \dots\dots\dots$

3 $a b + c = \dots\dots\dots$

- 12  If $\frac{m}{n}$ is a rational number and $\frac{m^2}{n^2} = 0.16$, find the value of $(\frac{m}{n})^3$

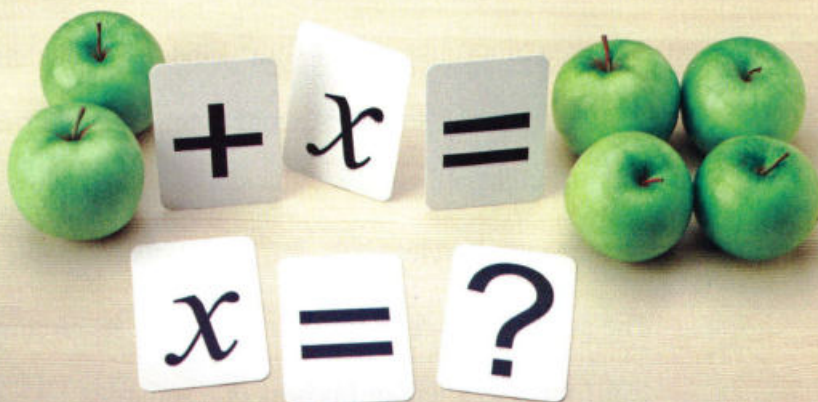
« ± 0.064 »

Exercise

7

Solving equations in \mathbb{Q}

From the school book



Interactive test

Remember

Understand

Apply

Problem Solving

1 Find the solution set of each of the following equations :

1 $x - 7 = 3$ where $x \in \mathbb{N}$

3 $5x = 20$ where $x \in \mathbb{Q}$

5 $-4 + y = 13$ where $y \in \mathbb{N}$

7 $x - 7 = 0$ where $x \in \mathbb{Z}$

9 $x - 6\frac{1}{4} = 12\frac{1}{2}$ where $x \in \mathbb{Q}$

2 $x + 17 = 13$ where $x \in \mathbb{N}$

4 $\frac{2}{5}x = \frac{1}{5}$ where $x \in \mathbb{Q}$

6 $m - (-3) = 1$ where $m \in \mathbb{Z}$

8 $y - (-5) = -3$ where $y \in \mathbb{Q}$

10 $8.91 + x = 11.09$ where $x \in \mathbb{Q}$

2 Solve each of the following equations :

1 $2x - 1 = 5$ where $x \in \mathbb{Q}$

3 $3x - 13 = 26$ where $x \in \mathbb{N}$

5 $8 + 2x = 14$ where $x \in \mathbb{Z}$

7 $8 - 2x = -2$ where $x \in \mathbb{Z}$

9 $2x + 3x + 25 = 5$ where $x \in \mathbb{Z}$

2 $8x + 4 = 12$ where $x \in \mathbb{Q}$

4 $2x + 14 = 14$ where $x \in \mathbb{N}$

6 $\frac{5}{6}x - 4 = 11$ where $x \in \mathbb{Q}$

8 $2 - 5x = 0$ where $x \in \mathbb{Q}$

10 $6x - 2x + 7 = 4$ where $x \in \mathbb{Z}$

3 Solve each of the following equations in \mathbb{Q} :

1 $2(X - 3) = 4$

3 $7(X - 2) - 3(X + 1) = 3$

5 $4(X - 1) - (X + 3) = 0$

7 $2(X - 3) + 3(X - 2) - 4X = -3$

2 $3X + 2(5X - 3) = 7$

4 $3(X + 2) + 7(X - 1) = 12$

6 $5(X - 2) + 2(X + 4) = -16$

8 $3y + 6(y + 3) - (8y - 16) = 60$

4 Find in \mathbb{Q} the solution set of each of the following equations :

1 $2X + 5 = X + 9$

3 $X + 3 = 18 - 3X$

5 $4(X + 1) = 2(X - 1)$

7 $a + 5a - 2 = 2(3 - a)$

9 $\frac{X+1}{3} = \frac{X-1}{4}$

2 $5X - 4 = 2X + 11$

4 $3X + 6 = 30 - 5X$

6 $3(X - 2) = 5X - 10$

8 $3(2X - 8) - (2X + 2) = X - 3$

10 $\frac{5}{4+4X} = \frac{3}{1-2X}$

5 Complete the following :

1 If $X + 5 = 7$, then $X = \dots\dots\dots$

2 If $3t = 6$, then the value of : $6t = \dots\dots\dots$

3 If $2X = 5$, then the value of : $4X = \dots\dots\dots$

4 If $X + 9 = 11$, then the value of : $7X = \dots\dots\dots$

5 If $2t + 3 = 15$, then the value of : $\frac{1}{3}t = \dots\dots\dots$

6 If $z - 1\frac{1}{4} = 5\frac{1}{2}$, then the value of : $4z - 18 = \dots\dots\dots$

7 If $\frac{p}{4} = \frac{2}{3}$, then the value of : $\frac{p}{2} = \dots\dots\dots$

8 If the age of a man now is X years, then his age 5 years ago is $\dots\dots\dots$

9 If the age of a man now is y years, then his age after 4 years is $\dots\dots\dots$

10 If the age of a man after 5 years is X years, then his age now is $\dots\dots\dots$

11 If the age of Youssef after 4 years is X years, then his age 2 years ago is $\dots\dots\dots$

12 A rectangle with length equals triple its width. If the length = X cm, then its width = $\dots\dots\dots$ cm.

13 The rectangle whose width = X cm. and its length is twice its width, then its perimeter = $\dots\dots\dots$ cm.

14 Two integers, their sum is 5, if one of them is X , then the other one is $\dots\dots\dots$

15 Two integers, the difference between them is 2, if the small one is X , then the great one is $\dots\dots\dots$

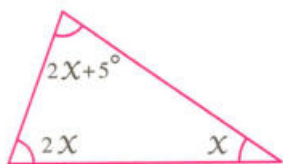
6 Choose the correct answer from those given :

- 1 If $2x = 2$, then $3x - 1 = \dots\dots\dots$
 (a) 2 (b) 3 (c) 4 (d) 5
- 2 If $2x = 0$, then $x = \dots\dots\dots$
 (a) 2 (b) 3 (c) 5 (d) zero
- 3 If $2ab = 10$, then $3ab = \dots\dots\dots$
 (a) 5 (b) 6 (c) 15 (d) 30
- 4 If $0.2 + a = 5$, then $\frac{a}{4} = \dots\dots\dots$
 (a) 4.8 (b) 1.3 (c) 1.2 (d) 19.2
- 5 If $5x + 8x + 2x + 4x = 114$, then $5x + 3 = \dots\dots\dots$
 (a) 33 (b) 35 (c) 47 (d) $8x$
- 6 The S.S. of the equation $\frac{2a}{3} = 8 + 4a$ in \mathbb{Q} is $\dots\dots\dots$
 (a) $\{-2.4\}$ (b) $\{2.4\}$ (c) $\{-3\frac{1}{3}\}$ (d) $\{0\}$
- 7 Which of the following equations is equivalent to the equation $x + 3 = 12$?
 (a) $x - 3 = -12$ (b) $x + (-3) = 12$
 (c) $x - (-3) = 12$ (d) $x - (-3) = -12$
- 8 Which of the following equations is equivalent to the equation $x - 12 = 15$?
 (a) $x + 12 = -15$ (b) $\frac{1}{3}x - 4 = 5$ (c) $x - 4 = -5$ (d) $x + 4 = 5$

Geometric Applications

7 Find the measure of each angle in each of the following triangles :

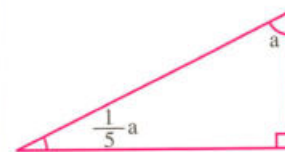
1



2



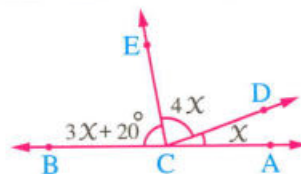
3



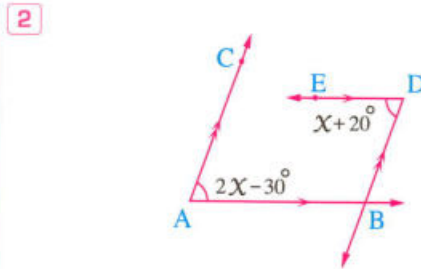
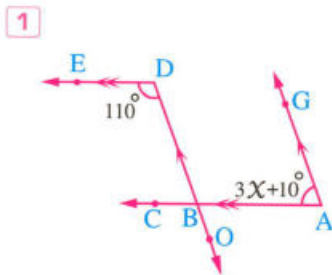
8 In the opposite figure :

If $C \in \overleftrightarrow{AB}$, find $m(\angle DCE)$

« 80° »



9 In each of the following figures find the value of X :



10 The length of a rectangle exceeds its width by 4 metres and its perimeter is 68 metres.

Find the dimensions of the rectangle.

« 19 m. , 15 m. »

11 The length of a rectangle is less than twice its width by 4 cm., if its perimeter equals the perimeter of a square of side length equals 7 cm.

Find the dimensions of the rectangle.

« 6 cm. , 8 cm. »

12 The length of a rectangle is twice its width. If the length decreases by 5 cm. and the width increases by 6 cm. , then the rectangle becomes a square.

Find the area of the rectangle.

« 242 cm.² »

Life Applications

13 Two integers , the smaller number is $2X$ and the greater number is $7X$, if the difference between them is 25 , find the two integers.

« 10 , 35 »

14 Two natural numbers, one of them is twice the other and their sum is 108

Find the two numbers.

« 36 , 72 »

15 The difference between two natural numbers is 5 and their sum is 21

What are the two numbers ?

« 13 , 8 »

16 Find the number which if added to its triple the result is 32

« 8 »

17 Find the number which if we subtract 9 from its triple , the result will be 6

« 5 »

18 Three consecutive natural numbers their sum is 213

What are these numbers ?

« 70 , 71 , 72 »

19 The sum of three consecutive even numbers is 966 , find them. « 320 , 322 , 324 »

20 Find three consecutive odd numbers if their sum is 357 « 117 , 119 , 121 »

21 A man's age now is three times his son's age and after two years , the sum of their ages will be 52 years. What is the age of each now ? « 12 years , 36 years »

22 Three brothers , Amgad , Bassim and Ayman , the sum of their ages is 89 years.
If Amgad was born before Bassim by 2 years and Bassim was born before Ayman by 6 years , what is the age of each of them now ? « 25 years , 31 years , 33 years »

23 The price of one metre of wool exceeds by 2 pounds than the price of one metre of silk. If the price of 3 metres of wool and 4 metres of silk is 671 pounds. Find the price of one metre of each kind. « 97 pounds , 95 pounds »

For excellent pupils

24 Find in \mathbb{Q} the S.S. of each of the following equations :

1 $5 - \frac{6}{x} = -1$

2 $-\frac{3}{5} + \frac{x}{10} = -\frac{1}{5} - \frac{x}{5}$

25 Find in \mathbb{Q} the S.S. of each of the following equations :

1 $(x+3)^2 - (x-2)^2 = 15$

2 $(2x+3)(2x-1) - (2x-1)^2 = 14$

26 If the S.S. of the equation $12x+3=39$ in \mathbb{Q} equals the S.S. of the equation

$a x - 12 = a$ in \mathbb{Q} , find the value of a

« 6 »

27 If $a+1$ is a solution of the equation $(x+a)(x-a) = x^2 - ax + 3$ in \mathbb{Q} ,
find the value of a

« 3 »

28 Three brothers were born in 1980 , 1984 and 1986 , the required is finding the year in which the sum of their ages became 41 years.

« 1997 »



Exercise

8

Solving inequalities in \mathbb{Q}

From the school book



Interactive test

Remember

Understand

Apply

Problem Solving

- 1 Which number would you add to each side of the inequality to obtain x in one side of it ?

1 $x + 5 > 9$

2 $x - 4 < 6$

3 $x - 7 < 3$

4 $x + 9 > 12$

5 $x - 1.5 \leq 3.2$

6 $4.8 \leq x + 0.6$

7 $1\frac{1}{2} > x - 2\frac{1}{2}$

8 $x + \frac{1}{3} > -\frac{1}{6}$

- 2 Find the solution set of the inequality $x + 3 \leq 6$ in each of the following cases :

1 $x \in \mathbb{Z}$

2 $x \in \mathbb{N}$

, then represent the solution set on the number line.

- 3 Find the solution set of each of the following inequalities in \mathbb{Q} :

1 $x + 2 > 5$

2 $x + 4 > 1$

3 $y - 5 > 7$

4 $19 < y + 14$

5 $-1 \geq x - 3$

6 $-5\frac{1}{2} > a + 1\frac{1}{4}$

7 $-2x < 12$

8 $\frac{2}{3}x \geq 1$

9 $-\frac{1}{4}x \leq \frac{1}{4}$

- 4 Solve each of the following inequalities in \mathbb{Q} :

1 $3x - 2 < 1$

2 $2x + 3 < 9$

3 $4x + 2 \geq -10$

4 $3x - 2 \geq 5$

5 $3x - 9 < 0$

6 $1 + 2x \leq -3$

7 $9 - 6x < 15$

8 $2 - 3x \leq 4$

9 $\frac{3x-2}{5} \geq \frac{1}{2}$

10 $8x - 3x + 1 \leq 29$

11 $4n - 2(n - 1) \geq 0$

12 $-3m + 6(m - 4) > 9$

5 Solve each of the following inequalities in \mathbb{Q} :

1 $6d + 1 \leq 5d - 3$

3 $3x - 2 < 5x - 8$

5 $5x + 1 \geq 2(x + 2)$

7 $3(x + 2) \geq -2(x + 1)$

9 $3(7y - \frac{1}{3}) \leq 20y - 1$

2 $6x + 2 \geq 14 + 5x$

4 $8 - 2x \leq 5x$

6 $3(x + 2) < -x + 4$

8 $2 - 3(x - 5) \geq x + 7$

10 $\frac{x}{2} + 3 \leq 2x + 1$

6 Find the S.S. of each of the following inequalities :

1 $9 \leq 4x + 1 \leq 17$, $x \in \mathbb{Z}$

3 $9 > x + 6 > 2$, $x \in \mathbb{N}$

2 $9 \leq 3x + 2 < 12$, $x \in \mathbb{Q}$

7 Complete :

1 If $x > y$, then $x + z \dots\dots y + z$

3 If $x < y$ and $y < z$, then $x < \dots\dots$

5 If $a - 3 < 0$, then $\dots\dots > \dots\dots$

7 If $b < 0$, then $b + 3 \dots\dots 3$

8 If $x > y$ and z is positive ($z > 0$) , then $xz \dots\dots yz$

9 If $x < y$ and z is negative ($z < 0$) , then $xz \dots\dots yz$

2 If $x < y$, then $x + z \dots\dots y + z$

4 If $z > y$ and $y > x$, then $z > \dots\dots$

6 If $a + 5 > 0$, then $\dots\dots > \dots\dots$

8 Choose the correct answer from those given :

1 If $-x < 5$, then $\dots\dots$

(a) $x > 5$

(b) $x > -5$

(c) $x < 5$

(d) $x < -5$

2 If $x \in \mathbb{N}$, then the S.S. of the inequality $-x > 3$ is $\dots\dots$

(a) $\{4, 5, \dots\}$

(b) $\{-4, -5, \dots\}$

(c) $\{-3\}$

(d) \emptyset

3 $\frac{x}{3} < 4$ is equivalent to $\dots\dots$

(a) $x > \frac{4}{3}$

(b) $x < \frac{4}{3}$

(c) $x > 12$

(d) $x < 12$

4 If $x \in \mathbb{Z}$, then the S.S. of the inequality $20 < 5x < 25$ is $\dots\dots$

(a) $\{4\}$

(b) $\{5\}$

(c) $\{4, 5\}$

(d) \emptyset

5 The S.S. of the inequality $-2x < \text{zero}$ in \mathbb{Q} is $\dots\dots$

(a) \emptyset

(b) \mathbb{Q}_+

(c) \mathbb{Q}_-

(d) \mathbb{Z}_+

6 The number of solutions of the inequality $\frac{1}{5} < x < \frac{2}{3}$, where $x \in \mathbb{Q}$ is $\dots\dots$

(a) zero

(b) 1

(c) 2

(d) an infinite number.

- 7 If $x > y$, then $\frac{1}{x}$ $\frac{1}{y}$, where $x \neq 0, y \neq 0$
 (a) $>$ (b) $<$ (c) $=$ (d) \geq
- 8 The number 2 belongs to the S.S. of the inequality where x is an integer.
 (a) $x > 2$ (b) $x < 2$ (c) $-x > -3$ (d) $-x > 3$
- 9 If $x > 5$, then $-x$
 (a) < -9 (b) ≥ -5 (c) < -5 (d) > -5

- 9 Show by using examples that if $a > b$ and $c > d$, then it is not always correct that $a - c > b - d$

- 10 Put (\checkmark) for the correct statement and (\times) for the incorrect statement, when a statement is false, give an example that shows why it is false (given that $x > y$):

- | | | | |
|------------------|--------------------|--------------------|------------------|
| 1 $y < x$ () | 2 $x > 0$ () | 3 $y^2 \geq 0$ () | 4 $y^2 > y$ () |
| 5 $xy > 0$ () | 6 $x + y > y$ () | 7 $y^2 > x$ () | 8 $y^2 < xy$ () |
| 9 $xy < x^2$ () | 10 $x^3 < y^2$ () | | |

Life Application

- 11 Hany wants to buy a pair of shoes and some shirts, if Hany has L.E. 200, the price of the pair of shoes is L.E. 70 and the price of one shirt is L.E. 40
 What is the greatest number of shirts Hany can buy?

« 3 »



For excellent pupils

- 12 If the S.S. of the inequality $a \leq 3x - 5 \leq b$ in \mathbb{Q} is $\{x : x \in \mathbb{Q}, 2 \leq x \leq 5\}$, find the values of a and b

« 1, 10 »

- 13 If $-4 \leq x \leq 5$ and $2 \leq y \leq 7$, where $x \in \mathbb{Q}$ and $y \in \mathbb{Q}$, find:

- | | |
|---|---------|
| 1 The greatest possible value of the expression $x + y$ | « 12 » |
| 2 The greatest possible value of the expression $y - x$ | « 11 » |
| 3 The smallest possible value of the expression xy | « -28 » |
| 4 The smallest possible value of the expression $x^2 + y^2$ | « 0 » |

2 Statistics and Probability

Exercises of the unit :

9. Samples :

- Systematic sample.
- Random sample.

10. Probability :

- Experimental probability.
- Theoretical probability.



Scan

the QR code
to solve an interactive
test on each
lesson





Exercise

9

Samples

 From the school book




● Remember

● Understand

● Apply

● Problem Solving

- 1**  A factory's canteen service wanted to find the preferences of their 427 employees during their 15 – minute break. Each employee was given a number from 1 to 427
A 10% sample of the 427 were to be surveyed and asked to select a preference from :

 - Hot beverage.
 - Cold drink with biscuits.
 - Hot soup with bread.
 - Fruit with fresh water.

The sample were determined by selecting 43 sample numbers in the range using calculator. Identify the sample numbers using a calculator.
- 2** A school makes a study about how the pupils come to school. If the number of pupils in the school is 320 , each pupil is given a number from 1 to 320 A sample of 10% from this number is selected as a sample to ask them how they come to school :

 - On foot
 - Public transport
 - Taxi
 - Bike
 - Private car

Determine the numbers of the sample using the calculator.
- 3** A company makes a study about the best places which the workers in the company prefer to spend their annual holiday among :

 - Port Said
 - Alexandria
 - Matrouh
 - The North Coast
 - Ismailia

If the number of the workers in the company is 250 workers and a sample of 10% from the number of workers is selected to make a survey on it , determine the numbers of the sample using the calculator.
- 4** It is noticed that 230 persons use a public bus daily and the public transport authority wanted to collect some informations concerning with the using daily of this service. It is necessary to form a random sample representing 10% from the users of this bus to make a survey on them. Determine the numbers of this sample using the calculator.

Exercise

10

Probability

From the school book

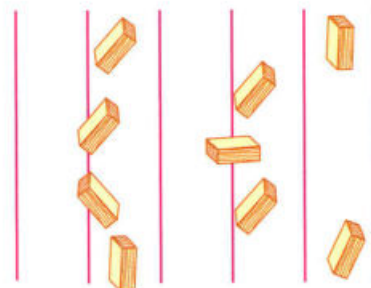
Remember Understand Apply Problem Solving



Interactive test

First Problems on experimental probability :

- 1 (a) Draw six parallel lines with a distance of 2 cm. between each of them on an A_4 sheet of paper.
 (b) Bring a piece of wood of length 2 cm.
 (c) Slightly toss the piece of wood in the air so that it falls from a suitable height onto the A_4 sheet.
 (d) Repeat the trial 50 times.
 (e) Record the number of times that the piece of wood falls across the line and also between the lines.
 (f) Deduce the probability of the piece of wood falling between the lines.



	Across	Between	Total
Tally			
Frequency			50

- 2 (a) Drop a drawing pin 100 times from a suitable height.
 (b) Record the number of times it lands with its point up and its point down :

	Up	Down	Total
Tally			
Frequency			100



- (c) Deduce the probability of the drawing pin landing point "UP" and point "Down"

Second Problems on theoretical probability :**1 As throwing a fair die and observing the upper face , complete the following :**

- 1 The probability of appearance a number greater than 2 =
- 2 The probability of appearance a number less than 3 =
- 3 The probability of appearance an even number =
- 4 The probability of appearance the number 4 =
- 5 The probability of appearance the number 7 =
- 6 The probability of appearance a number less than or equal to 6 =
- 7 The probability of appearance a prime number =
- 8 The probability of appearance a prime even number =
- 9 The probability of appearance a number divisible by 5 =
- 10 The probability of appearance the number 5 or the number 6 =

2 Complete the following :

- 1 The probability of occurring the impossible event = and the probability of occurring the certain event =
- 2 If a coin is flipped once , then the probability of appearance of a head =
- 3 10 cards numbered from 1 to 10. If a card is drawn randomly , then the probability that the card is numbered by an odd number =
- 4 In the experiment of throwing a fair die once and observing the upper face , the probability that the apparent number is less than 1 =
- 5 A box contains 48 oranges and 4 oranges of them are bad. If an orange is drawn randomly , then the probability that the drawn orange is bad = and the probability that the drawn orange is good =
- 6 If the probability of occurring an event is $\frac{5}{8}$, then the probability that the event doesn't occur is
- 7 An activity room has 3 doors numbered from 1 to 3. If a student went out using one of them , then the probability that the student went out using the door number 2 is
- 8 If the probability that a person get infected (in a city whose number of inhabitants is 200000) with a disease is 0.003 , then the expected number of infected persons with the disease in this city is persons.

3 Choose the correct answer from those given :

- 1 Which of the following is the probability of occurrence of an event ?
(a) 1.2 (b) -0.4 (c) 315% (d) 75%
- 2 As throwing a fair die once, the probability of appearance of a number greater than 4 is
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1
- 3 A basket contains cards numbered from 1 to 20. If a card is drawn randomly, what is the probability that the number written on it is divisible by 6 ?
(a) $\frac{3}{20}$ (b) $\frac{4}{20}$ (c) $\frac{5}{20}$ (d) $\frac{6}{20}$
- 4 A bag has 5 red balls and 3 white balls. If the balls are similar and a person draws a ball randomly, then the probability that the drawn ball is white =
(a) $\frac{3}{5}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{5}{3}$
- 5 A letter is selected randomly from the name "ZAMALEK". The probability of selecting the letter A is
(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$
- 6 Rashad is in grade 7 in a class of 36 students. 16 of them are girls. If a student is selected randomly from the class, what is the probability that the student is a boy ?
(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) $\frac{5}{9}$ (d) $\frac{1}{36}$
- 7 A class has 25 boys and 20 girls. A pupil of them is selected randomly, then the probability that the pupil is a girl =
(a) $\frac{1}{20}$ (b) $\frac{4}{9}$ (c) $\frac{1}{25}$ (d) $\frac{5}{9}$
- 8 If the probability of success of a student is 70%, then the probability of his failure is
(a) 0.7 (b) 0.07 (c) 0.3 (d) 0.03

4 A card is drawn from a bag of 25 cards numbered from 1 to 25

Calculate the probability that the drawn card carries :

- 1 A number divisible by 5
- 2 A number ≥ 20
- 3 A perfect square number.

5 One card is selected randomly from 8 cards numbered from 1 to 8

Write down the sample space. Then find the probability of each of the following events :

- 1 Getting an even number.
- 2 Getting an odd number.
- 3 Getting a number greater than or equal to 6
- 4 Getting a number divisible by 3

6 A letter is selected randomly from the word "SAMEH"

Calculate the probability of selecting the letter :

- 1 S 2 E 3 R

7 A bag contains 5 red balls , 3 yellow balls and 2 black balls.

If all balls are alike and a ball is drawn from the bag randomly , find :

- 1 The probability that the drawn ball is yellow.
2 The probability that the drawn ball is yellow or red.
3 The probability that the drawn ball is not yellow.



8 A card is chosen randomly from ten cards numbered from 1 to 10

What is the probability that the chosen card shows :

- 1 An odd number. 2 A prime number.
3 An even number. 4 An odd number greater than 3

9 If a fair die is tossed once , what is the probability of each of the following events :

- 1 Appearance of an even number less than or equal to 4
2 Appearance of a number between zero and 10
3 Appearance of a number divisible by 7
4 Appearance of a number not divisible by 2



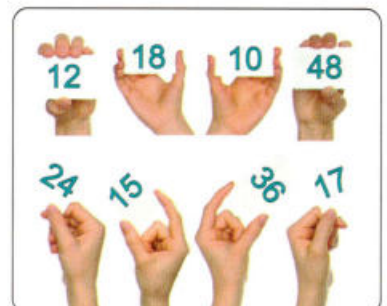
10 A fair die is rolled once and the number of dots on the upper face is observed. Write down the sample space , then find the probability of each of the following events :

- 1 Getting a number greater than 6
2 Getting a number satisfying the inequality : $1 \leq X \leq 6$
3 Getting a number satisfying the inequality : $2 < X < 4$



11 8 cards , numbered by the opposite numbers , are put in a bag. Bassim drew a card from these cards randomly. Find :

- 1 The probability that the card carries a number whose tens digit is even.
2 The probability that the card carries a number whose units digit is odd.
3 The probability that the card carries a number multiple of 4



- 12** A cube is designed such that each two opposite faces carry one of the digits 1, 2 and 3. The cube is rolled and the apparent face is observed :



- 1 Write down the sample space.
- 2 What is the probability such that the number on the upper face is 2 ?
- 3 What is the probability such that the number on the upper face is odd ?

- 13** A bag contains 30 similar marbles. Hani drew a marble randomly and he found it red. If the probability of drawing a red marble = $\frac{2}{5}$, find the number of red marbles in the bag.

- 14** A box contains 80 similar balls. Some of them are red and the rest is blue. If the probability of drawing a red ball is $\frac{1}{4}$, find the number of blue balls.

- 15** The set $\{2, 3, 5\}$ is used in writing a 2-digit number. Find the probability of each of the following events :

- | | |
|----------------------------------|---------------------------------------|
| 1 The tens digit is odd. | 2 The units digit is odd. |
| 3 The sum of the two digits is 7 | 4 The product of the two digits is 15 |

- 16** Wael has a bag containing 22 marbles, 12 of them are black and the rest is red. If two marbles of them are drawn without returning them to the bag and they were red. Then he drew a third marble without looking at it. What is the probability that the last marble is black ?

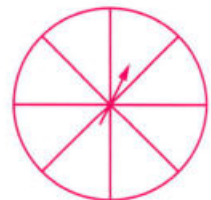
- 17** A class has 50 students. The number of girls is less than the number of boys by 10. If one student is chosen randomly, find the probability that the student is a boy.

- 18** Choose the correct answer from those given :

- 1 A bag contains 3 white balls, 2 black and one red. If a ball is drawn randomly from the bag, then the probability that the drawn ball is not black equals
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$
- 2 A bag has a number of similar balls. Half of them is red, the third is black and the rest is white. A ball is drawn randomly, then the probability that the drawn ball is white =
 (a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) zero

- 3 A box contains coloured balls (red, green, blue and yellow). If the box contains 20 yellow balls and the probability of drawing a yellow ball from the box randomly is $\frac{1}{4}$. What is the number of all balls in the box ?
 (a) 5 (b) 25 (c) 60 (d) 80
- 4 The number of pupils in a class (7 grade) is 36 pupils. The probability of choosing a pupil whose age is less than or equal to 13 is $\frac{1}{6}$. What is the number of pupils whose ages are more than 13 years old ?
 (a) 23 (b) 24 (c) 30 (d) 32
- 5 In a mixed school, if the ratio between the number of boys to the number of girls is 7 : 9 A student is selected randomly from the students of this school. The probability that the selected student is a boy equals
 (a) zero (b) $\frac{7}{16}$ (c) $\frac{9}{16}$ (d) 7
- 6 A small box contains 25 tickets numbered from 1 to 25 A large box contains 50 tickets numbered from 1 to 50 Without looking at them, a ticket is picked from one of the two boxes. Which box would give the larger chance of picking a ticket with the number 17 ?
 (a) The larger box. (b) The smaller box.
 (c) Both would give the same chance. (d) The given information is not enough.

- 19 The opposite spinning game is divided into 8 sectors of the same area. $\frac{1}{8}$ of the sectors is coloured in red, and $\frac{1}{4}$ of the sectors is coloured in green, $\frac{3}{8}$ of the sectors is coloured in blue and the rest in yellow.
 If the pointer of the spinner is spinned, what is the probability that the pointer stops at the yellow or the red colour ?



- 20 A class contains 40 students, 30 of them succeeded in maths, 24 succeeded in science and 20 succeeded in both. A student is chosen randomly.

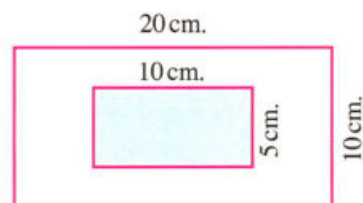
Find the probability that this student :

- | | |
|-----------------------|-------------------------------------|
| 1 Succeeded in maths. | 2 Succeeded in science. |
| 3 Failed in science. | 4 Failed in both maths and science. |

- 21** Two players play in a football team. During training , one of them shot 21 penalty kicks and he scored 18 goals and the other shot 32 penalty kicks and he scored 25 goals. Which of them should you choose to shoot a penalty kick ? Why ?
- 22** Maryam and Souad played together with two dice. If the product of the two apparent numbers on the upper face is even , then Souad wins the game.
If the product of those numbers is odd , then Maryam wins :
- 1** On your opinion , is this system of the game fair ? Why ?
 - 2** If it is not fair , determine which one of the two girls has the greater chance to win ? Why ?

23 In the opposite figure :

If a person shot towards the drawn board ,
find the probability of shooting the shaded part.



For excellent pupils

- 24** A bag contains a number of similar balls , 5 of them are white and the rest are red.
If the probability of drawing a red ball is $\frac{2}{3}$
Find the total number of balls.
- 25** A card is drawn from a group of cards numbered from 1 to n.
If the probability that the drawn card carries a number greater than 8 is $\frac{1}{3}$, then find the value of n

SKILLS

TIMSS Problems



Accumulative basic skills

1 Choose the correct answer from the given ones :

1 $3x^2 + 2x + 2x^2 - x = \dots\dots\dots$

- (a) $6x$ (b) $6x^2$ (c) $5x^2 + x$ (d) $7x^2 - x$

2 If $y = \frac{a+b}{c}$, $a = 8$, $b = -6$ and $c = -2$, then $y = \dots\dots\dots$

- (a) -1 (b) 1 (c) -7 (d) 7

3 At dividing $113 + 113 + 113 + 113$ by 4 , then the remainder = $\dots\dots\dots$

- (a) zero (b) 1 (c) 4 (d) 13

4 $4(3 + x) = \dots\dots\dots$

- (a) $12 + x$ (b) $7 + x$ (c) $12 + 4x$ (d) $12x$

5 $\frac{4}{10} + \frac{3}{100} = \dots\dots\dots$

- (a) 0.34 (b) 0.43 (c) 4.3 (d) 3.4

6 If three times of a number equals 27 , then $\frac{1}{9}$ of this number is $\dots\dots\dots$

- (a) 1 (b) 3 (c) 9 (d) 27

7 Which of the following is equal to $\frac{3}{5}$?

- (a) 6% (b) 60% (c) $\frac{9}{10}$ (d) 0.53

8 If the fractions $\frac{4}{14}$ and $\frac{x}{21}$ are equal, then $x = \dots\dots\dots$

- (a) 6 (b) 7 (c) 11 (d) 14

9 $2 \times 4 \times 6 \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{6} = \dots\dots\dots$

- (a) 48 (b) 2304 (c) 1 (d) zero

10 A worker cut a part of a pipe equals $\frac{1}{5}$ of this pipe, if the length of the cut part equals 3 m. , then the length of the pipe completely equals $\dots\dots\dots$

- (a) 8 m. (b) 12 m. (c) 15 m. (d) 18 m.

- 11 Which of the following does express the number 36 as the product of its prime factors ?
 (a) 6×6 (b) 4×9 (c) $4 \times 3 \times 3$ (d) $2 \times 2 \times 3 \times 3$
- 12 $5 \times 4 \times 3 \times 2 \times 1 \times 0 = \dots\dots\dots$
 (a) 120 (b) 60 (c) 20 (d) zero
- 13 Double of the square of the number (half) is $\dots\dots\dots$
 (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) 2
- 14 If the number of boys in one of the parties is m and the number of girls is n and each person has 2 balloons, then which of the following expressions does express the number of balloons in this party ?
 (a) $2(m + n)$ (b) $2 + (m + n)$ (c) $2m + n$ (d) $m + 2n$
- 15 The smallest number of the following numbers is $\dots\dots\dots$
 (a) 0.52 (b) 0.5 (c) 0.056 (d) 0.562

2 Complete the following :

- 1 $24.65 + 5.748 = \dots\dots\dots$
- 2 $-2\frac{3}{4} \div 1\frac{3}{8} = \dots\dots\dots$
- 3 Third of the third = $\dots\dots\dots$
- 4 $\frac{(19)^2 - 9 \times 19 + 19}{19} = \dots\dots\dots$
- 5 $\frac{3x}{8} + \frac{x}{4} + \frac{x}{2} = \dots\dots\dots$ (in the simplest form)
- 6 If $y = 100 - \frac{100}{1+m}$, at $m = 9$, then $y = \dots\dots\dots$
- 7 If $a + b = 25$, then $2a + 2b = \dots\dots\dots$
- 8 $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\dots\dots\dots$ (in the same pattern)
- 9 If $3y = 6$, then $5y = \dots\dots\dots$
- 10 If $\frac{1}{2}x = 5y = 10$, then $xy = \dots\dots\dots$
- 11 If $x + y = yx = 5$, then $x^2y + y^2x = \dots\dots\dots$
- 12 If $x - y = 3$, $x + y = 5$, then $x^2 - y^2 = \dots\dots\dots$
- 13 If $x^2 = 16$, $y^2 = 9$, $xy = 12$, then $(x - y)^2 = \dots\dots\dots$
- 14 If the degree of the algebraic term $5x^n y^2$ is 5, then $n = \dots\dots\dots$
- 15 A piece of wood of length 40 cm., it is cut into three parts of lengths $2x - 5$, $x + 7$ and $x + 6$ in centimetres, then the length of the longest part = $\dots\dots\dots$ cm.

Second | Geometry and Measurement

UNIT 3 Geometry and Measurement — 48

Accumulative Basic skills — 113
"TIMSS Problems"



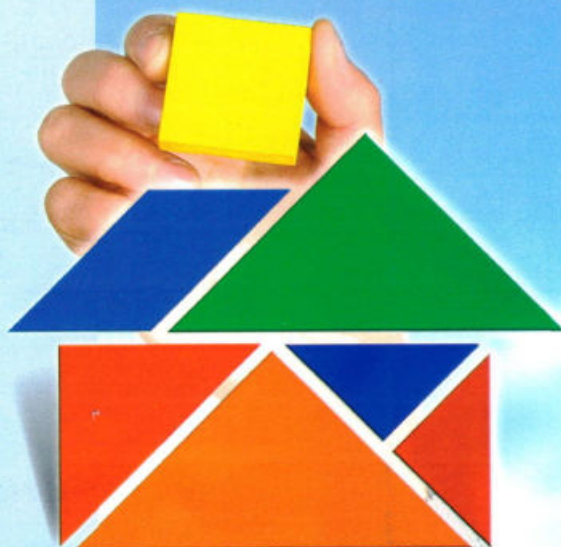
3 Geometry and Measurement

Exercises of the unit :

1. Deductive proof.
2. The polygon.
3. The parallelogram and its properties.
4. The special cases of the parallelogram.
5. The triangle : Theorem (1), exterior angle of the triangle.
6. Theorem (2) , theorem (3).
7. Pythagoras' theorem.
8. Geometric transformations.
9. Reflection in a straight line.
10. Reflection in a point.
11. Translation.
12. Rotation.

Scan

the QR code
to solve an interactive
test on each
lesson





Exercise

1

Deductive proof

From the school book



Remember

Understand

Apply

Problem Solving



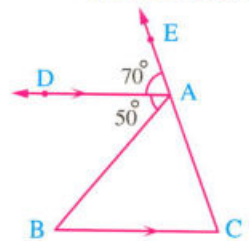
Interactive test

1 In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle DAB) = 50^\circ$ and $m(\angle DAE) = 70^\circ$

Find the measures of the angles of $\triangle ABC$

Complete the following table by writing the reason of each step of the solution steps :



Mathematical Statement

The reason

$m(\angle DAB) = 50^\circ$, $m(\angle DAE) = 70^\circ$

1

$m(\angle CAB) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$

2

$\overline{AD} \parallel \overline{BC}$

3

$m(\angle C) = m(\angle DAE) = 70^\circ$

4

$m(\angle B) = m(\angle DAB) = 50^\circ$

5

2 In the opposite figure :

$m(\angle AMB) = 50^\circ$, $m(\angle EMD) = 80^\circ$, \overline{MC} bisects $\angle BMD$ and $m(\angle CMD) = 65^\circ$

Complete the following proof to find $m(\angle AME)$

Given

R.T.F.

Proof

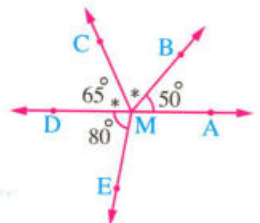
$\therefore \overline{MC}$ bisects \angle (given)

$\therefore m(\angle BMC) = m(\angle$ ) = $^\circ$

$\therefore m(\angle AMB) + m(\angle BMC) + m(\angle CMD) + m(\angle DME) + m(\angle AME) =$ $^\circ$

$\therefore m(\angle AME) =$ $^\circ -$ $^\circ =$ $^\circ$

(The req.)

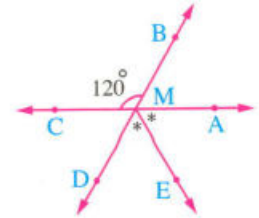


3 In the opposite figure :

$$\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}, m(\angle BMC) = 120^\circ$$

and \overrightarrow{ME} bisects $\angle AMD$

Complete the following proof to find $m(\angle EMC)$



Given

R.T.F.

Proof

$$\because \overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$$

$$\therefore m(\angle BMC) = m(\angle \dots\dots\dots) \text{ (V.O.A.)}$$

$$\therefore m(\angle \dots\dots\dots) = 120^\circ$$

$$\because \overrightarrow{ME} \text{ bisects } \angle AMD$$

$$\therefore m(\angle \dots\dots\dots) = m(\angle \dots\dots\dots)$$

$$\therefore m(\angle EMD) = \frac{\dots\dots\dots}{\dots\dots\dots} = \dots\dots\dots^\circ$$

$$\because M \in \overrightarrow{BD}$$

$$\therefore m(\angle BMC) + m(\angle \dots\dots\dots) = 180^\circ$$

$$\therefore m(\angle DMC) = \dots\dots\dots^\circ - \dots\dots\dots^\circ = \dots\dots\dots^\circ$$

$$\because m(\angle EMC) = m(\angle \dots\dots\dots) + m(\angle \dots\dots\dots)$$

$$\therefore m(\angle EMC) = \dots\dots\dots^\circ + \dots\dots\dots^\circ = \dots\dots\dots^\circ$$

(The req.)

4 In the opposite figure :

$$AB = AC, BD = CD$$

Complete the following proof to prove that \overrightarrow{AD} bisects $\angle BAC$

Given

R.T.P.

Proof

$$\because \text{In } \triangle ADB, \dots\dots\dots :$$

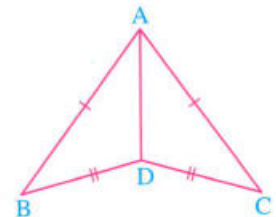
$$\begin{cases} AB = \dots\dots\dots & \text{(given)} \\ \dots\dots\dots = CD & \text{(given)} \\ \overline{AD} \dots\dots\dots \end{cases}$$

$$\therefore \triangle ADB \equiv \triangle \dots\dots\dots, \text{ then we deduce that :}$$

$$m(\angle \dots\dots\dots) = m(\angle \dots\dots\dots)$$

$$\therefore \overrightarrow{AD} \text{ bisects } \angle \dots\dots\dots$$

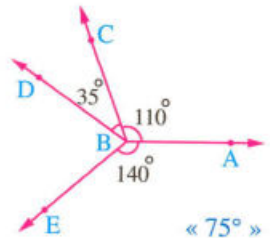
(Q.E.D.)



5 In the opposite figure :

$m(\angle ABC) = 110^\circ$, $m(\angle CBD) = 35^\circ$
and $m(\angle ABE) = 140^\circ$

Find : $m(\angle EBD)$



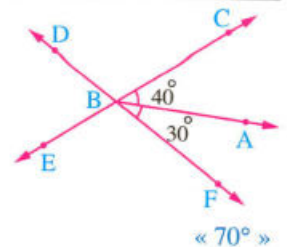
« 75° »

6 In the opposite figure :

$\overrightarrow{CE} \cap \overrightarrow{FD} = \{B\}$,

$m(\angle ABC) = 40^\circ$ and $m(\angle ABF) = 30^\circ$

Find : $m(\angle DBE)$



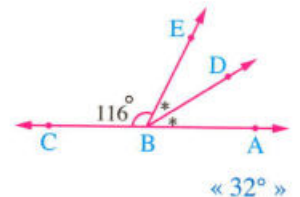
« 70° »

7 In the opposite figure :

$B \in \overrightarrow{AC}$, $m(\angle CBE) = 116^\circ$

and \overrightarrow{BD} bisects $\angle ABE$

Find : $m(\angle ABD)$



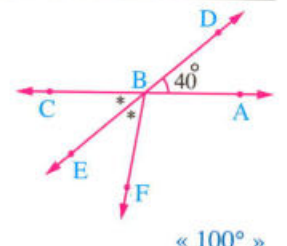
« 32° »

8 In the opposite figure :

$\overrightarrow{AC} \cap \overrightarrow{DE} = \{B\}$, $m(\angle ABD) = 40^\circ$

and \overrightarrow{BE} bisects $\angle CBF$

Find : $m(\angle ABF)$



« 100° »

9 In the opposite figure :

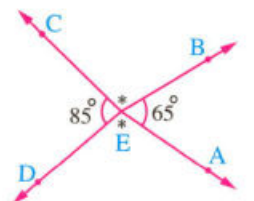
$\overrightarrow{EA} \cap \overrightarrow{EB} \cap \overrightarrow{EC} \cap \overrightarrow{ED} = \{E\}$

If $m(\angle BEC) = m(\angle AED)$

, $m(\angle AEB) = 65^\circ$, $m(\angle CED) = 85^\circ$

Find : $m(\angle BEC)$

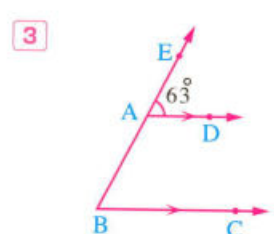
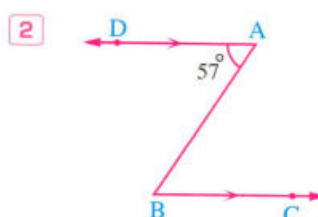
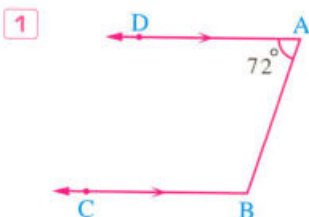
Are A , E and C on the same straight line ? Why ?



« 105° »

10 In each of the following figures ,

If $\overrightarrow{AD} \parallel \overrightarrow{BC}$ Find : $m(\angle ABC)$, giving reason.



11 In the opposite figure :

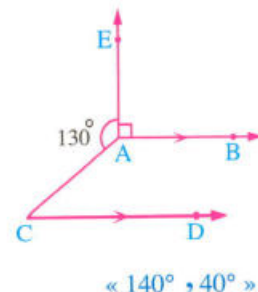
$$\overrightarrow{AB} \parallel \overrightarrow{CD}$$

$$m(\angle EAC) = 130^\circ$$

$$\text{and } m(\angle EAB) = 90^\circ$$

Find : 1 $m(\angle BAC)$

2 $m(\angle C)$

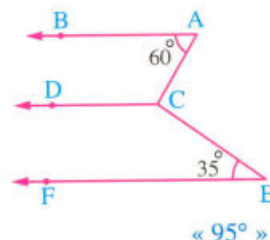


12 In the opposite figure :

$$\overrightarrow{AB} \parallel \overrightarrow{CD}, \overrightarrow{AB} \parallel \overrightarrow{EF}$$

$$m(\angle A) = 60^\circ \text{ and } m(\angle E) = 35^\circ$$

Find : $m(\angle ACE)$

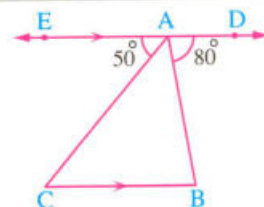


13 In the opposite figure :

$$\overrightarrow{DE} \parallel \overrightarrow{BC}, A \in \overrightarrow{DE}, m(\angle DAB) = 80^\circ$$

$$\text{and } m(\angle EAC) = 50^\circ$$

Find the measures of the angles of $\triangle ABC$



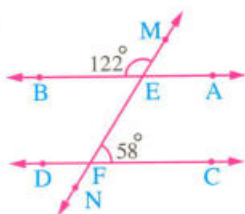
« $m(\angle BAC) = 50^\circ, m(\angle B) = 80^\circ, m(\angle C) = 50^\circ$ »

14 In each of the following figures,

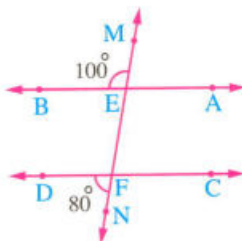
If \overleftrightarrow{MN} intersects \overleftrightarrow{AB} , \overleftrightarrow{CD} at E and F respectively,

Prove that : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

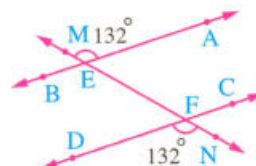
1



2



3

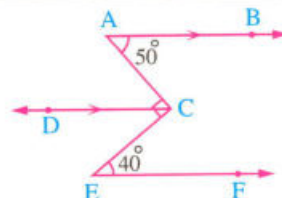


15 In the opposite figure :

$$\overrightarrow{AB} \parallel \overrightarrow{CD}, m(\angle A) = 50^\circ,$$

$$\angle ACE \text{ is right and } m(\angle E) = 40^\circ$$

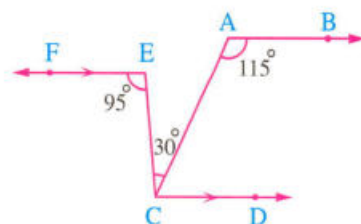
Prove that : $\overrightarrow{AB} \parallel \overrightarrow{EF}$



16 In the opposite figure :

$\overrightarrow{EF} \parallel \overrightarrow{CD}$, $m(\angle CEF) = 95^\circ$,
 $m(\angle ACE) = 30^\circ$, $m(\angle BAC) = 115^\circ$

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{EF}$



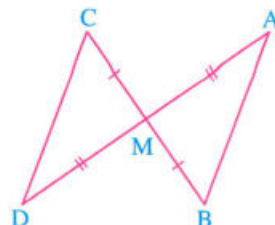
17 In the opposite figure :

$\overline{AD} \cap \overline{BC} = \{M\}$,
 $MA = MD$ and $MB = MC$

Prove that :

1 $AB = CD$

2 $\overline{AB} \parallel \overline{CD}$



18 Prove that :

- 1 A straight line which is perpendicular to one of two parallel lines in the same plane is also perpendicular to the other.
- 2 A straight line that is parallel to one of two parallel lines is also parallel to the other.

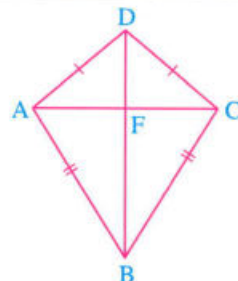
19 In the opposite figure :

$AD = CD$ and $AB = BC$

Use the properties of congruent triangles
 to show that :

1 \overrightarrow{DB} bisects $\angle ADC$

2 \overline{AC} and \overline{DB} are perpendicular to each other.



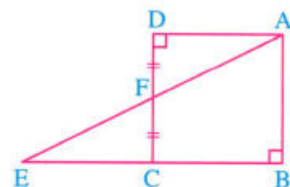
20 In the opposite figure :

ABCD is a square in which F

is the midpoint of \overline{CD}

and $\overline{AF} \cap \overline{BC} = \{E\}$

Prove that : $CE = CB$



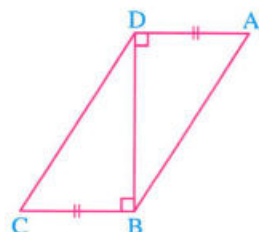
21 In the opposite figure :

$AD = BC$ and $m(\angle ADB) = m(\angle DBC) = 90^\circ$

Prove that :

1 $AB = CD$

2 $\overline{AB} \parallel \overline{CD}$

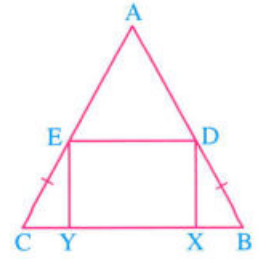


22 In the opposite figure :

$EC = DB$ and

$DXYE$ is a rectangle.

Prove that : $m(\angle ADE) = m(\angle AED)$



23 In the opposite figure :

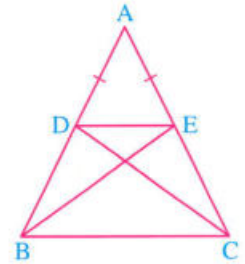
$AD = AE$ and

$m(\angle ADC) = m(\angle AEB)$

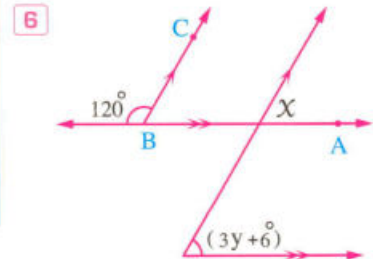
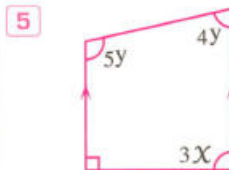
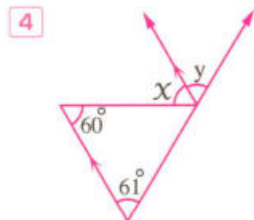
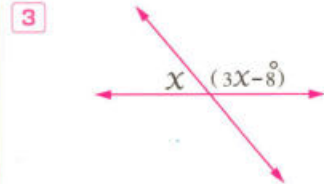
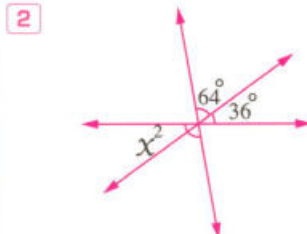
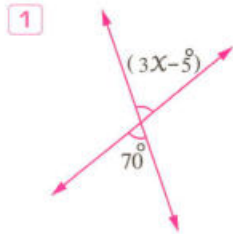
Show that :

1 $BE = CD$

2 $BD = CE$



24 Find the values of x and y in each of the following :



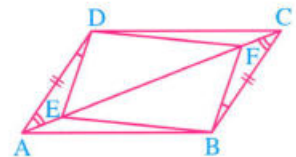
For excellent pupils

25 In the opposite figure :

1 Is $\triangle ADE$ congruent to $\triangle CBF$? Give your reason (s).

2 **Prove that :**

First : $\triangle DEF \equiv \triangle BFE$ **Second :** $\triangle ABE \equiv \triangle CDF$

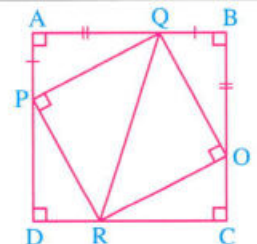


26 In the opposite figure :

1 Is $\triangle PAQ$ congruent to $\triangle QBO$? Give your reason (s).

2 **Show that :**

First : $\triangle PQR \equiv \triangle OQR$ **Second :** $\triangle PDR \equiv \triangle RCO$





Exercise 2

The polygon

From the school book



Interactive test

● Remember ● Understand ● Apply ● Problem Solving

1 Complete the following :

- 1 The regular polygon is the one in which :
(a) (b)
- 2 The sum of measures of the interior angles of the quadrilateral = °
- 3 The sum of measures of the interior angles of the pentagon = °
- 4 The sum of measures of the interior angles of the hexagon = °
- 5 The sum of measures of the interior angles of the heptagon = °
- 6 The measure of the interior angle of the regular pentagon = °
and the measure of the interior angle of the regular heptagon = °
- 7 The sum of measures of the exterior angles of the hexagon equals °
- 8 If the perimeter of a regular hexagon is 30 cm. , then its side length = cm.
and the measure of each interior angle in it = °
- 9 If the perimeter of a regular polygon = 80 cm. and its side length = 10 cm. ,
then the measure of each interior angle in it = °

2 Choose the correct answer from those given :

- 1 The sum of measures of the interior angles of a polygon of n sides equals
(a) $n \times 180^\circ$ (b) $(n - 2) \times 180^\circ$ (c) $\frac{(n - 2) \times 180^\circ}{2}$ (d) $\frac{(n - 2) \times 180^\circ}{2n}$

- 2 The measure of the interior angle of a regular polygon of n sides equals
 - (a) $\frac{(n-2) \times 90^\circ}{n}$ (b) $\frac{(n-2) \times 180^\circ}{2}$ (c) $\frac{(n-2) \times 180^\circ}{n}$ (d) $180^\circ \times (n-1)$
- 3 The measure of the interior angle of the regular polygon of 10 sides equals
 - (a) 72° (b) 108° (c) 144° (d) 150°
- 4 The measure of the interior angle of a regular polygon of 18 sides equals
 - (a) 130° (b) 140° (c) 150° (d) 160°
- 5 If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is
 - (a) 6 (b) 4 (c) 7 (d) 8
- 6 The sum of measures of the exterior angles of the triangle equals
 - (a) 90° (b) 180° (c) 360° (d) 720°
- 7 In the quadrilateral ABCD, if $m(\angle A) = 2m(\angle B) = m(\angle C) = 96^\circ$, then $m(\angle D) = \dots\dots\dots$
 - (a) 96° (b) 48° (c) 120° (d) 144°

3 Find the number of the diagonals of each of the following figures :

- 1 Triangle.
- 2 Quadrilateral.
- 3 Pentagon.

Hint : The number of diagonals of the polygon of n sides $= \frac{n(n-3)}{2}$

4 In each of the following, find the measure of the angle marked by (?) :

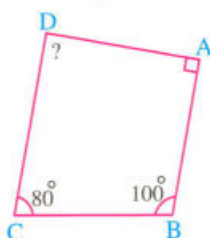


Fig. (1)

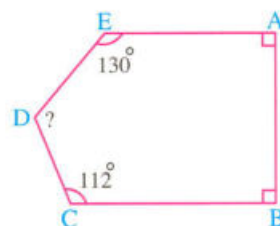


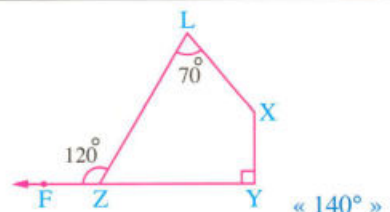
Fig. (2)

« 90° , 118° »

5 In the opposite figure :

$F \in \overrightarrow{YZ}$, $m(\angle L) = 70^\circ$,
 $m(\angle Y) = 90^\circ$ and $m(\angle LZF) = 120^\circ$

Find : $m(\angle X)$

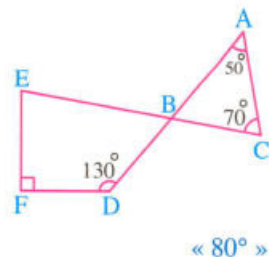


« 140° »

6 In the opposite figure :

$\overline{CE} \cap \overline{AD} = \{B\}$, $m(\angle A) = 50^\circ$
 $m(\angle C) = 70^\circ$, $m(\angle D) = 130^\circ$ and
 $m(\angle F) = 90^\circ$

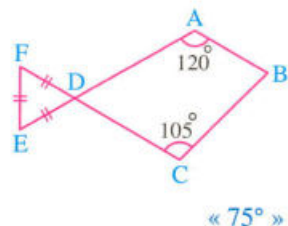
Find : $m(\angle E)$



7 In the opposite figure :

$\overline{AE} \cap \overline{CF} = \{D\}$,
 $\triangle DEF$ is an equilateral triangle ,
 $m(\angle A) = 120^\circ$ and $m(\angle C) = 105^\circ$

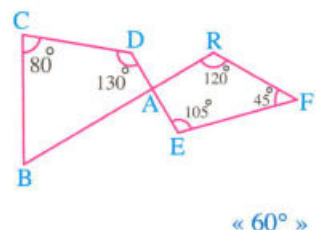
Find : $m(\angle B)$



8 In the opposite figure :

$\overline{ED} \cap \overline{RB} = \{A\}$, $m(\angle F) = 45^\circ$,
 $m(\angle R) = 120^\circ$, $m(\angle E) = 105^\circ$,
 $m(\angle D) = 130^\circ$ and $m(\angle C) = 80^\circ$

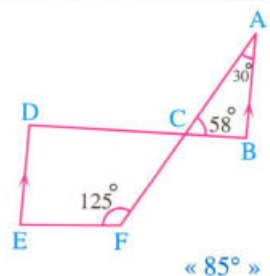
Find : $m(\angle B)$



9 In the opposite figure :

$\overline{BD} \cap \overline{AF} = \{C\}$, $\overline{AB} \parallel \overline{ED}$,
 $m(\angle A) = 30^\circ$ and $m(\angle ACB) = 58^\circ$,
 $m(\angle CFE) = 125^\circ$

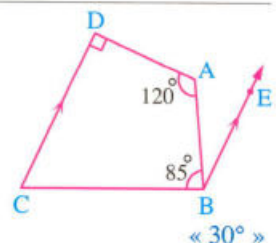
Find : $m(\angle E)$



10 In the opposite figure :

$m(\angle A) = 120^\circ$, $m(\angle D) = 90^\circ$,
 $m(\angle ABC) = 85^\circ$ and $\overline{BE} \parallel \overline{CD}$

Find : $m(\angle ABE)$



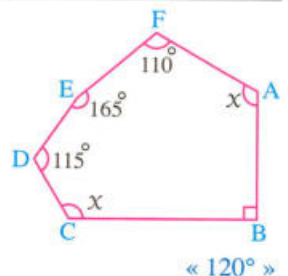
11 In the opposite figure :

ABCDEF is a hexagon.

$m(\angle A) = m(\angle C)$

Find :

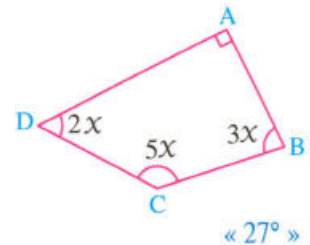
The value of x



12 In the opposite figure :

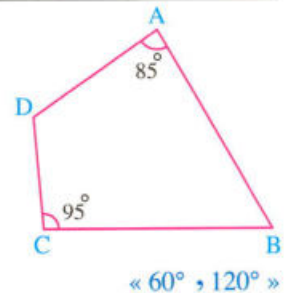
ABCD is a quadrilateral
in which : $m(\angle A) = 90^\circ$

Find : The value of x



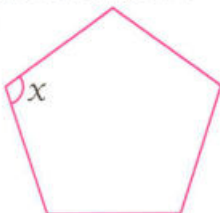
13 In the opposite figure :

$m(\angle A) = 85^\circ$, $m(\angle C) = 95^\circ$
and $m(\angle B) = \frac{1}{2} m(\angle D)$
Find the measure of each of them.

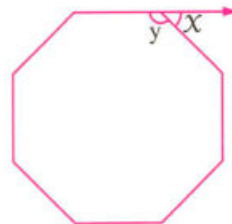


14 In each of the following , if the polygon is regular , find the measures of the unknown angles :

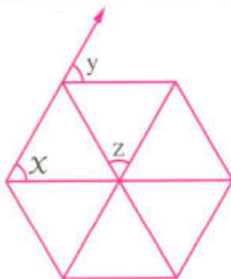
1



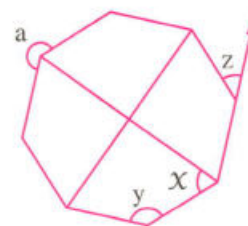
2



3

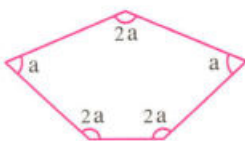


4

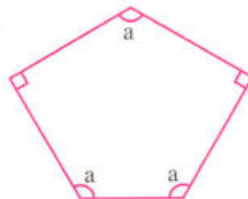


15 In each of the following , find the values of the unknown symbols :

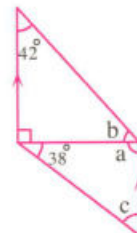
1



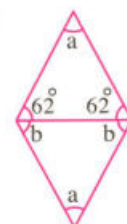
2



3

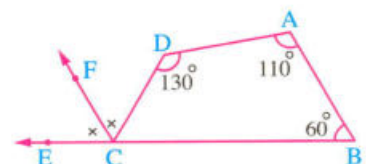


4



16 In the opposite figure :

$m(\angle A) = 110^\circ$, $m(\angle B) = 60^\circ$,
 $m(\angle D) = 130^\circ$, \overrightarrow{CF} bisects $\angle DCE$ and $C \in \overrightarrow{BE}$
Prove that : $\overrightarrow{CF} \parallel \overrightarrow{AB}$

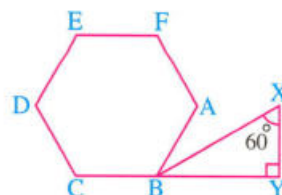


17 In the opposite figure :

ABCDEF is a regular hexagon ,

$Y \in \overrightarrow{CB}$, $\overline{XY} \perp \overline{YB}$ and $m(\angle X) = 60^\circ$

Prove that : \overline{BX} bisects $\angle ABY$

**18** If the ratio among the measures of the angles of a pentagon is $3 : 3 : 2 : 3 : 4$

, find the greatest measure of the angles of this pentagon.

« 144° »

19 If the measure of the exterior angle of a regular polygon is 30° , how many sides does it have ? What is the sum of the measures of its interior angles ?

« 12 , 1800° »

20 Is it possible that a regular polygon has an interior angle of measure 100° ? Why ?**21** A polygon of 9 sides. The sum of measures of eight angles of it is 1140°

1 Find the measure of the remained angle.

« 120° »

2 Is it possible that this polygon is regular ? Explain your answer.

22 A polygon has 15 sides :

1 Calculate the sum of the measures of its interior angles.

« 2340° »

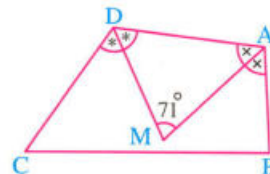
2 If the sum of the measures of five of its exterior angles is 200° , calculate the sum of the measures of the ten interior angles which are not adjacent to the five exterior angles.

« 1640° »

**For excellent pupils****23 In the opposite figure :**

\overline{AM} bisects $\angle BAD$, \overline{DM} bisects $\angle ADC$ and
 $m(\angle AMD) = 71^\circ$

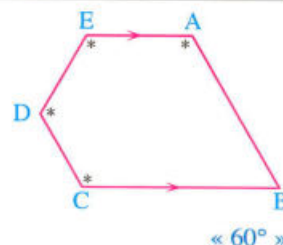
Prove that : $m(\angle B) + m(\angle C) = 142^\circ$

**24 In the opposite figure :**

$\overline{AE} \parallel \overline{BC}$,

$m(\angle A) = m(\angle E) = m(\angle D) = m(\angle C)$

Find : $m(\angle B)$



« 60° »

Exercise

3

The parallelogram and its properties

From the school book



Remember Understand Apply Problem Solving



Interactive test

1 Complete the following :

- 1 In a parallelogram , every two opposite sides are ,
- 2 In a parallelogram , every two opposite angles are
- 3 In a parallelogram , every two consecutive angles are
- 4 In a parallelogram , the two diagonals
- 5 The quadrilateral in which two sides only are parallel is called
- 6 A quadrilateral represents a parallelogram if (Write only one answer)
- 7 ABCD is a parallelogram in which $m(\angle A) = 50^\circ$, then $m(\angle B) = \dots\dots\dots^\circ$
- 8 In the parallelogram XYZL , if $m(\angle X) = \frac{1}{2} m(\angle Y)$, then $m(\angle Y) = \dots\dots\dots^\circ$

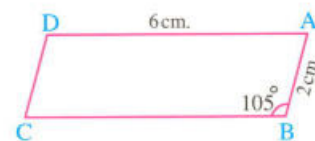
2 In the opposite figure :

ABCD is a parallelogram in which $AB = 2 \text{ cm.}$,

$AD = 6 \text{ cm.}$ and $m(\angle B) = 105^\circ$

Complete the following :

- 1 $BC = \dots\dots\dots \text{ cm.}$, $DC = \dots\dots\dots \text{ cm.}$
- 2 $m(\angle D) = \dots\dots\dots^\circ$, $m(\angle A) = \dots\dots\dots^\circ$ and $m(\angle C) = \dots\dots\dots^\circ$
- 3 The perimeter of the parallelogram ABCD = cm.



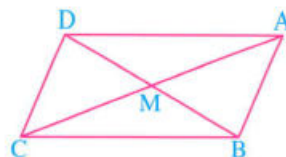
3 In the opposite figure :

ABCD is a parallelogram such that :

$$\overline{AC} \cap \overline{BD} = \{M\}, CD = 2 \text{ cm.},$$

$$MC = 2.5 \text{ cm. and } BD = 3.6 \text{ cm.}$$

Calculate the perimeter of $\triangle AMB$



« 6.3 cm. »

4 In the opposite figure :

XYZL is a parallelogram in which :

$$m(\angle Y) = 118^\circ, m(\angle XZY) = 27^\circ$$

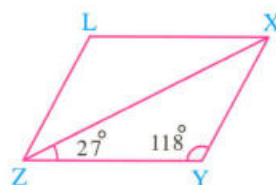
Find :

1 $m(\angle YXZ)$

2 $m(\angle LZX)$

3 $m(\angle LXZ)$

4 $m(\angle L)$



« 35°, 35°, 27°, 118° »

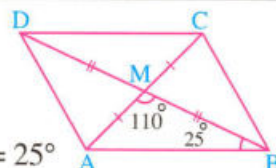
5 In the opposite figure :

ABCD is a quadrilateral whose diagonals intersect

at M, $MA = MC$, $MB = MD$, $m(\angle AMB) = 110^\circ$ and $m(\angle MBA) = 25^\circ$

1 Prove that : The figure ABCD is a parallelogram.

2 Find : $m(\angle ACD)$



« 45° »

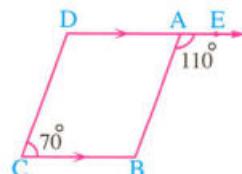
6 In the opposite figure :

ABCD is a quadrilateral in which :

$$\overline{AD} \parallel \overline{BC}, E \in \overline{DA}, m(\angle BAE) = 110^\circ$$

$$\text{and } m(\angle DCB) = 70^\circ$$

Prove that : ABCD is a parallelogram.



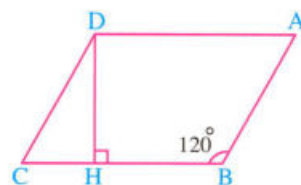
7 In the opposite figure :

ABCD is a parallelogram in which :

$$m(\angle B) = 120^\circ \text{ and } \overline{DH} \perp \overline{BC}$$

$$\text{where } \overline{DH} \cap \overline{BC} = \{H\}$$

Find : $m(\angle HDC)$



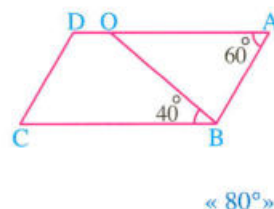
« 30° »

8 In the opposite figure :

ABCD is a parallelogram , in which :

$m(\angle A) = 60^\circ$, $m(\angle OBC) = 40^\circ$ where $O \in \overline{AD}$

Find : $m(\angle ABO)$



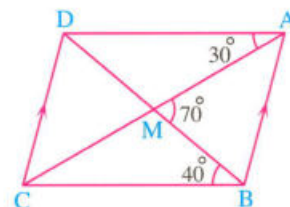
9 In the opposite figure :

ABCD is a quadrilateral where : $\overline{AC} \cap \overline{BD} = \{M\}$,

$\overline{AB} \parallel \overline{DC}$, $m(\angle AMB) = 70^\circ$, $m(\angle MBC) = 40^\circ$

and $m(\angle MAD) = 30^\circ$

Prove that : ABCD is a parallelogram.

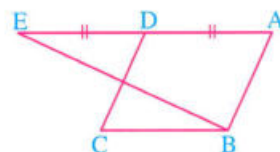


10 In the opposite figure :

ABCD is a parallelogram ,

$E \in \overline{AD}$ where : $AD = DE$

Prove that : \overline{DC} and \overline{BE} bisect each other.

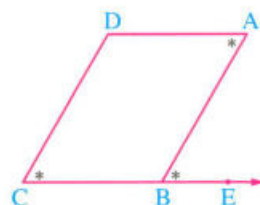


11 In the opposite figure :

ABCD is a quadrilateral ,

$E \in \overline{CB}$ and $m(\angle BCD) = m(\angle EBA) = m(\angle A)$

Prove that : ABCD is a parallelogram.

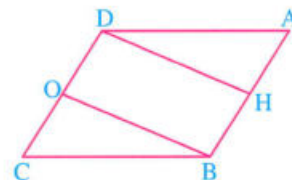


12 In the opposite figure :

ABCD is a parallelogram , H is the midpoint of \overline{AB}

and O is the midpoint of \overline{DC}

Prove that : HBOD is a parallelogram.



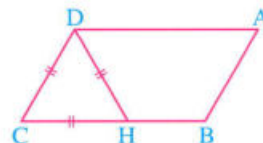
13 In the opposite figure :

ABCD is a parallelogram , $H \in \overline{BC}$ where :

$\triangle DHC$ is an equilateral triangle.

1 Prove that : $HC = AB$

2 Find : $m(\angle B)$ and $m(\angle HDA)$

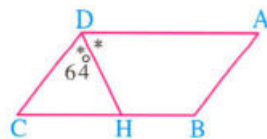


« 120° , 60° »

14 In the opposite figure :

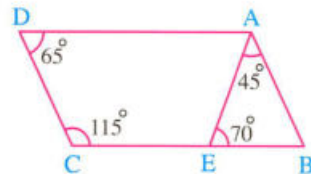
ABCD is a parallelogram , $H \in \overline{BC}$
 \overrightarrow{DH} bisects $\angle ADC$ and $m(\angle HDC) = 64^\circ$

Find : **1** $m(\angle DHB)$ **2** $m(\angle ABC)$

« 116° , 128° »**15 In the opposite figure :**

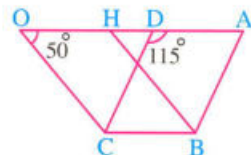
$E \in \overline{BC}$, $m(\angle BAE) = 45^\circ$,
 $m(\angle AEB) = 70^\circ$, $m(\angle D) = 65^\circ$
 and $m(\angle C) = 115^\circ$

Prove that : ABCD is a parallelogram.

**16 In the opposite figure :**

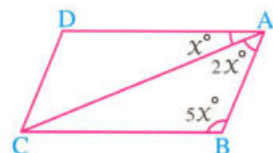
ABCD and HBCO are two parallelograms
 such that $m(\angle O) = 50^\circ$
 and $m(\angle ADC) = 115^\circ$

Find : $m(\angle ABH)$

« 65° »**17 In the opposite figure :**

ABCD is a parallelogram in which :
 $m(\angle DAC) = x^\circ$, $m(\angle BAC) = 2x^\circ$
 and $m(\angle ABC) = 5x^\circ$

Find : $m(\angle BCD)$ and $m(\angle ADC)$ in degrees.

« 67.5° , 112.5° »**18 Choose the correct answer from the given ones :**

- 1** ABCD is a parallelogram in which : $m(\angle A) = 50^\circ$, then $m(\angle C) = \dots\dots\dots$
 (a) 50° (b) 60° (c) 130° (d) 150°
- 2** ABCD is a parallelogram in which : $m(\angle A) + m(\angle C) = 140^\circ$
 , then $m(\angle B) = \dots\dots\dots$
 (a) 70° (b) 40° (c) 110° (d) 220°
- 3** If the lengths of two consecutive sides of a parallelogram are 3 cm.
 and 5 cm. , then its perimeter equals $\dots\dots\dots$ cm.
 (a) 12 (b) 14 (c) 16 (d) 18

- 4 If the perimeter of a parallelogram is 25 cm. and if one of its sides is of length 7 cm. , then the consecutive side is of length cm.

(a) 7 (b) 18 (c) 12.5 (d) 5.5

- 5 In the opposite figure :

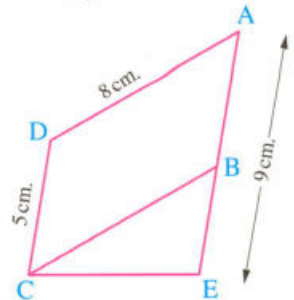
If ABCD is a parallelogram

, $E \in \overline{AB}$, $CD = 5$ cm. , $AE = 9$ cm.

, $AD = 8$ cm. , the perimeter of $\triangle BEC = 18$ cm.

, then the length of $\overline{EC} =$ cm.

(a) 8 (b) 6 (c) 5 (d) 4



For excellent pupils

- 19 ABCD is a parallelogram in which : E is the midpoint of \overline{AB} ,

F is the midpoint of \overline{CD} , if $\overline{AF} \cap \overline{DE} = \{M\}$, $\overline{BF} \cap \overline{CE} = \{N\}$

Prove that : 1 $\overline{ED} \parallel \overline{FB}$

2 FMEN is a parallelogram.

- 20 XYZL is a parallelogram in which : $m(\angle Y) = 3 m(\angle X)$

Find the measures of the interior angles of XYZL

« $m(\angle Y) = m(\angle L) = 135^\circ$ and $m(\angle X) = m(\angle Z) = 45^\circ$ »



Now

Solve the interactive tests by scanning the QR code

1



Download QR reader Application on your phone

2



Open the application ,then scan QR code in each exercise



Exercise

4

The special cases of the parallelogram

From the school book



● Remember

● Understand

● Apply

● Problem Solving



Interactive test

1 Complete the following :

- 1 The parallelogram whose two diagonals are perpendicular is called
- 2 The parallelogram whose two diagonals are is called a rectangle.
- 3 The parallelogram whose two diagonals are equal in length and perpendicular is called
- 4 The quadrilateral whose sides are equal in length is called
- 5 The quadrilateral whose diagonals bisect each other is called
- 6 The rectangle is a with a right angle.
- 7 The rhombus is a in which its diagonals are perpendicular.
- 8 The square is a with a right angle.
- 9 The rhombus whose two diagonals are equal in length is called
- 10 The rectangle whose two diagonals are perpendicular is called
- 11 The rectangle whose two adjacent sides have the same length is called
- 12 If $\overline{XY} \parallel \overline{ZL}$, $XY = ZL$, then the quadrilateral $XYZL$ is called
- 13 If $ABCD$ is a rhombus, then \perp
- 14 The perimeter of the square = , the perimeter of the rectangle = and the perimeter of the rhombus =
- 15 The rhombus whose perimeter is 42 cm., its side length = cm.

2 Choose the correct answer from the given ones :

- 1 The two diagonals of a rectangle
 - (a) are perpendicular.
 - (b) are equal in length.
 - (c) are perpendicular and equal in length.
 - (d) bisect its interior angles.
- 2 The two diagonals of a rhombus are
 - (a) perpendicular and not equal in length.
 - (b) equal in length and not perpendicular.
 - (c) perpendicular and equal in length.
 - (d) not equal in length and not perpendicular.
- 3 The two diagonals of the square are
 - (a) just perpendicular.
 - (b) just equal in length.
 - (c) perpendicular and equal in length.
 - (d) not equal in length and not perpendicular.
- 4 The adjacent sides are equal in length in a parallelogram, then the figure is a
 - (a) square.
 - (b) rhombus.
 - (c) rectangle.
 - (d) trapezium.
- 5 If ABCD is a rectangle in which $AC = 5$ cm., then $BD =$ cm.
 - (a) 2.5
 - (b) 5
 - (c) 10
 - (d) 20
- 6 If ABCD is a square, then $m(\angle CAB) =$
 - (a) 90°
 - (b) 45°
 - (c) 60°
 - (d) 30°
- 7 If ABCD is a parallelogram in which $m(\angle A) = m(\angle B)$, then ABCD is a
 - (a) rectangle.
 - (b) rhombus.
 - (c) square.
 - (d) trapezium.
- 8 If ABCD is a rhombus in which $m(\angle ACB) = 32^\circ$, then $m(\angle D) =$
 - (a) 32°
 - (b) 64°
 - (c) 116°
 - (d) 26°

3 In the opposite figure :

ABCD is a rectangle in which :

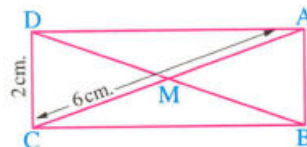
$AC = 6$ cm. , $CD = 2$ cm.

and $\overline{AC} \cap \overline{BD} = \{M\}$

Complete : 1 $AB =$ cm.

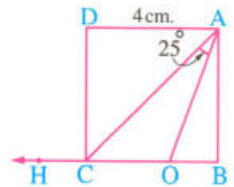
2 $DM =$ cm.

3 The perimeter of $\triangle ABM =$ cm.



4 In the opposite figure :

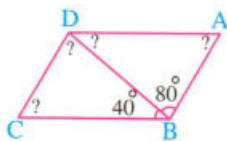
ABCD is a square in which $AD = 4 \text{ cm.}$,
 $O \in \overline{BC}$ such that : $m(\angle OAC) = 25^\circ$
 and $H \in \overline{BC}$



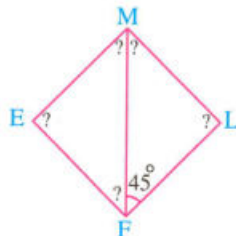
Complete the following :

- 1 The perimeter of the square = cm.
- 2 $m(\angle ACH) = \dots\dots\dots^\circ$
- 3 $m(\angle AOC) = \dots\dots\dots^\circ$

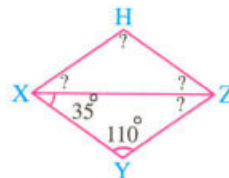
5 Find the measures of the angles marked by (?) in each of the following figures :



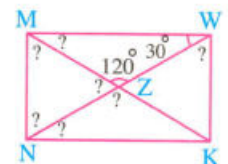
a parallelogram
Fig. (1)



a square
Fig. (2)



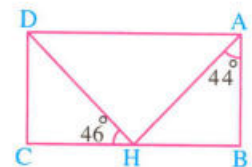
a rhombus
Fig. (3)



a rectangle
Fig. (4)

6 In the opposite figure :

ABCD is a rectangle , $H \in \overline{BC}$ such that :
 $m(\angle DHC) = 46^\circ$ and $m(\angle BAH) = 44^\circ$
 Calculate : $m(\angle AHD)$

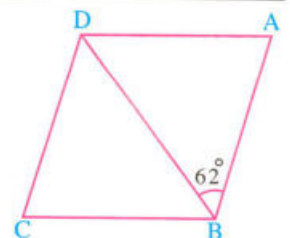


« 88° »

7 In the opposite figure :

ABCD is a rhombus,
 \overline{BD} is a diagonal in it ,
 $m(\angle ABD) = 62^\circ$

Find with proof : $m(\angle A)$

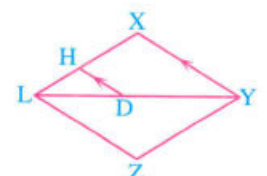


« 56° »

8 In the opposite figure :

XYZL is a rhombus , $D \in \overline{YL}$
 Draw $\overline{DH} \parallel \overline{YX}$ such that :
 $\overline{DH} \cap \overline{XL} = \{H\}$

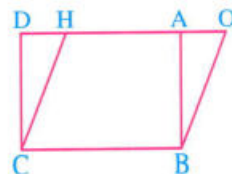
Prove that : $m(\angle HDL) = m(\angle HLD)$



9 In the opposite figure :

ABCD is a rectangle
and OBCH is a parallelogram.

Prove that : $DH = AO$

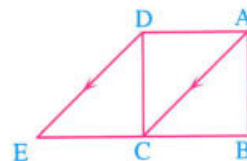


10 In the opposite figure :

ABCD is a square, $E \in \overrightarrow{BC}$, $\overline{AC} \parallel \overline{DE}$

1 Prove that : ACED is a parallelogram.

2 Find : $m(\angle ACE)$



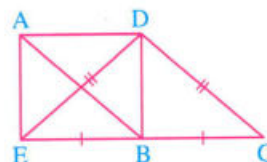
« 135° »

11 In the opposite figure :

ABCD is a parallelogram,

$E \in \overrightarrow{CB}$ where $BC = BE$, if $DE = DC$

, prove that : The figure AEBD is a rectangle.



12 In the opposite figure :

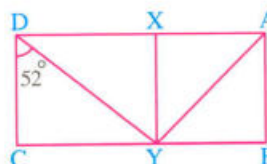
ABCD is a rectangle, $X \in \overline{AD}$

and $Y \in \overline{BC}$ such that :

AXYB is a square.

If $m(\angle YDC) = 52^\circ$,

then find with proof : $m(\angle AYD)$



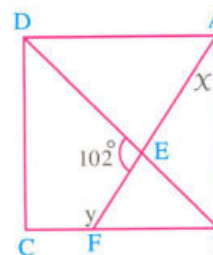
« 97° »

13 In the opposite figure :

ABCD is a square.

Find in degrees the value

of each of x and y



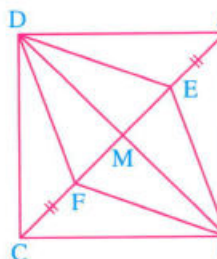
« 33° , 123° »

14 In the opposite figure :

ABCD is a square, its diagonals intersect at M,

$E \in \overline{AC}$, $F \in \overline{AC}$ such that : $AE = CF$

Prove that : EBFD is a rhombus.





For excellent pupils

- 15 Eslam drew a parallelogram, a rhombus, a rectangle and a square, then he hid parts of them as in the opposite figure and he asked his friend Bassem to recognize each figure.

Help Bassem to name each drawn figure.



1



2



3



4

- 16 Use "Some" or "All" to get a correct statement :

- 1 squares are rectangles.
- 2 quadrilaterals are parallelograms.
- 3 squares are rhombuses.
- 4 parallelograms are rectangles.
- 5 rectangles are parallelograms.
- 6 rhombuses are squares.

Now at all bookstores

EL-MOASSER

in

Science

for all educational stages



The triangle

From the school book



Interactive test

Remember

Understand

Apply

Problem Solving

1 Complete the following :

- 1 The sum of measures of the interior angles of a triangle = $\dots\dots\dots^\circ$
- 2 The measure of the exterior angle of a triangle is equal to the sum of $\dots\dots\dots$
- 3 If the measure of an angle in a triangle equals the sum of measures of the other two angles in the triangle , then the triangle is $\dots\dots\dots$
- 4 If the measure of an angle in a triangle is greater than the sum of measures of the other two angles , then the triangle is $\dots\dots\dots$
- 5 In $\triangle ABC$: If $m(\angle A) + m(\angle C) = m(\angle B)$, then $m(\angle B) = \dots\dots\dots^\circ$
- 6 In $\triangle ABC$: If $m(\angle B) > m(\angle A) + m(\angle C)$, then $\angle B$ is $\dots\dots\dots$
- 7 It is possible that each of the interior angles of a triangle is of measure $\dots\dots\dots^\circ$

2 Choose the correct answer from the given ones :

- 1 The triangle contains two $\dots\dots\dots$ angles at least.
 (a) acute (b) obtuse (c) right (d) reflex
- 2 The sum of measures of the interior angles of a triangle equals the measure of $\dots\dots\dots$ angle.
 (a) a right (b) a straight (c) an acute (d) a reflex
- 3 In $\triangle XYZ$: If $m(\angle X) = 50^\circ$, $m(\angle Y) = 100^\circ$, then $m(\angle Z) = \dots\dots\dots$
 (a) 30° (b) 50° (c) 80° (d) 100°
- 4 In $\triangle ABC$: If $m(\angle A) + m(\angle B) = 110^\circ$, then $m(\angle C) = \dots\dots\dots$
 (a) 110° (b) 90° (c) 70° (d) 55°

- 5 If the measures of two angles in a triangle are 35° and 45° , then the triangle is
 (a) acute-angled. (b) right-angled. (c) obtuse-angled. (d) equilateral.
- 6 The measure of the exterior angle of the equilateral triangle at any one of its vertices equals
 (a) 60° (b) 120° (c) 150° (d) 30°

3 In each of the following figures, find the measure of the angle marked by (?) :

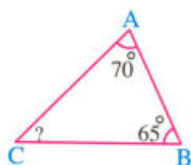


Fig. (1)

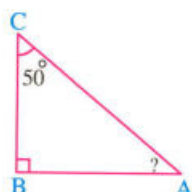


Fig. (2)

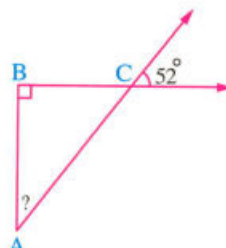


Fig. (3)

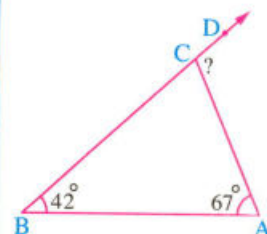


Fig. (4)

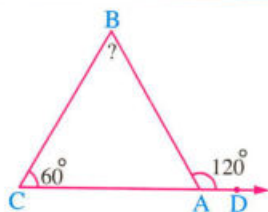


Fig. (5)

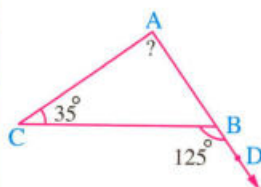


Fig. (6)

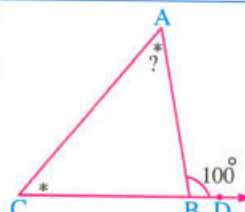


Fig. (7)

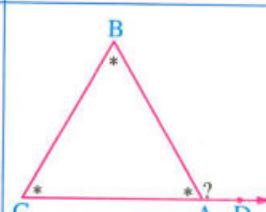


Fig. (8)

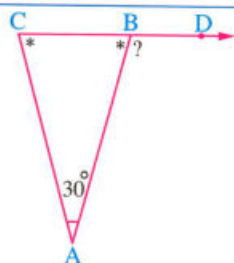


Fig. (9)

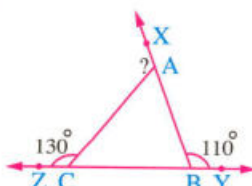


Fig. (10)

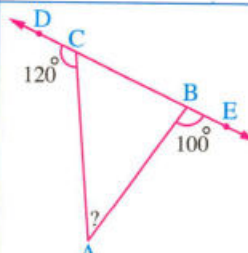


Fig. (11)

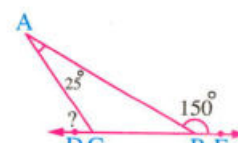


Fig. (12)

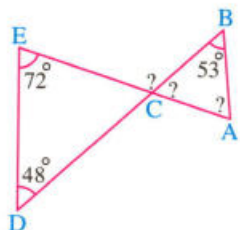


Fig. (13)

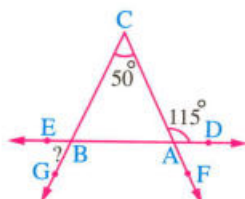


Fig. (14)

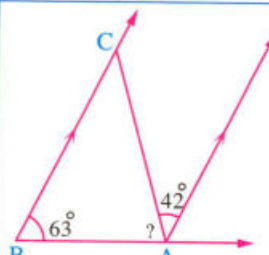


Fig. (15)

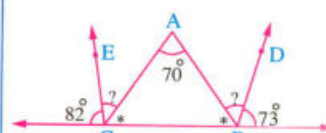


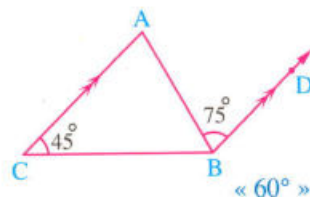
Fig. (16)

4 In the opposite figure :

$\overrightarrow{BD} \parallel \overrightarrow{CA}$, $m(\angle C) = 45^\circ$

and $m(\angle ABD) = 75^\circ$

Find : $m(\angle ABC)$



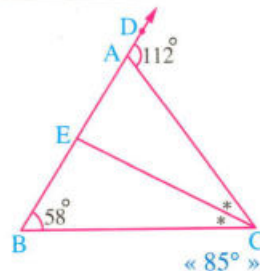
5 In the opposite figure :

ABC is a triangle in which : $m(\angle B) = 58^\circ$,

$E \in \overline{AB}$ such that \overrightarrow{CE} bisects $\angle ACB$,

$D \in \overrightarrow{BA}$ and $m(\angle CAD) = 112^\circ$

Find : $m(\angle AEC)$



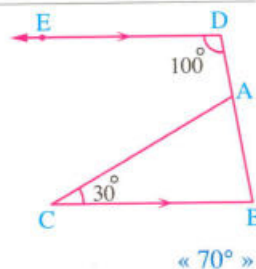
6 In the opposite figure :

$\overrightarrow{DE} \parallel \overrightarrow{BC}$, $m(\angle D) = 100^\circ$,

$m(\angle C) = 30^\circ$

and $A \in \overline{DB}$

Find : $m(\angle BAC)$



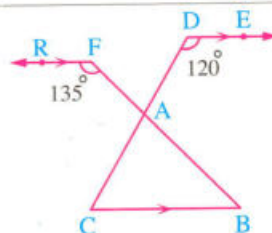
7 In the opposite figure :

$\overrightarrow{DE} \parallel \overrightarrow{FR} \parallel \overrightarrow{BC}$,

$m(\angle CDE) = 120^\circ$ and $m(\angle RFB) = 135^\circ$

Calculate the measures of the angles of $\triangle ABC$

« $m(\angle B) = 45^\circ$, $m(\angle C) = 60^\circ$, $m(\angle A) = 75^\circ$ »



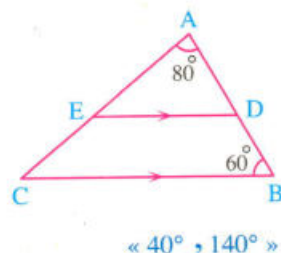
8 In the opposite figure :

ABC is a triangle in which :

$m(\angle A) = 80^\circ$ and $m(\angle B) = 60^\circ$

$\overrightarrow{DE} \parallel \overrightarrow{BC}$ where : $D \in \overline{AB}$ and $E \in \overline{AC}$

Find : $m(\angle AED)$ and $m(\angle DEC)$



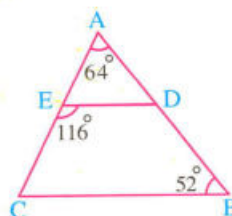
9 In the opposite figure :

ABC is a triangle in which $m(\angle A) = 64^\circ$,

$m(\angle B) = 52^\circ$, $m(\angle DEC) = 116^\circ$,

$E \in \overline{AC}$ and $D \in \overline{AB}$

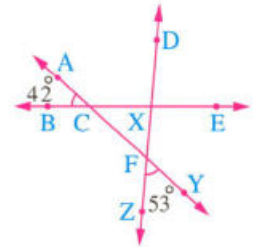
Prove that : $\overrightarrow{DE} \parallel \overrightarrow{BC}$



10 In the opposite figure :

Prove that :

$$m(\angle DXE) = 85^\circ ,$$

then calculate $m(\angle DXC)$ and $m(\angle EXF)$ « 95° , 95° »

11 In the opposite figure :

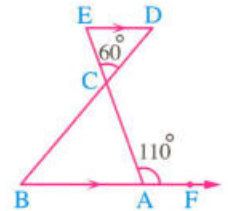
$$\overrightarrow{ED} \parallel \overrightarrow{BA} , m(\angle CAF) = 110^\circ ,$$

$$\overrightarrow{DB} \cap \overrightarrow{AE} = \{C\} ,$$

$$m(\angle DCE) = 60^\circ \text{ and } F \in \overrightarrow{BA}$$

Find the measures of each of the angles of the two triangles DCE and ABC

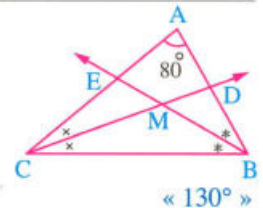
$$\ll m(\angle E) = 70^\circ , m(\angle D) = 50^\circ , m(\angle B) = 50^\circ , m(\angle ACB) = 60^\circ , m(\angle BAC) = 70^\circ \gg$$



12 In the opposite figure :

 \overrightarrow{BM} bisects $\angle ABC$ and \overrightarrow{CM} bisects $\angle ACB$

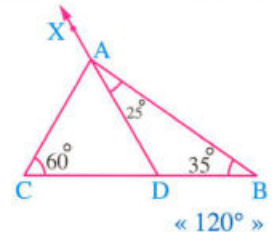
$$\text{If } m(\angle A) = 80^\circ ,$$

Find : $m(\angle EMD)$ « 130° »

13 In the opposite figure :

ABC is a triangle , $m(\angle B) = 35^\circ$, $m(\angle C) = 60^\circ$,

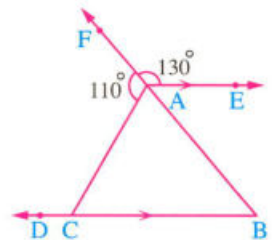
$$m(\angle BAD) = 25^\circ , D \in \overrightarrow{BC} \text{ and } X \in \overrightarrow{DA}$$

Find : $m(\angle XAC)$ « 120° »

14 In the opposite figure :

ABC is a triangle , $\overrightarrow{AE} \parallel \overrightarrow{BC}$, $D \in \overrightarrow{BC}$, $F \in \overrightarrow{BA}$,

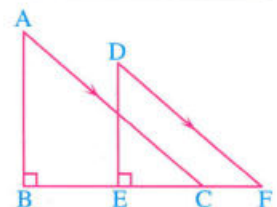
$$m(\angle FAE) = 130^\circ \text{ and } m(\angle FAC) = 110^\circ$$

Find : $m(\angle ACD)$ « 120° »

15 In the opposite figure :

The points F , C , E and B are collinear ,

$$m(\angle B) = m(\angle DEC) = 90^\circ \text{ and } \overrightarrow{AC} \parallel \overrightarrow{DF}$$

Prove that : $m(\angle A) = m(\angle D)$ 

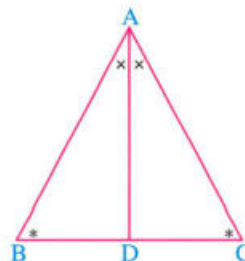
16 In the opposite figure :

ABC is a triangle ,

$$m(\angle B) = m(\angle C)$$

and \overrightarrow{AD} is the bisector of $\angle BAC$

Prove that : $AB = AC$

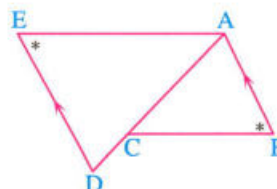


17 In the opposite figure :

$$\overline{AB} \parallel \overline{ED}$$

$$\text{and } m(\angle ABC) = m(\angle AED)$$

Prove that : $\overline{BC} \parallel \overline{AE}$

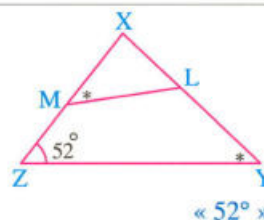


18 In the opposite figure :

XYZ is a triangle in which $m(\angle Z) = 52^\circ$, $L \in \overline{XY}$

and $M \in \overline{XZ}$ such that : $m(\angle Y) = m(\angle XML)$

Find : $m(\angle XLM)$

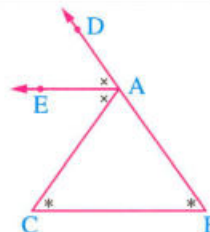


19 In the opposite figure :

ABC is a triangle in which : $m(\angle B) = m(\angle C)$,

$D \in \overline{BA}$ and \overrightarrow{AE} bisects $\angle DAC$

Prove that : $\overrightarrow{AE} \parallel \overline{BC}$

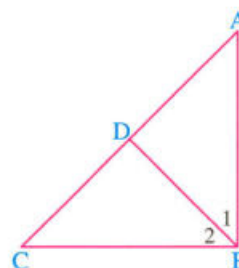


20 In the opposite figure :

ABC is a triangle in which : $D \in \overline{AC}$,

$$m(\angle 1) = m(\angle A) \text{ and } m(\angle 2) = m(\angle C)$$

Prove that : $\angle ABC$ is a right angle.



For excellent pupils

21 ABC is a triangle in which : $m(\angle A) = 2 m(\angle C)$ and $m(\angle B) = 4 m(\angle C)$

Prove that : $\angle B$ is an obtuse angle.

22 ABC is a triangle in which : $m(\angle C) = 28^\circ$, $m(\angle A) = 4x^\circ$

$$\text{, } m(\angle B) = (2x + 2)^\circ$$

Find : $m(\angle A)$ and $m(\angle B)$

« 100° , 52° »



Exercise

6

Theorem 2 and its corollary, and theorem 3

From the school book



Interactive test

● Remember

● Understand

● Apply

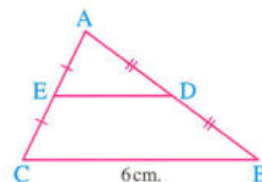
● Problem Solving

1 Complete the following :

- 1 The ray drawn from the midpoint of a side of a triangle parallel to another side
- 2 The line segment joining the midpoints of two sides of a triangle is the third side.
- 3 The length of the line segment joining the midpoints of two sides of a triangle equals

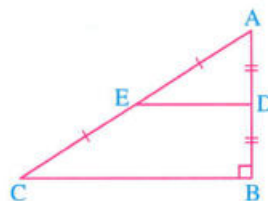
4 In the opposite figure :

If D and E are the midpoints of \overline{AB} and \overline{AC} respectively
 , $BC = 6$ cm.
 , then $DE = \dots\dots\dots$ cm.



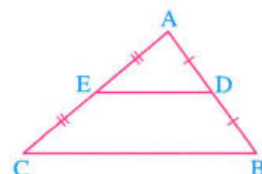
5 In the opposite figure :

If $m(\angle B) = 90^\circ$, D and E are the
 midpoints of \overline{AB} and \overline{AC} respectively
 , then $m(\angle ADE) = \dots\dots\dots^\circ$



6 In the opposite figure :

If D and E are the midpoints of \overline{AB} and \overline{AC} respectively
 , and the perimeter of the triangle $ABC = 24$ cm.
 , then the perimeter of the triangle $ADE = \dots\dots\dots$ cm.

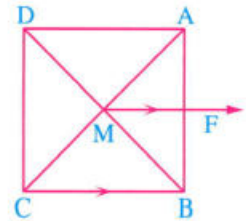


7 In the opposite figure :

If the perimeter of the square $ABCD = 20$ cm.

, $\overline{MF} \parallel \overline{CB}$ where $F \in \overline{AB}$

, then $AF = \dots\dots\dots$ cm.

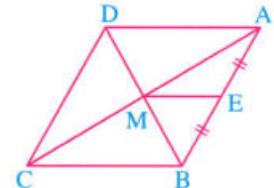


8 In the opposite figure :

\therefore The perimeter of the rhombus $ABCD = 24$ cm. ,

E is the midpoint of \overline{AB}

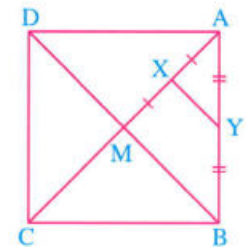
$\therefore ME = \dots\dots\dots$ cm.



9 In the opposite figure :

\therefore $ABCD$ is a square , X and Y are the midpoints of \overline{AM} and \overline{AB} respectively and $AC = 12$ cm.

$\therefore XY = \dots\dots\dots$ cm. , $m(\angle AYZ) = \dots\dots\dots^\circ$



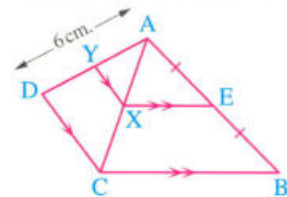
2 In the opposite figure :

$AE = EB$, $\overline{EX} \parallel \overline{BC}$,

$\overline{XY} \parallel \overline{CD}$

and $AD = 6$ cm.

Find the length of : \overline{AY}



« 3 cm. »

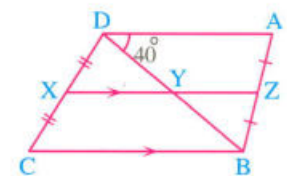
3 In the opposite figure :

X is the midpoint of \overline{CD}

, Z is the midpoint of \overline{AB}

, $\overline{XY} \parallel \overline{CB}$, $m(\angle ADB) = 40^\circ$

Find : $m(\angle ZYB)$



« 40° »

4 In the opposite figure :

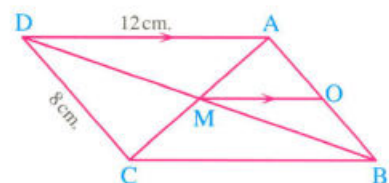
$ABCD$ is a parallelogram , $\overline{AC} \cap \overline{BD} = \{M\}$

Draw $\overline{MO} \parallel \overline{AD}$ to cut \overline{AB} at O

If $AD = 12$ cm. and $DC = 8$ cm.

, then find : **1** The perimeter of $ABCD$

2 The length of \overline{AO}

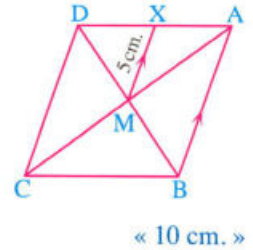


« 40 cm. , 4 cm. »

5 In the opposite figure :

ABCD is a parallelogram , its diagonals intersect at M
Draw $\overrightarrow{MX} \parallel \overrightarrow{BA}$ to intersect \overline{AD} at X

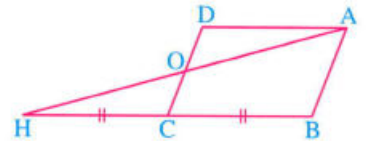
- 1 Prove that : X is the midpoint of \overline{AD}
- 2 If $MX = 5 \text{ cm.}$, then find the length of \overline{CD}



« 10 cm. »

6 In the opposite figure :

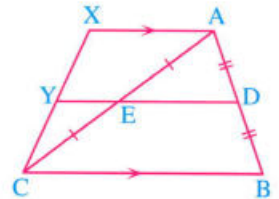
ABCD is a parallelogram ,
 $BC = CH$, $H \in \overline{BC}$
Draw \overline{AH} to cut \overline{DC} at O
Prove that : $AO = OH$



7 In the opposite figure :

$AD = DB$, $AE = EC$,
 $\overline{AX} \parallel \overline{BC}$, $\overline{DE} \cap \overline{XC} = \{Y\}$

Prove that : Y is the midpoint of \overline{XC}

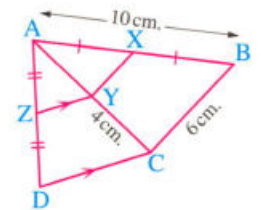


8 In the opposite figure :

ABCD is a quadrilateral in which :
X and Z are the midpoints of \overline{AB}
and \overline{AD} respectively and $Y \in \overline{AC}$ such that :
 $\overline{YZ} \parallel \overline{CD}$ and $YC = 4 \text{ cm.}$

If $BC = 6 \text{ cm.}$ and $AB = 10 \text{ cm.}$

- , then find :
- 1 The length of \overline{AY}
 - 2 The perimeter of $\triangle AXY$

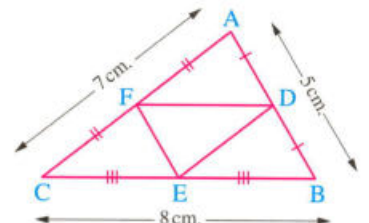


« 4 cm. , 12 cm. »

9 In the opposite figure :

$AB = 5 \text{ cm.}$, $BC = 8 \text{ cm.}$,
 $AC = 7 \text{ cm.}$, D , E and F are the midpoints of
 \overline{AB} , \overline{BC} and \overline{CA} respectively.

Calculate the perimeter of : $\triangle DEF$



« 10 cm. »

10 In the opposite figure :

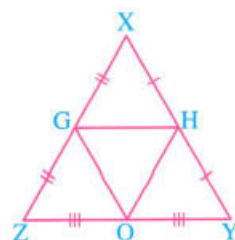
XYZ is a triangle in which :

H , O and G are the midpoints of \overline{XY} , \overline{YZ} and \overline{ZX} respectively.

If the perimeter of ΔHOG is 18 cm. ,

then find the perimeter of : ΔXYZ

« 36 cm. »

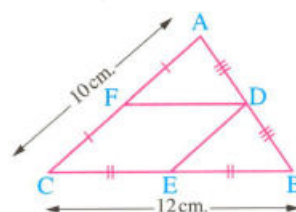


11 In the opposite figure :

ABC is a triangle in which D , E and F are the midpoints of \overline{AB} , \overline{BC} and \overline{CA} respectively ,
BC = 12 cm. , AC = 10 cm.

Find the perimeter of the figure DECF

« 22 cm. »



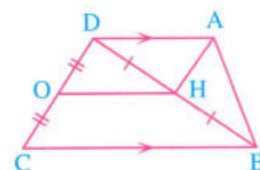
12 In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $AD = \frac{1}{2} BC$,

H is the midpoint of \overline{BD} ,

O is the midpoint of \overline{CD}

Prove that : AHOD is a parallelogram.

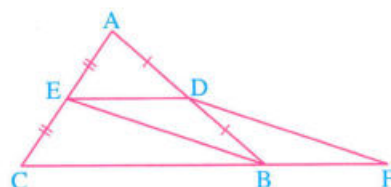


13 In the opposite figure :

D and E are the midpoints of \overline{AB} and \overline{AC} respectively ,

$F \in \overline{CB}$ where $BF = \frac{1}{2} BC$

Prove that : BEDF is a parallelogram.



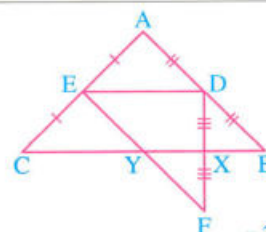
14 In the opposite figure :

D is the midpoint of \overline{AB} , E is the midpoint of \overline{AC} ,

$\overline{DF} \cap \overline{BC} = \{X\}$, $DX = XF$, BC = 12 cm.

Find the length of : \overline{XY}

« 3 cm. »



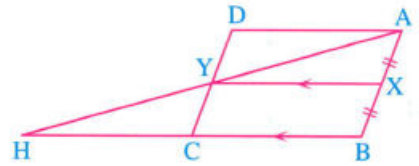
15 In the opposite figure :

ABCD is a parallelogram, X is the midpoint of \overline{AB}

Draw $\overrightarrow{XY} \parallel \overline{BC}$ to cut \overline{DC} at Y

Draw \overrightarrow{AY} to cut \overline{BC} at H

Prove that : C is the midpoint of \overline{BH}

**16 In the opposite figure :**

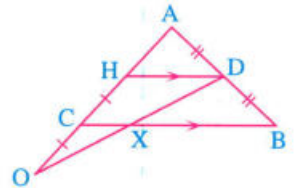
ABC is a triangle, D is the midpoint of \overline{AB} ,

$\overrightarrow{DH} \parallel \overline{BC}$, $O \in \overline{AC}$ such that $HC = CO$

Prove that : $CO = \frac{1}{3} AO$

If we draw \overline{DO} to cut \overline{BC} at X,

then prove that : $OX = XD$

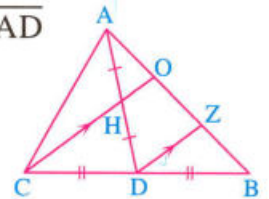
**17 In the opposite figure :**

ABC is a triangle, D is the midpoint of \overline{BC} and H is the midpoint of \overline{AD}

Draw \overline{CH} to cut \overline{AB} at O,

then draw $\overrightarrow{DZ} \parallel \overline{CO}$ to cut \overline{AB} at Z

Prove that : $AO = OZ = ZB$

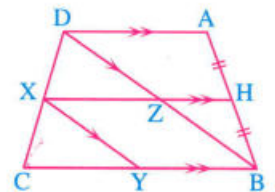
**18 In the opposite figure :**

ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$

Let H be the midpoint of \overline{AB} ,

$\overline{HX} \parallel \overline{BC}$ and $\overline{XY} \parallel \overline{DB}$

Prove that : Y is the midpoint of \overline{BC}

**19** ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, E is the midpoint of \overline{AB} , draw $\overrightarrow{EX} \parallel \overline{BC}$ to cut \overline{DB} at X, \overline{DC} at Y, and draw $\overrightarrow{YZ} \parallel \overline{DB}$ to cut \overline{BC} at Z

Prove that : $XD = YZ$

20 ABC is a triangle in which $AB = 9$ cm., $AC = 8$ cm., $D \in \overline{AB}$,

$E \in \overline{AB}$ such that $AD = DE = EB$ and \overrightarrow{DX} , \overrightarrow{EY} are drawn parallel to \overline{BC} and cutting \overline{AC} at X and Y respectively, where $DX = 4$ cm.

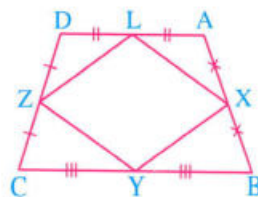
Calculate : The perimeter of the shape DEYX

« $17\frac{2}{3}$ cm. »

21 In the opposite figure :

ABCD is a quadrilateral in which X, Y, Z and L are the midpoints of \overline{AB} , \overline{BC} , \overline{CD} and \overline{DA} respectively.

Prove that : XYZL is a parallelogram.

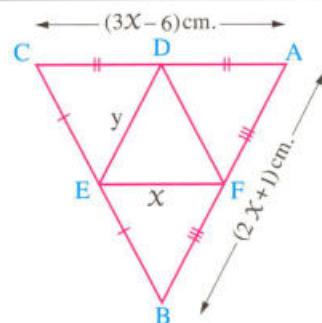


22 ABC is a triangle in which $AB = AC$, X, Y and Z are the midpoints of \overline{AB} , \overline{BC} and \overline{CA} respectively. **Prove that :** AXYZ is a rhombus.

23 Connecting with algebra :

In the opposite figure :

Find the value of each of : x and y



« 6 cm., 6.5 cm. »

Life Application

24 Sara wants to design a kite whose two diagonals are of lengths 64 cm. and 90 cm. She wants to put a stripe to decorate the kite such that the stripe joins the midpoints of the sides of the kite. How long is the stripe ?



« 154 cm. »

For excellent pupils

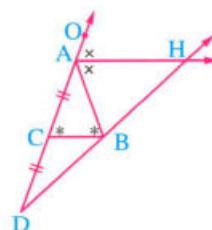
25 In the opposite figure :

ABC is a triangle in which : $m(\angle ABC) = m(\angle ACB)$

, $D \in \overline{AC}$ such that $AC = CD$ and $O \in \overline{CA}$

Let \overline{AH} bisect $\angle BAO$ such that : $\overline{AH} \cap \overline{DB} = \{H\}$

Prove that : $DB = BH$





Exercise

7

Pythagoras' theorem

From the school book



Remember Understand Apply Problem Solving



Interactive test

1 In each of the following figures, find the length of the unknown side :

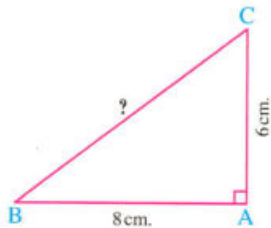


Fig. (1)

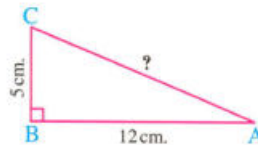


Fig. (2)

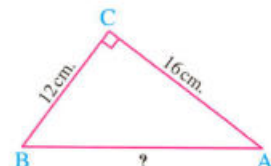


Fig. (3)

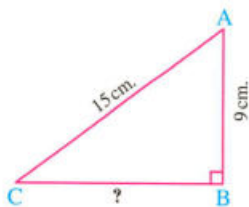


Fig. (4)

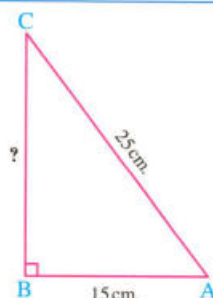


Fig. (5)

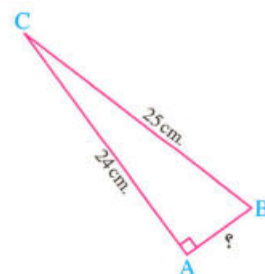


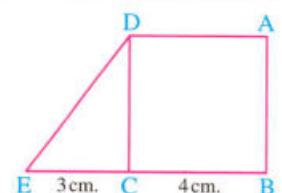
Fig. (6)

2 In the opposite figure :

ABCD is a square whose side length = 4 cm.

and $E \in \overline{BC}$ where $CE = 3$ cm.

Find : The length of \overline{DE}



« 5 cm. »

3 In the opposite figure :

$\overline{AD} \perp \overline{BC}$, $BD = 9$ cm. ,
 $DC = 16$ cm. and $AC = 20$ cm.

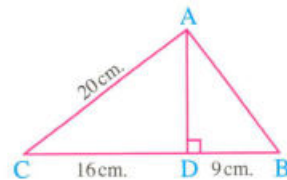
Find :

1 AD

2 AB

3 The area of $\triangle ABC$

« 12 cm. , 15 cm. , 150 cm.² »



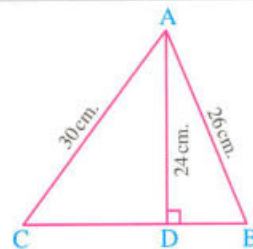
4 In the opposite figure :

ABC is a triangle and $\overline{AD} \perp \overline{BC}$

If $AD = 24$ cm. , $AB = 26$ cm. and $AC = 30$ cm.

Find BC and calculate the area of $\triangle ABC$

« 28 cm. , 336 cm.² »



5 In the opposite figure :

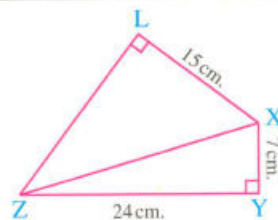
XYZL is a quadrilateral in which :

$m(\angle XYZ) = m(\angle XLZ) = 90^\circ$,

$XY = 7$ cm. , $YZ = 24$ cm. and $XL = 15$ cm.

Find : The length of each of \overline{XZ} and \overline{ZL}

« 25 cm. , 20 cm. »



6 In the opposite figure :

$m(\angle B) = m(\angle ACD) = 90^\circ$, $AB = 9$ cm. ,

$BC = 12$ cm. and $DC = 20$ cm.

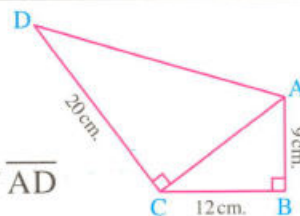
Find : 1 The length of \overline{AC}

2 The length of \overline{AD}

3 The perimeter of the figure ABCD

4 The area of the figure ABCD

« 15 cm. , 25 cm. , 66 cm. , 204 cm.² »



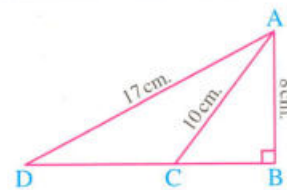
7 In the opposite figure :

$\triangle ABD$ is a right-angled triangle at B

, $AB = 8$ cm. , $AD = 17$ cm.

and $C \in \overline{BD}$ such that $AC = 10$ cm.

Find : The length of each of \overline{CB} , \overline{BD} and \overline{CD}



« 6 cm. , 15 cm. , 9 cm. »

8 In the opposite figure :

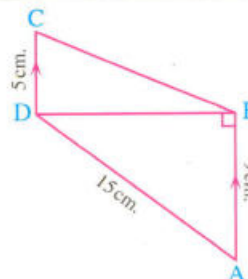
$m(\angle ABD) = 90^\circ$, $\overline{BA} \parallel \overline{CD}$

, $AB = 9$ cm. , $AD = 15$ cm.

and $DC = 5$ cm.

Calculate : The length of \overline{BC}

« 13 cm. »



9 In the opposite figure :

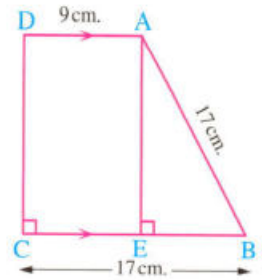
ABCD is a trapezium , $\overline{AD} \parallel \overline{BC}$

, $m(\angle DCB) = 90^\circ$, $\overline{AE} \perp \overline{BC}$

If $AB = BC = 17$ cm. and $AD = 9$ cm.

, **find** : The length of \overline{DC}

and calculate : the area of the trapezium ABCD



« 15 cm. , 195 cm². »

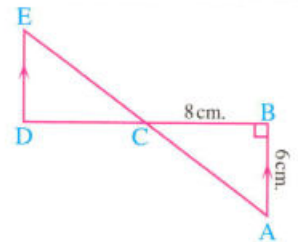
10 In the opposite figure :

$\overline{BD} \cap \overline{AE} = \{C\}$, $\overline{AB} \parallel \overline{DE}$

, $AB = 6$ cm. , $BC = 8$ cm.

and C is the midpoint of \overline{BD}

Calculate : The length of \overline{CE}



« 10 cm. »

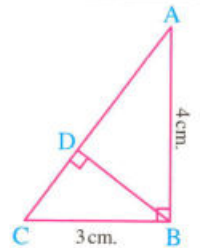
11 In the opposite figure :

ABC is a right-angled triangle at B

, $\overline{BD} \perp \overline{AC}$, $AB = 4$ cm.

and $BC = 3$ cm.

Calculate : The length of \overline{BD}



« 2.4 cm. »

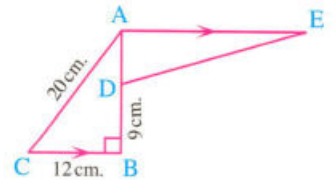
12 In the opposite figure :

ABC is a triangle , $m(\angle B) = 90^\circ$

, $\overline{AE} \parallel \overline{BC}$, If $BC = 12$ cm. , $AC = 20$ cm.

, $D \in \overline{AB}$ where $BD = 9$ cm. and $AE = 2 BC$

Find : The length of each of \overline{AD} and \overline{ED}



« 7 cm. , 25 cm. »

13 Complete the following :

- 1 In the right-angled triangle , the area of the square on the hypotenuse equals
- 2 If XYZ is a right-angled triangle at X , $XY = 12$ cm. and $XZ = 9$ cm. , then $YZ = \dots\dots\dots$ cm.
- 3 If ABC is a right-angled triangle at B , $AB = 20$ cm. and $AC = 25$ cm. , then $BC = \dots\dots\dots$ cm.

4 In the opposite figure :

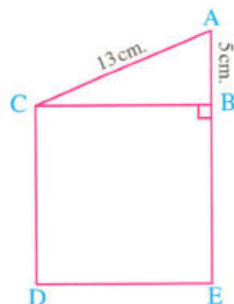
If $m(\angle ABC) = 90^\circ$

, $AB = 5$ cm.

and $AC = 13$ cm.

, then the area of

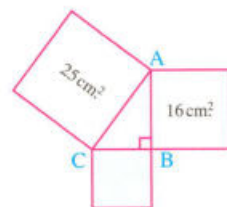
the square $BEDC = \dots\dots\dots \text{cm}^2$



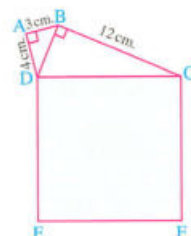
5 A rectangle is of length 8 cm. and width 6 cm. ,
then the length of its diagonal equals $\dots\dots\dots$ cm.

6 If the area of a rectangle equals 60 cm^2 and its width is 5 cm. ,
then the length of its diagonal = $\dots\dots\dots$ cm.

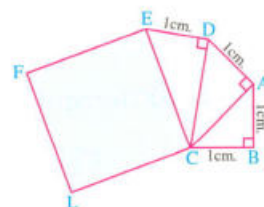
7 If $\triangle ABC$ is right-angled at B ,
then the side length of the shaded
square = $\dots\dots\dots$ cm.



8 If $\triangle ABD$ is right-angled at A
and $\triangle BCD$ is right-angled at B ,
then the area of the shaded
square = $\dots\dots\dots \text{cm}^2$



9 If the triangles ABC , ACD and DCE
are right-angled at B , A and D respectively ,
 $AB = BC = AD = DE = 1$ cm.
, then the area of the shaded
square = $\dots\dots\dots \text{cm}^2$



14 Choose the correct answer from those given :

1 In the opposite figure :

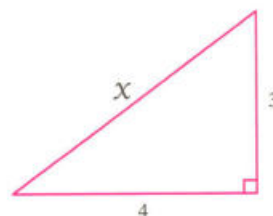
Which of the following relations is true ?

(a) $x = 4^2 + 3^2$

(b) $x^2 = 4^2 - 3^2$

(c) $x^2 + 9 = 16$

(d) $x^2 = 25$



2 In the opposite figure :

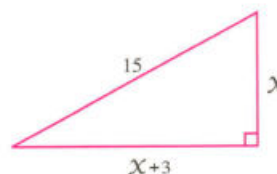
Which of the following relations is true ?

(a) $x + 3 + x = 15$

(b) $x^2 + 3x = 108$

(c) $(x + 3)^2 = 15 - x^2$

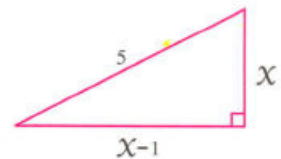
(d) $x^2 + 6x + 9 = 225$



3 In the opposite figure :

Which of the following relations is true ?

- (a) $x^2 + (x - 1)^2 = 5$ (b) $x + (x - 1) = 25$
 (c) $x^2 - x = 12$ (d) $(x - 1)^2 - x^2 = 25$



4 If ABCD is a square , then $(AC)^2 = \dots\dots\dots$

- (a) AB (b) $(AB)^2$ (c) $2 (AB)^2$ (d) $4 (AB)^2$

Life Applications

15 A window cleaner has a ladder which is 5 metres long.

He places it so that it reaches a window till 4 metres from the ground.

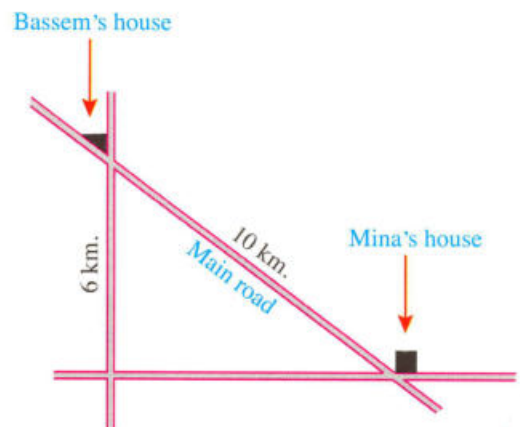
How far is the wall from the foot of the ladder ?



« 3 m. »

16 If Mina wants to go to the house of his friend Bassem.

What is the distance saved if he takes the main road instead of the other two roads ?



« 4 km. »

For excellent pupils

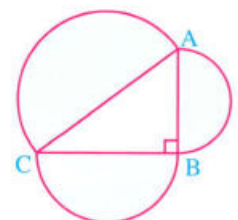
17 If $\triangle ABC$ is right-angled at B , D is the midpoint of \overline{BC} ,

prove that : $(AC)^2 - (AD)^2 = 3 (BD)^2$

18 In the opposite figure :

Prove that the sum of areas of the two semicircles drawn on the two sides of the right angle in a right-angled triangle equals the area of the semicircle drawn on the hypotenuse.

[Given the area of the circle = πr^2]





Exercise

8

Geometric transformations

From the school book

Remember

Understand

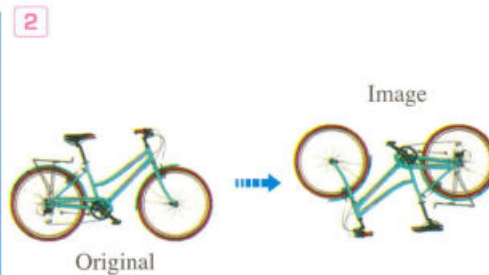
Apply

Problem Solving

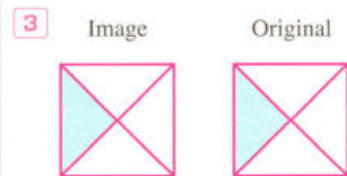
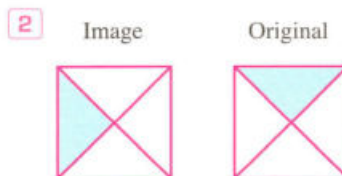
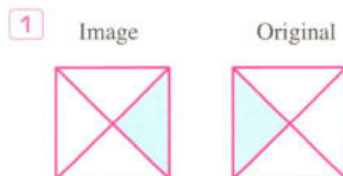


Interactive test

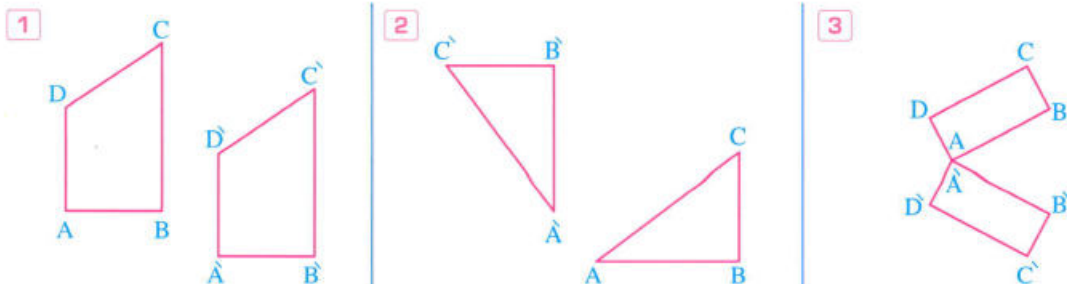
- 1 Describe the type of the geometric transformation (reflection, translation or rotation) in each of the following :



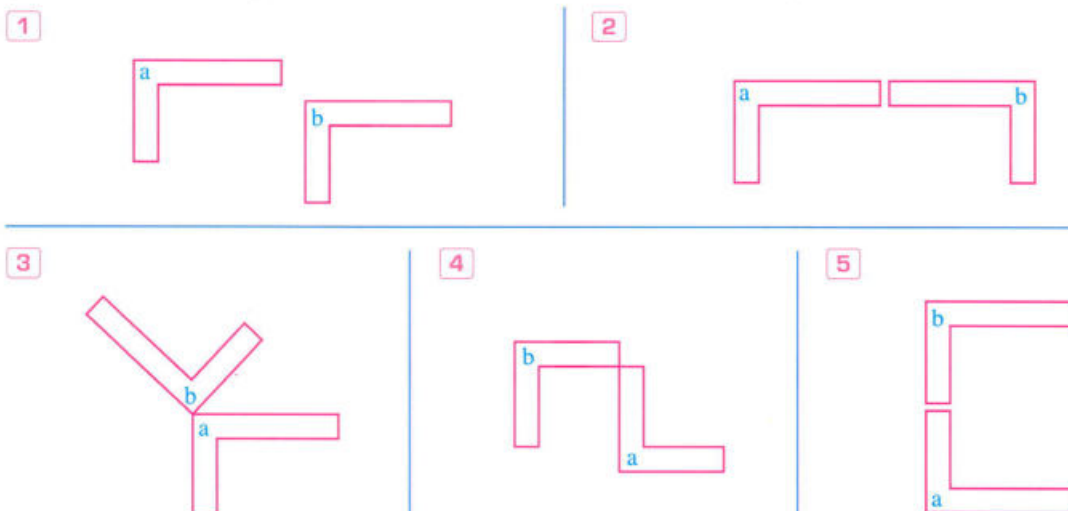
- 2 Write the type of the geometric transformation (reflection, translation or rotation) :



- 3 Describe the type of transformation in each of the following figures (reflection, translation, rotation) :

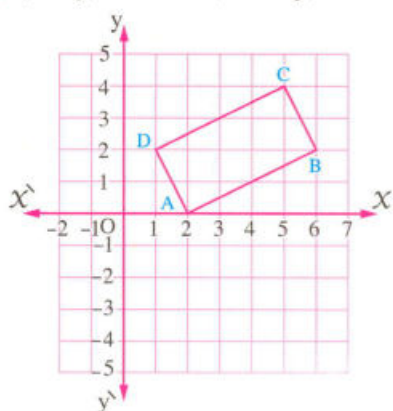


- 4 Figure b is the image of figure a by a geometric transformation. Identify each transformation as (a translation, a reflection or a rotation) :

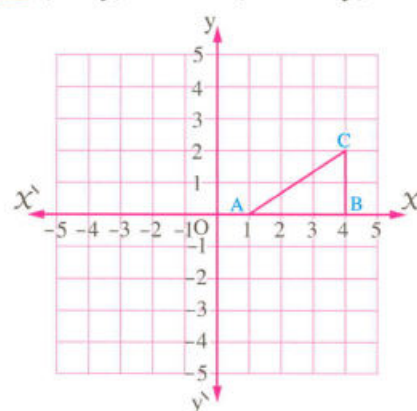


- 5 Draw the image of each figure according to the shown transformation, then describe each type :

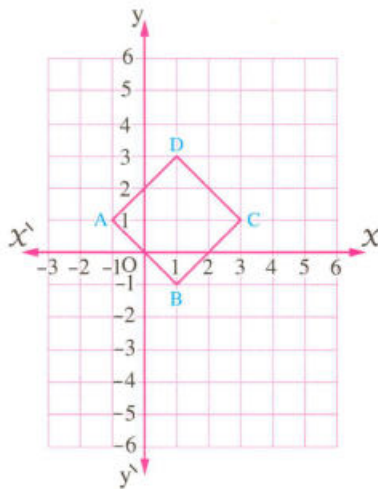
1 $(X, y) \longrightarrow (-X, y)$



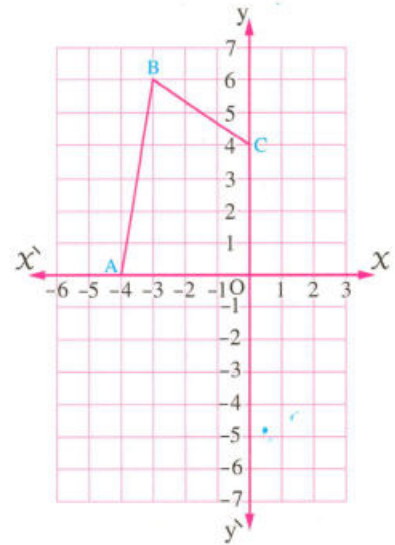
2 $(X, y) \longrightarrow (-X, -y)$



3 $(X, y) \longrightarrow (X + 2, y + 3)$



4 $(X, y) \longrightarrow (y, -X)$



- 6 Draw the image of $\triangle ABC$ where A (1, 2), B (3, 2) and C (3, 5) by each of the following transformations :

1 $(X, y) \longrightarrow (X, -y)$

2 $(X, y) \longrightarrow (X + 1, y - 3)$

3 $(X, y) \longrightarrow (-y, X)$

- 7 Draw the image of the polygon ABCDEO according to each transformation and describe the type :

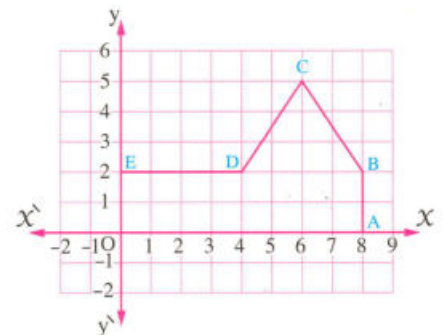
1 $(X, y) \longrightarrow (-X, y)$

2 $(X, y) \longrightarrow (X, y + 5)$

3 $(X, y) \longrightarrow (-X, -y)$

4 $(X, y) \longrightarrow (X - 5, y)$

5 $(X, y) \longrightarrow (X, -y)$



For excellent pupils

- 8 Draw $\triangle ABC$ whose image is $\triangle \hat{A} \hat{B} \hat{C}$ by the transformation $(X, y) \longrightarrow (-y, X)$ where \hat{A} (1, -1), \hat{B} (3, 1) and \hat{C} (4, -1), then describe the transformation type.



Exercise

9

Reflection in a straight line

From the school book



Remember

Understand

Apply

Problem Solving



Interactive test

First Problems on reflection in the plane :

1 Draw the image of each of A , \overline{AB} and $\triangle ABC$ by reflection in the straight line L :

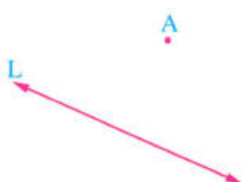


Fig. (1)

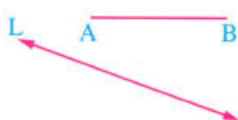


Fig. (2)

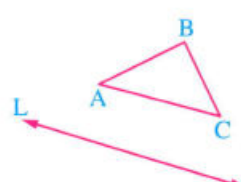
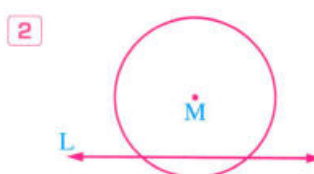
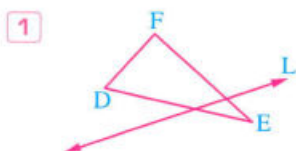


Fig. (3)

2 Copy the figures below in your notebook , then draw the images of $\triangle DEF$ and the circle M by reflection in L :



3 Draw the triangle ABC in which : $AB = 6$ cm. , $m(\angle A) = 90^\circ$ and $m(\angle B) = 30^\circ$, then draw its image by reflection in \overleftrightarrow{AB}

4 Draw the image of $\triangle ABC$ in which : $AB = 3$ cm. , $BC = 4$ cm. and $AC = 5$ cm. by reflection in the straight line containing the shortest side.

5 Draw the image of $\triangle XYZ$ in which : $XY = 3$ cm. , $YZ = 5$ cm. and $ZX = 7$ cm. by reflection in the straight line containing the longest side.

6 Draw the rectangle ABCD in which : $AB = 6$ cm. and $CB = 4$ cm. , then draw its image by reflection in \overleftrightarrow{AD} . Say the name of the resulting figure which consists of the rectangle and its image , then find its perimeter. « 32 cm. »

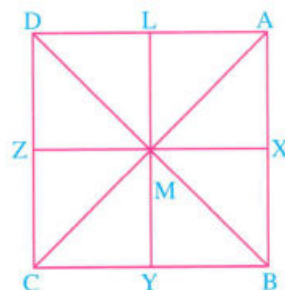
7 Draw $\triangle ABC$ where $BC = 3$ cm. , $AB = 4$ cm. and $AC = 5$ cm. If the point D is the image of the point C by reflection in \overleftrightarrow{AB} Find :
 1 The perimeter of $\triangle ACD$
 2 The area of $\triangle ACD$ « 16 cm. , 12 cm² »

8 In the opposite figure :

ABCD is a square. M is the point of intersection of its diagonals. X , Y , Z and L are the midpoints of its sides \overline{AB} , \overline{BC} , \overline{CD} and \overline{DA} respectively.

Complete the following :

- 1 The image of the point A by reflection in \overleftrightarrow{LY} is
- 2 The image of the \overline{AM} by reflection in \overleftrightarrow{XM} is
- 3 The image of the $\triangle ALM$ by reflection in \overleftrightarrow{LY} is
- 4 The image of the $\triangle ALM$ by reflection in \overleftrightarrow{AM} is
- 5 The image of the $\triangle AMB$ by reflection in \overleftrightarrow{LY} is
- 6 The image of the $\triangle AMB$ by reflection in \overleftrightarrow{XZ} is
- 7 The image of the square AXML by reflection in \overleftrightarrow{LY} is
and by reflection in \overleftrightarrow{AM} is
- 8 The image of the square ABCD by reflection in \overleftrightarrow{LY} is
- 9 $\triangle MZD$ is the image of $\triangle MZC$ by reflection in
- 10 $\triangle AXM$ is the image of $\triangle CYM$ by reflection in

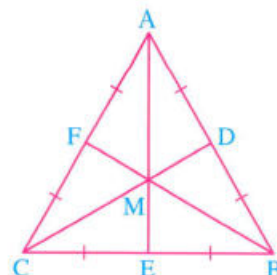


9 In the opposite figure :

$\triangle ABC$ is an equilateral triangle , where D , E and F are the midpoints of \overline{AB} , \overline{BC} and \overline{AC} respectively and $\overline{AE} \cap \overline{BF} \cap \overline{CD} = \{M\}$:

Complete :

- 1 The axes of symmetry of $\triangle ABC$ are
- 2 \overline{AB} is the reflected image of \overline{AC} by reflection in
- 3 The reflected image of \overline{AF} by reflection in \overleftrightarrow{BF} is
and the reflected image of \overline{CF} in \overleftrightarrow{AE} is
- 4 The reflected image of $\triangle AMD$ by reflection in \overleftrightarrow{AE} is
 $\therefore m(\angle AMD) = m(\angle \dots\dots\dots)$ because reflection in a line reserves
- 5 The reflected image of $\triangle AMB$ by reflection in \overleftrightarrow{AE} is
- 6 $\triangle BMC$ is the reflected image of by reflection in \overleftrightarrow{CD} and the reflected image of by reflection in \overleftrightarrow{BF}
 $\therefore BM = AM$, and $CM = AM$ because the reflection reserves



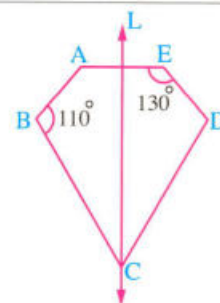
10 Complete the following :

- 1 The reflection in a plane reserves :,,,
- 2 If the reflection in a straight line transforms the figure to itself , then this straight line is called
- 3 **The number of axes of symmetry of :**

(a) The equilateral triangle is	(b) The isosceles triangle is
(c) The scalene triangle is	(d) The parallelogram is
(e) The rectangle is	(f) The rhombus is
(g) The square is	(h) The trapezium which is not isosceles is
(i) The isosceles trapezium is	(j) The circle is

11 In the opposite figure :

If the straight line L
is the axis of symmetry
of the figure ABCDE ,
calculate : $m(\angle BCD)$



« 60° »

- 12** By using geometric instruments , draw the rectangle ABCD , where $AB = 3$ cm. and $BC = 4$ cm. locate \hat{A} as the reflected image of A by reflection in \overleftrightarrow{CD} and locate \hat{C} as the reflected image of C by reflection in \overleftrightarrow{AB}

Prove that :

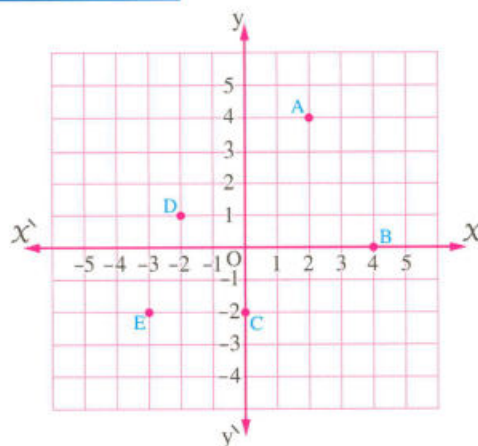
- 1 $m(\angle \hat{C}AC) = 2 m(\angle CAB)$
- 2 $\overleftrightarrow{AC} \parallel \overleftrightarrow{\hat{A}\hat{C}}$

Second Problems on reflection in the Cartesian plane :

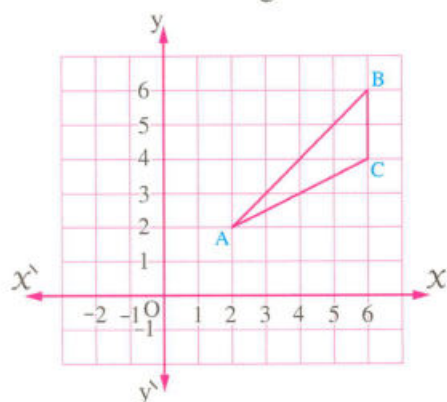
1 In the opposite figure :

Write the coordinates of the image of each point by reflection in :

- 1 The x -axis
- 2 The y -axis

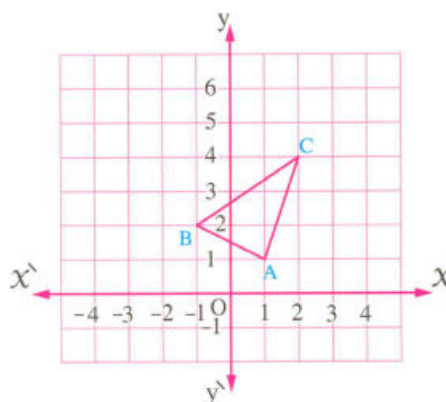


2 Copy each of the following figures on a lattice and draw the image of the figure by a geometric transformation as shown below each figure , then write the coordinates of each vertex of the figure.



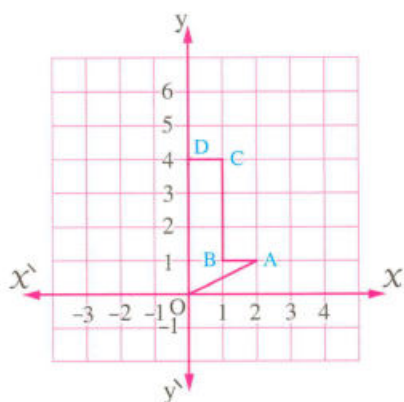
reflection in the x -axis

Fig. (1)



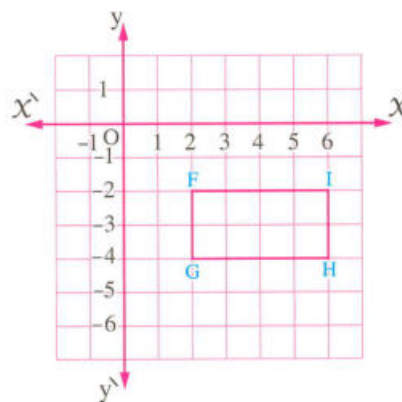
reflection in the y -axis

Fig. (2)







reflection in the y -axis

Fig. (3)



reflection in \overleftrightarrow{FG}

Fig. (4)

- 3 Draw \overline{AB} where A (4 , 3) and B (1 , - 2) , then draw its image by reflection in :
- 1 The X -axis. 2 The y -axis.
-
- 4 If A (3 , 1) and B (3 , - 2) , draw \overline{DC} which is the image of \overline{AB} by reflection in the y -axis and name the figure ABCD and calculate its perimeter. « 18 length units »
-
- 5  Draw the image of $\triangle ABC$ where A (- 6 , - 1) , B (- 2 , - 1) and C (- 5 , - 6) by reflection in the X -axis.
-
- 6 Draw the image of $\triangle OBC$ where O (0 , 0) , B (3 , 0) and C (- 1 , 2) by reflection in the y -axis.
-
- 7 On a square lattice , draw $\triangle ABC$ where A (2 , - 2) , B (3 , 4) and C (- 3 , 2) , then draw $\triangle \hat{A}\hat{B}\hat{C}$ which is the image of $\triangle ABC$ by reflection in the y -axis , then draw $\triangle \hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}$ which is the image of $\triangle \hat{A}\hat{B}\hat{C}$ by reflection in the X -axis.
-
- 8 On a square lattice , draw the rectangle whose vertices are A (3 , 2) , B (8 , 2) , C (8 , 6) and D (3 , 6) , then draw its image by reflection in the y -axis.
-
- 9  Graph the square ABCD and its image by reflection in the X -axis , then compare the lengths of the sides and the area where A (0 , 2) , B (- 5 , 0) , C (- 3 , - 5) and D (2 , - 3)
-
- 10 ABCD is a rectangle in which : A (1 , 1) , B (1 , 3) and C (- 3 , 3) Determine the coordinates of D from the graph , then draw the image of the rectangle ABCD by reflection in the X -axis.
-
- 11  Draw the image of the square ABCD where A (2 , 3) and B (2 , - 1) by reflection in the y -axis. What do you notice ?
-
- 12  Draw the image of the rectangle XYZL where X (2 , 2) and Y (- 3 , 2) with width 3 units by reflection in the X -axis.

13 Complete the following table :

No.	The point	Its image by reflection in the X -axis	Its image by reflection in the y -axis
1	$(3, -2)$
2	$(1, 2)$
3	$(-2, 4)$
4	$(0, 5)$
5	$(3, 0)$
6	$(0, 0)$

14 Complete the following :

- The image of the point $(1, 3)$ by reflection in the X -axis is
- The image of the point $(-2, 5)$ by reflection in the y -axis is
- The image of the point $(2, -3)$ by reflection in the is $(2, 3)$
- The image of the point $(-1, -4)$ by reflection in the is $(1, -4)$
- The image of the point $(0, 3)$ by reflection in the is itself.
- The image of the point $(-5, 0)$ by reflection in the is itself.
- The image of the point $(2, 1)$ by reflection in the X -axis followed by reflection in the y -axis is
- The image of the point $(2, -3)$ by reflection in the y -axis followed by reflection in the X -axis is
- If $\hat{A}(-2, 3)$ is the image of the point $A(2, 3)$ by reflection in y -axis, then the image of the point \hat{A} by reflection in the y -axis is



For excellent pupils

15 Determine on a square lattice the points $A(5, 4)$, $B(5, 1)$, $C(2, 1)$, $\hat{A}(4, 5)$, $\hat{B}(1, 5)$ and $\hat{C}(1, 2)$

- If $\Delta \hat{A}\hat{B}\hat{C}$ is the image of ΔABC by reflection in the straight line L , draw this straight line.
- If the figure $ABB\hat{A}$ is the image of the figure $CB\hat{B}\hat{C}$ by reflection in the straight line M , draw this straight line.



Exercise

10

Reflection
in a point

From the school book



Interactive test

Remember

Understand

Apply

Problem Solving

First Problems on reflection in the plane :**1** Choose the correct answer from the given ones :

- 1 If $\overleftrightarrow{A'B'}$ is the image of \overleftrightarrow{AB} by reflection in M , then $\overleftrightarrow{A'B'}$ \overleftrightarrow{AB}

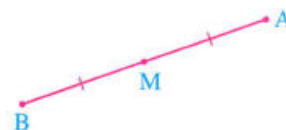
(a) $>$ (b) $<$ (c) $=$ (d) \neq

- 2 In the opposite figure :

The image of \overleftrightarrow{AB}

by reflection in the point M is

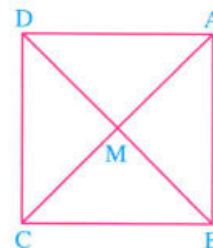
(a) \overleftrightarrow{AM} (b) \overleftrightarrow{BA} (c) \overleftrightarrow{BM}



- 3 In the opposite figure :

ABCD is a square whose diagonals intersect at M , the image of $\triangle ABM$ by reflection in M is \triangle

(a) $\triangle ADM$ (b) $\triangle BCM$
(c) $\triangle DCM$ (d) $\triangle CDM$



- 4 If \hat{A} is the image of A by reflection in M and $MA = 5$ cm. , then $AA' =$

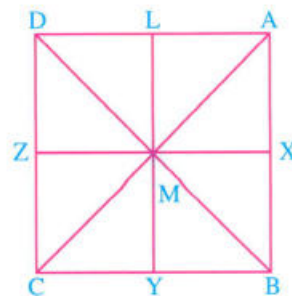
(a) 5 cm. (b) 7 cm. (c) 10 cm. (d) 15 cm.

2 In the opposite figure :

ABCD is a square whose diagonals intersect at M
 , X , Y , Z and L are the midpoints of \overline{AB} , \overline{BC} , \overline{CD}
 and \overline{DA} respectively

Complete the following :

- 1 The image of the point A by reflection in M is
- 2 The image of the point X by reflection in M is
- 3 The image of \overline{AL} by reflection in M is
- 4 The image of \overline{MZ} by reflection in M is
- 5 The image of \overline{BM} by reflection in M is
- 6 The image of \overline{AX} by reflection in X is
- 7 The image of $\triangle ALM$ by reflection in M is
- 8 The image of $\triangle BXM$ by reflection in M is
- 9 The image of $\triangle AMB$ by reflection in M is
- 10 The image of the square AXML by reflection in M is



3 Draw $\triangle ABC$ in which $AB = BC = 4$ cm. and $AC = 5$ cm. , then draw its image by reflection in the point B

4 In each of the following figures, draw $\triangle A'B'C'$ as the image of $\triangle ABC$ by reflection in the point B and mention the name of the figure $AC'A'C$ giving reason.

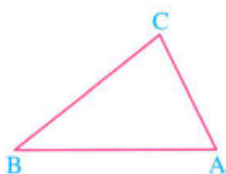


Fig. (1)

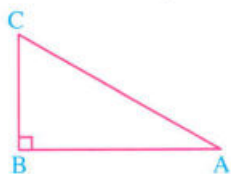


Fig. (2)

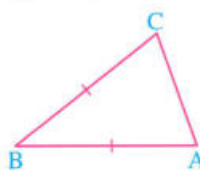


Fig. (3)

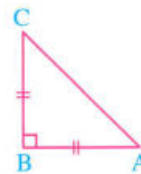


Fig. (4)

5 Draw $\triangle ABC$ in which $BC = 3$ cm. , $AB = 4$ cm. and $m(\angle B) = 90^\circ$, then draw $\triangle A'B'C'$ as the image of $\triangle ABC$ by reflection in C Prove that the quadrilateral $ABA'B'$ is a parallelogram.

6 Draw the square ABCD whose side length is 5 cm. , then draw its image by reflection in the point M where M is the point of intersection of its diagonals. What do you observe ?

- 7 ABC is a triangle , F is the midpoint of \overline{AC} Draw D as the image of B by reflection in F
What is the type of the figure ABCD and what is the type of the triangle ABC required to transfer the figure ABCD to :

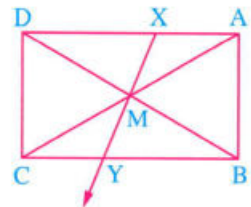
- 1 Rectangle. 2 Rhombus.

8  In the opposite figure :

ABCD is a rectangle , M is the point of intersection of its diagonals , $X \in \overline{AD}$ and $\overline{XM} \cap \overline{BC} = \{Y\}$

Prove that :

- 1 Y is the reflected image of X in M
2 The figure AXC Y is a parallelogram.

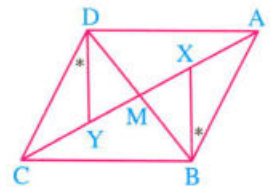


9  In the opposite figure :

ABCD is a parallelogram , M is the point of intersection of its diagonals and $X \in \overline{AC}$, $Y \in \overline{AC}$
such that $m(\angle ABX) = m(\angle CDY)$

Prove that :

- 1 ΔABX is the image of ΔCDY by reflection in M
2 The figure XBYD is a parallelogram.



Second Problems on reflection in the Cartesian plane :

1 Choose the correct answer from those given :

- 1 The image of the point $(-3, 2)$ by reflection in the origin point is
(a) $(3, 2)$ (b) $(-3, -2)$ (c) $(3, -2)$ (d) $(-3, 2)$
- 2 The point $(5, -2)$ is the image of the point by reflection in the origin point.
(a) $(5, -2)$ (b) $(-5, -2)$ (c) $(-5, 2)$ (d) $(5, 2)$
- 3 The point whose image by reflection in the origin point is itself is
(a) $(0, 1)$ (b) $(1, 0)$ (c) $(0, 0)$ (d) $(-1, 0)$
- 4 The image of the point $(3, -2)$ by reflection in the origin point followed by reflection in X-axis is
(a) $(3, -2)$ (b) $(-3, -2)$ (c) $(-3, 2)$ (d) $(3, 2)$

- 2 On a square lattice , draw $\triangle ABC$ where $A(3, 1)$, $B(1, 4)$ and $C(0, 0)$, then draw its image by reflection in the point C
- 3 In XY -coordinate plane , draw $\triangle ABC$, where $A(-2, 4)$, $B(5, 0)$ and $C(3, -3)$, then draw the reflected image of $\triangle ABC$ in the origin point.
- 4 $ABCD$ is a rectangle where $A(2, 5)$, $B(6, 5)$, $C(6, 8)$ and $D(2, 8)$, then draw the image of the rectangle $ABCD$ by reflection in the origin point.



For excellent pupils

- 5 In the opposite figure :

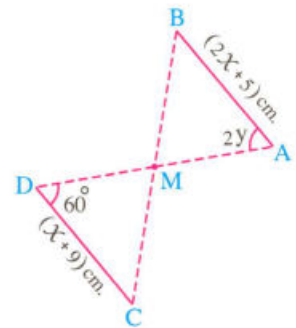
If \overline{CD} is the image of \overline{BA}

by reflection in the point M and $BA = (2x + 5)$ cm. ,

$CD = (x + 9)$ cm. , $m(\angle A) = 2y$ and $m(\angle D) = 60^\circ$

Find : 1 The length of \overline{CD}

2 The value of y



« 13 cm. , 30° »

For the next year,

ask for



in

**Math, Science
& English**

for 2nd prep.





Exercise

11

Translation

From the school book



● Remember

● Understand

● Apply

● Problem Solving



Interactive test

First Problems on translation in the plane :

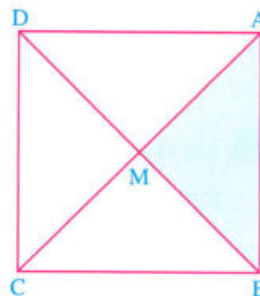
- 1 Draw a line segment \overline{AB} where $AB = 5$ cm. , then draw the image of \overline{AB} by a translation of magnitude of 8 cm. in the direction of \overrightarrow{AB}
- 2 Using the geometric instruments , draw the square ABCD whose side length is 4 cm. , then draw its image by translation of magnitude of 4 cm. in the direction of \overrightarrow{AB}
- 3 Draw $\triangle ABC$ in which $AB = 4$ cm. , $BC = 6$ cm. and $CA = 5$ cm. , then draw the image of $\triangle ABC$ by a translation of magnitude of 3 cm. in the direction of \overrightarrow{CB}

4 In the opposite figure :

ABCD is a square whose side length is 4 cm.

M is the point of intersection of its diagonals. Draw :

- 1 The image of $\triangle MAB$ by the translation of distance 2 cm. in the direction of \overrightarrow{AD}
- 2 The image of $\triangle AMB$ by the translation AM in the direction of \overrightarrow{AM}

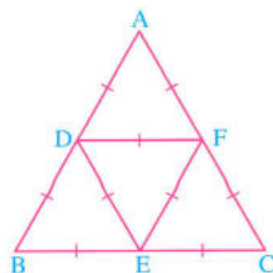


5 In the opposite figure :

The triangles $\triangle ADF$, $\triangle BDE$, $\triangle DEF$ and $\triangle EFC$ are congruent.

Complete :

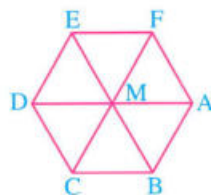
- 1 The image of $\triangle ADF$ by a translation of magnitude of AD in the direction of \overrightarrow{AD} is
- 2 $\triangle FEC$ is the image of $\triangle DBE$ by a translation of magnitude in the direction of



6 In the opposite figure :

ABCDEF is a regular hexagon. **Complete the following :**

- 1 The image of the point D by translation DM in the direction of \overrightarrow{DM} is
- 2 The image of \overline{AF} by translation ED in the direction of \overrightarrow{ED} is
- 3 The image of $\triangle MCD$ by translation EF in the direction of \overrightarrow{EF} is
- 4 The translation which makes $\triangle DME$ the image of $\triangle MAF$ is

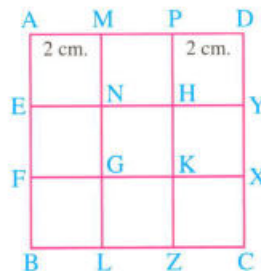


7 In the opposite figure :

ABCD is a square and all the interior squares are congruent.

Complete :

- 1 The image of \overline{AE} by a translation of magnitude of 2 cm. in the direction of \overrightarrow{GK} is
- 2 The image of the square AENM by a translation of magnitude of 4 cm. in the direction of \overrightarrow{PK} is
- 3 The square MNHP is the image of the square GLZK by a translation of magnitude in the direction of



8 $\triangle ABC$ is right-angled at B where $AB = 3$ cm. and $BC = 4$ cm. If $\triangle A'B'C'$ is the image of $\triangle ABC$ by translation of a distance 3 cm. in the direction of \overrightarrow{CB}

Prove that : The figure $A'B'C'C$ is a parallelogram.

9 Draw $\triangle ABC$ which is right-angled at B, in which $AB = BC = 3$ cm., then draw the image of $\triangle ABC$ by translation of a distance 3 cm. in the direction of \overrightarrow{AB} , then prove that : The figure $BB'C'C$ is a square.

- 10** ABCD is a rectangle, where $E \in \overline{AD}$, draw the translated image of $\triangle ABE$ by translation of a magnitude DA in the direction of \overrightarrow{AD} . If \hat{E} is the image of E by the same translation, **prove that** : The figure BC \hat{E} E is a parallelogram.
- 11** ABCD is a parallelogram, $\overline{BE} \perp \overline{AD}$ cutting it at E, draw $\triangle \hat{A}\hat{B}\hat{D}$ as the image of $\triangle ABE$ by translation of a distance ED in the direction of \overrightarrow{AD} , **then prove that** : The figure EBB \hat{D} is a rectangle.

Second Problems on translation in the Cartesian plane :

1 Complete the following :

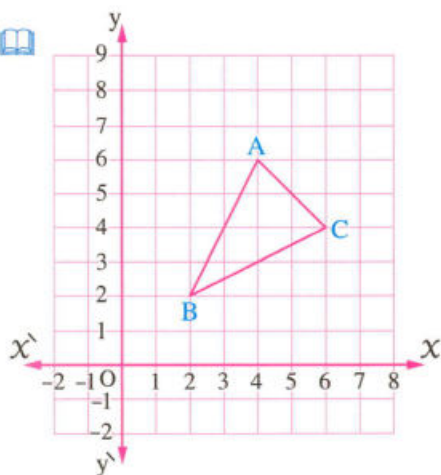
- 1** The image of the point (2, 5) by translation $(X, y) \longrightarrow (X + 2, y + 1)$ is
- 2** The image of the point (3, 2) by translation $(X, y) \longrightarrow (X + 3, y - 2)$ is
- 3** The image of the point (-5, 4) by translation $(X, y) \longrightarrow (X + 4, y - 5)$ is
- 4** The image of the point (-2, -5) by translation $(X, y) \longrightarrow (X - 2, y)$ is
- 5** The image of the point (3, -2) by translation $(X, y) \longrightarrow (X, y + 3)$ is

2 Choose the correct answer from those given :

- 1** The image of the point (-1, 2) by translation of magnitude of 3 units in the positive direction of the X-axis is
 (a) (-1, 5) (b) (2, 2) (c) (-2, 2) (d) (-1, 3)
- 2** The image of the point (-3, 4) by translation of magnitude of 4 units in the negative direction of the y-axis is
 (a) (-3, 0) (b) (-7, 4) (c) (-3, 8) (d) (-1, 4)
- 3** If \hat{A} (3, -3) is the image of A by translation $(X, y) \longrightarrow (X - 1, y - 4)$, then the point A is
 (a) (2, -7) (b) (4, 1) (c) (-4, -1) (d) (2, 1)
- 4** The image of the point (-1, 4) by the translation (3, -2) followed by reflection in the X-axis is
 (a) (2, 2) (b) (-2, 2) (c) (-2, -2) (d) (2, -2)
- 5** If the point (a, -1) is the image of (2, 4) by the translation $(X, y) \longrightarrow (X + 1, y - b)$, then (a, b) =
 (a) (3, 3) (b) (1, 3) (c) (3, 5) (d) (1, -5)
- 6** If \hat{A} is the image of the point A (2, 3) by reflection in the y-axis, then A is the image of \hat{A} by the translation
 (a) $(X, y) \longrightarrow (X + 4, y)$ (b) $(X, y) \longrightarrow (X, y + 6)$
 (c) $(X, y) \longrightarrow (X - 4, y)$ (d) $(X, y) \longrightarrow (X, y - 6)$

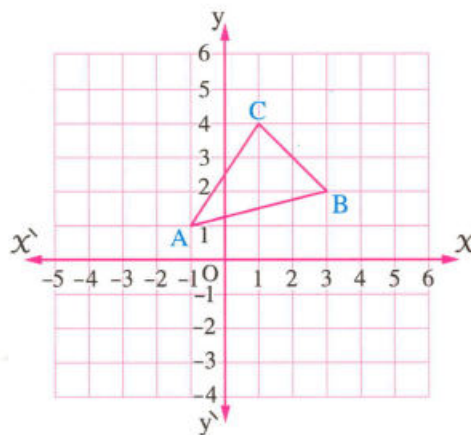
3 Draw the image of each of the following figures by the translation shown under each figure :

1



$$(X, y) \longrightarrow (X + 2, y + 3)$$

2



$$(X, y) \longrightarrow (X + 2, y)$$

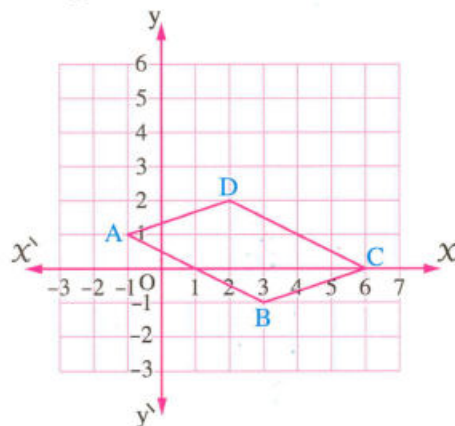
4 Copy the graph, then draw the image of the parallelogram ABCD under each of the following translations :

1 $(X, y) \longrightarrow (X + 5, y + 2)$

2 $(X, y) \longrightarrow (X - 8, y - 1)$

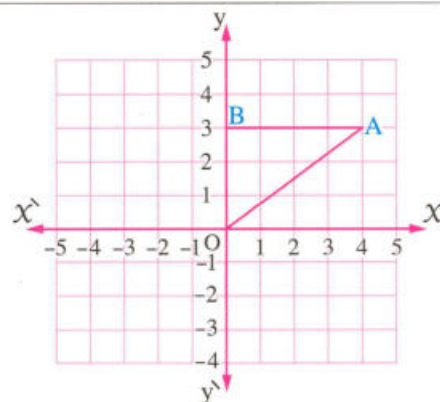
3 $(X, y) \longrightarrow (X + 2, y - 4)$

4 $(X, y) \longrightarrow (X - 4, y + 2)$

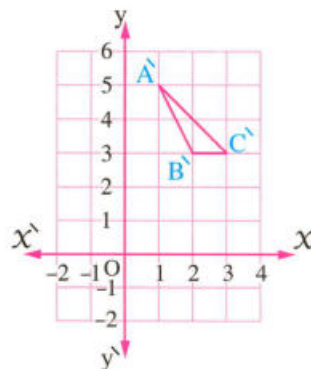


5 Using the square lattice, draw $\triangle OBC$ where O is the origin point, B (3, 0), C (0, 2), then draw its image by the translation $(X, y) \longrightarrow (X - 4, y + 1)$

6 Draw the image of $\triangle AOB$ by the translation of magnitude = AO and in the direction of \overrightarrow{AO}



- 7 Using the lattice, find the image of each of the following points by the translation of LM in the direction of \overrightarrow{LM} where : L (1 , 3) and M (4 , 5)
- 1 B (-2 , 3) 2 C (5 , 4) 3 D (3 , 0)
-
- 8 On a square lattice, draw $\triangle ABC$ where A (2 , 1) , B (1 , -1) and C (0 , 1) , then draw its image by a translation of 2 AB in the direction of \overrightarrow{AB}
-
- 9 A square has vertices A (1 , 1) , B (4 , 2) , C (3 , 5) and D (0 , 4)
- 1 Graph the square and its image under the translation which maps vertex A onto vertex B
- 2 Write the mapping rule for the translation.
-
- 10 Use the translation : $(X , y) \longrightarrow (X + 2 , y + 3)$ to locate the point whose image is (2 , 3)
-
- 11 If the image of the point A (1 , 1) by translation in the Cartesian plane is \hat{A} (2 , 2) , find the images of the points O (0 , 0) , B (-1 , 3) and C (-3 , 5) by the same translation.
-
- 12 If A (-3 , 1) and B (1 , -2) , write the mapping rule of the translation that makes B the image of A
-
- 13 If A (3 , 2) , B (5 , 1) , find :
- 1 \hat{C} which is the image of C (1 , -1) under translation of AB in the direction of \overrightarrow{AB}
- 2 D whose image is \hat{D} (2 , 1) under translation of AB in the direction of \overrightarrow{AB}
-
- 14 The point \hat{A} (3 , -3) is the image of the point A by the translation $(X , y) \longrightarrow (X - 1 , y - 4)$ Locate A , then by the same translation , draw the image of $\triangle ABC$ where B (5 , 0) and C (-1 , -2)
-
- 15 In the opposite figure :
- Copy the graph , then draw the triangle ABC whose image is $\triangle \hat{A}\hat{B}\hat{C}$ by the translation $(X , y) \longrightarrow (X + 2 , y + 3)$



16 State whether the graph shows a reflection or a translation :

1 Name the line of reflection.

2 Describe the translation.

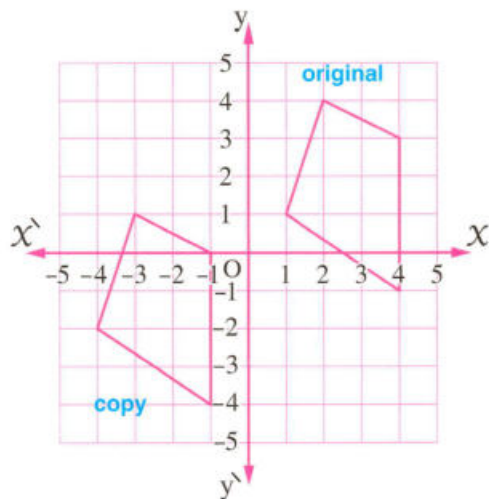


Fig. (1)

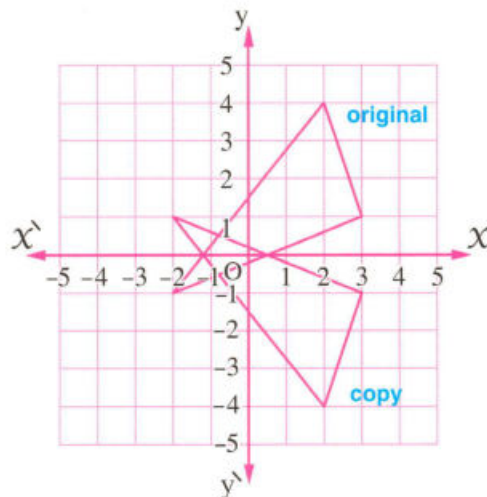


Fig. (2)

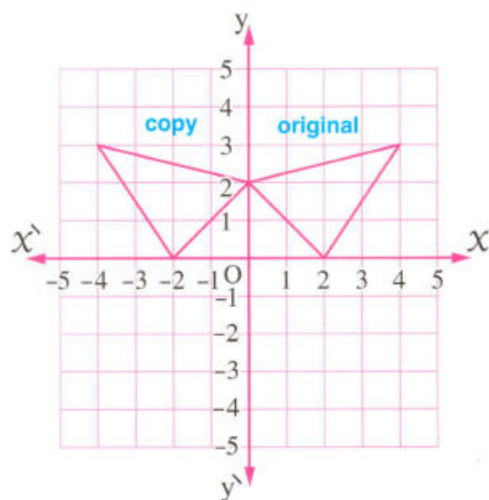


Fig. (3)

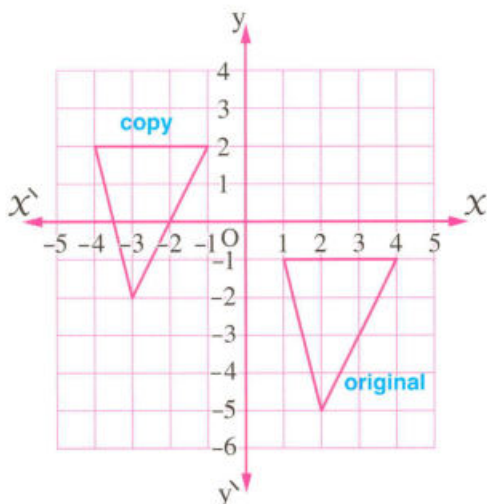


Fig. (4)

For excellent pupils

17 Draw $\triangle ABC$ on a square lattice where $A(4, 4)$, $B(4, 2)$ and $C(1, 2)$, then draw its image by the translation of magnitude 3 AB in the direction of \overrightarrow{AB}

18 If $A(2, 1)$ is the image of B by reflection in X -axis followed by reflection in y -axis, state the translation which makes A the image of the point B



Exercise

12

Rotation

From the school book



Remember

Understand

Apply

Problem Solving



Interactive test

First Problems on rotation in the plane :

- 1 Use the geometric tools to draw \overline{AB} with length 3 cm. , then draw its image by rotation $R(B, 135^\circ)$
- 2 Draw the equilateral triangle ABC with side length 6 cm. Draw the image of the triangle ABC by rotation $R(A, 60^\circ)$
- 3 Draw the triangle ABC in which $AB = 5$ cm. , $BC = 6$ cm. and $CA = 7$ cm. , then draw the image of $\triangle ABC$ by rotation :
 - 1 $R(A, 180^\circ)$
 - 2 $R(A, 360^\circ)$
- 4 Draw the triangle XYZ in which $XY = XZ = 3$ cm. and $YZ = 4$ cm. , then draw the image of $\triangle XYZ$ in each of the two cases :
 - 1 By rotation about X with an angle of measure 90°
 - 2 By rotation about X with an angle of measure 270°
- 5 Draw $\triangle ABC$ in which $AB = 5$ cm. , $AC = 3$ cm. , $m(\angle A) = 40^\circ$, then draw \hat{C} the image of C by rotation $R(A, 40^\circ)$, \hat{B} the image of B by rotation $R(A, -40^\circ)$
- 6 Draw the square ABCD with side length 5 cm. , then draw the image of the square ABCD :
 - 1 By rotation $R(B, 90^\circ)$
 - 2 By rotation $R(A, 180^\circ)$

7 Using the geometric tools, draw the square ABCD with side length 4 cm., then draw its image by rotation about its centre (The point of diagonals intersection) with an angle of measure 90°

8 Draw the rectangle ABCD in which $BC = 6$ cm., $AB = 4$ cm.

Draw the image of the rectangle ABCD :

1 By rotation $R(A, 90^\circ)$

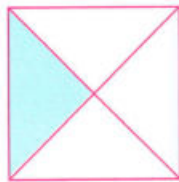
2 By rotation $R(M, 180^\circ)$ where M is the point of intersection of its diagonals.

9 Choose the correct answer from those given :

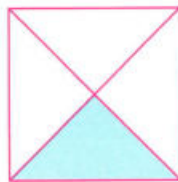
1 Which of these figures represents the rotation of the opposite square about its centre with an angle of measure 270° ?



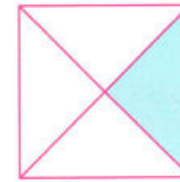
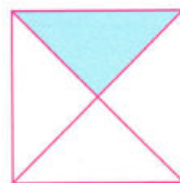
(a)



(b)

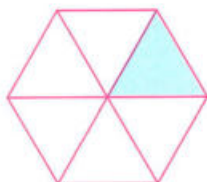


(c)



(d)

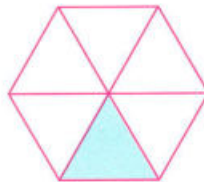
2 Which of these figures represents the rotation of the opposite regular hexagon about its centre with an angle of measure (-120°) ?



(a)



(b)



(c)



(d)

3 In the opposite figure :

If B is the midpoint of \overline{AC} , then the image of \overline{AC} by rotation about B with an angle of measure 180° is

(a) \overline{AC}

(b) \overline{AB}

(c) \overline{CA}

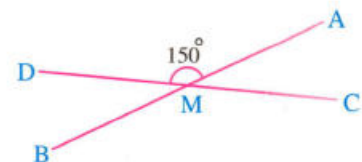
(d) \overline{CB}



4 In the opposite figure :

\overline{CD} is the image of \overline{AB} under
a rotation about M
with an angle of measure

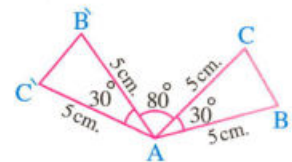
- (a) 75° (b) 30° (c) -30° (d) -150°



5 In the opposite figure :

$\triangle AB'C'$ is the image of $\triangle ABC$
by a rotation about A
with an angle of measure

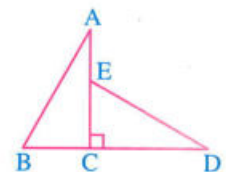
- (a) -110° (b) 80° (c) 110° (d) 140°



6 In the opposite figure :

$\triangle ABC$ is the image of $\triangle DEC$
which is right-angled
at C by rotation about C with an angle of measure

- (a) 90° (b) -90° (c) 180° (d) 360°



10 In the opposite figure :

The radius length of circle M is 3 cm. ,
 \overline{AC} and \overline{BD} are two perpendicular diameters in it.

Complete :

- 1** By the rotation $R(M, 90^\circ)$, the image of the point A is
and the image of the point B is

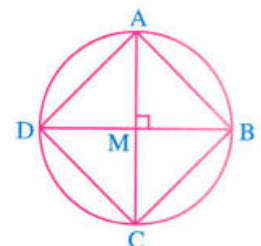
\therefore The image of \overline{AB} is and the image of \overline{AB} is

- 2** By rotation $R(M, -90^\circ)$, the image of \overline{AB} is and the image of
 \overline{AB} is and the image of \overline{AB} is

- 3** By rotation $R(M, 180^\circ)$, the image of the point A is and the image of the
point B is

\therefore The image of \overline{AB} is

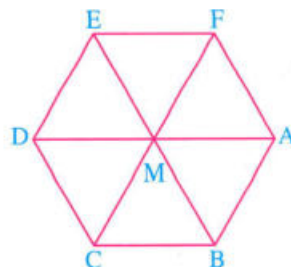
- 4** By rotation $R(M, -180^\circ)$, the image of \overline{AB} is



11 In the opposite figure :

ABCDEF is a regular hexagon whose centre is M

Complete the following :



- 1 The image of the point E by rotation about M with an angle of measure 120° is
- 2 The image of \overline{AF} by rotation about M with an angle of measure 180° is
- 3 The image of \overline{DE} by rotation about M with an angle of measure -60° is
- 4 The image of $\triangle MCD$ by rotation about M with an angle of measure 300° is
- 5 $\triangle ABM$ is the image of $\triangle CDM$ by rotation about with an angle of measure $^\circ$
- 6 $\triangle BMC$ is the image of by rotation about M with an angle of measure (-120°)

12 Referring to the opposite figure ,

choose the correct answer from those given :



Fig. (1)



Fig. (2)



Fig. (3)

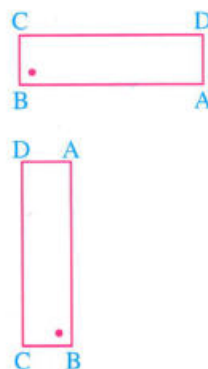


Fig. (4)

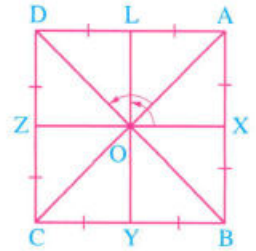
- 1 The image of the figure by reflection in \overleftrightarrow{AD} is
 (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)
- 2 The image of the figure by rotation about A with an angle of measure 90° is
 (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)
- 3 The image of the figure by translation to the right is
 (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)
- 4 The image of the figure by rotation about A with an angle of measure 180° is
 (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)

13 In the opposite figure :

ABCD is a square, O is the point of intersection of its diagonals, X, Y, Z and L are the midpoints of \overline{AB} , \overline{BC} , \overline{CD} and \overline{DA} respectively.

Find :

- 1 The image of $\triangle AXO$ by reflection in \overleftrightarrow{AO} followed by another reflection in \overleftrightarrow{LO}
- 2 The image of $\triangle AXO$ by rotation $R(O, 90^\circ)$



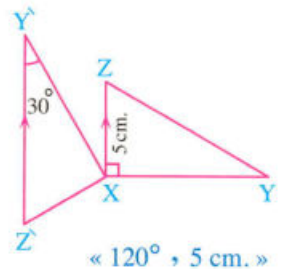
14 ABC is a right-angled triangle at B with $AB = 5$ cm. and $BC = 12$ cm. **Find :**

- 1 X as the image of B by translation 9 cm. in the direction of \overleftrightarrow{BA}
- 2 Y as the image of B by rotation $R(A, -90^\circ)$
- 3 The length of \overline{XY}

15 In the opposite figure :

If the point X is the centre of rotation such that \hat{Y} is the image of Y and \hat{Z} is the image of Z, $\overline{XZ} \parallel \overline{\hat{Y}\hat{Z}}$ **Find :**

- 1 The measure of the angle of rotation.
- 2 The length of $\overline{X\hat{Z}}$



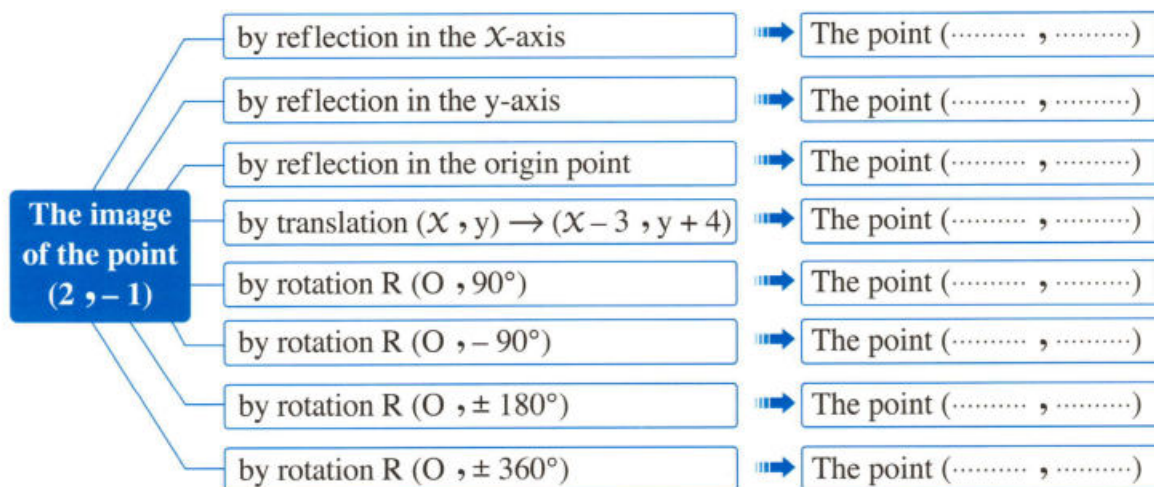
Second Problems on rotation in the Cartesian plane :

1 Complete the following :

- 1 The image of the point $(2, -3)$ by rotation about the origin point with an angle of measure 90° is and with an angle of measure 180° is
- 2 The image of the point $(-1, 0)$ by rotation about the origin point with an angle of measure 90° is and with an angle of measure 360° is
- 3 The point $(3, -2)$ is the image of the point $(2, 3)$ by rotation about the origin point with an angle of measure°
- 4 The image of the point by rotation about the origin point with an angle of measure 90° is $(-1, 4)$
- 5 The image of the point by rotation about the origin point with an angle of measure (-180°) is $(5, -2)$

- 6 The image of the point $(-3, 7)$ by rotation with an angle of measure 90° about the origin point followed by reflection in y -axis is
- 7 The image of the point $(-2, 0)$ by translation $(X, y) \longrightarrow (X + 3, y - 1)$ followed by rotation about the origin point with an angle of measure 90° is
- 8 The rotation with an angle of measure 90° about the origin point maps the point $(X, -y)$ onto the point
- 9 The image of (a, b) is the same point by rotation about the origin point with an angle of measure $^\circ$
- 10 If the image of the point (X, y) by rotation about the origin point with an angle of measure 90° is (a, b) , then $a + y = \dots\dots\dots$

2 Complete the following diagram :

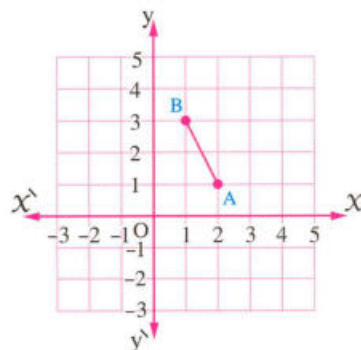


3 In the opposite figure :

The point $A(2, 1)$ and $B(1, 3)$

Draw the image of \overline{AB}

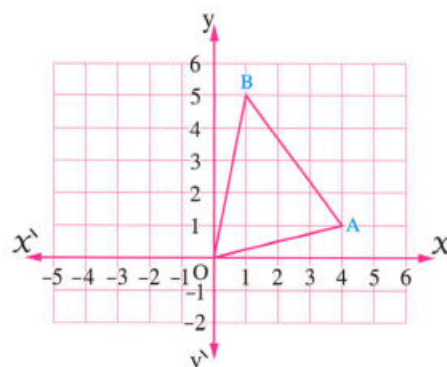
by rotation about the origin point
with an angle of measure 90°



- 4 On the lattice, draw the image of $\triangle OAB$ by rotation about the origin with an angle of measure :

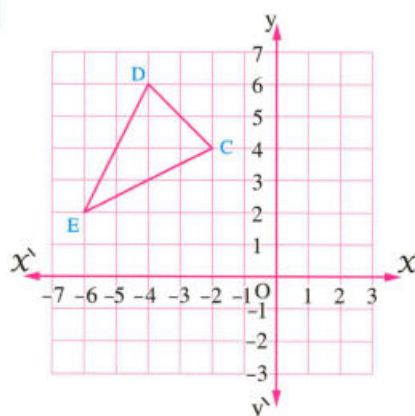
1 90°

2 180°



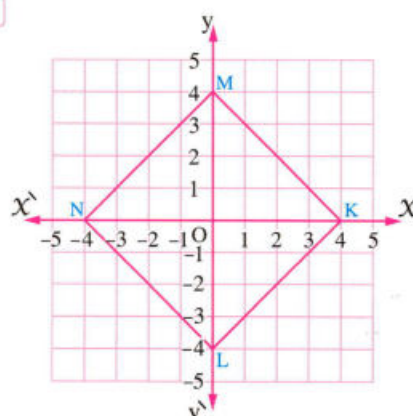
- 5 Copy each figure on a graph paper. Draw their images under the transformation indicated. Give the coordinates of the images vertices in each case :

1



Rotation of 90°
Clockwise about O

2



Rotation of 90°
anticlockwise about O

- 6 Draw on graph paper $\triangle ABC$ where A (3, -1), B (5, 2) and C (-2, 4), then draw its image by rotation about the origin point with an angle of measure 180°
- 7 In an orthogonal Cartesian coordinate system, determine the two points A (3, 0) and B (0, 2), then draw the image of $\triangle AOB$ by rotation about O with an angle of measure 90° where O is the origin point.

- 8 On a lattice, draw the quadrilateral ABCD where A (0, 4), B (4, 4), C (7, 0) and D (0, 0), then draw its image by rotation :

1 About the origin point where : $(x, y) \longrightarrow (-y, x)$

2 $R(O, -180^\circ)$

- 9 If the image of the point C by rotation with an angle of measure 90° about the origin is $\hat{C}(-4, 5)$, locate the point C , then draw its image $\hat{\hat{C}}$ by rotation about the origin with an angle of measure 180°

- 10 On a lattice, draw $\triangle ABC$ where $A(4, 4)$, $B(4, 2)$ and $C(1, 2)$, then draw its image by rotation about the point B with an angle of measure 180°

- 11 On graph paper, draw the rectangle $ABCD$ with vertices $A(0, 0)$, $B(0, 2)$, $C(4, 2)$ and $D(4, 0)$

(a) Draw three images formed by rotating the rectangle about the origin through an angle of measure

1 90°

2 180°

3 270°

(b) What are the coordinates of the centre of the rectangle?

(c) Draw three images formed by rotating the rectangle about its centre through an angle of measure

1 90°

2 180°

3 270°

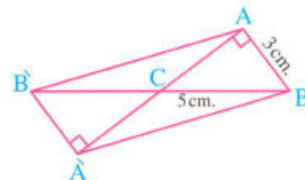
For excellent pupils

- 12 In the opposite figure :

ABC is a right-angled triangle at A , $AB = 3$ cm. and $BC = 5$ cm.

If $\triangle \hat{C}\hat{A}\hat{B}$ is the image of $\triangle CAB$ by rotation about C with an angle of measure 180°

Find the area of $\triangle A\hat{A}\hat{B}$



« 12 cm^2 »

SKILLS

TIMSS Problems

Accumulative basic skills

1 Choose the correct answer from the given ones :

- 1** A square is of area 144 cm^2 , then its perimeter = cm.
 (a) 12 (b) 48 (c) 288 (d) 576
- 2** A rectangle, its length is 6 cm. and its perimeter is 16 cm. , then its area is cm^2 .
 (a) 10 (b) 8 (c) 12 (d) 16
- 3** The supplementary of the angle whose measure is 30° , is an angle of measure
 (a) 30° (b) 60° (c) 120° (d) 150°
- 4** Which of the following figures is suitable to form a circle ?



(a)



(b)



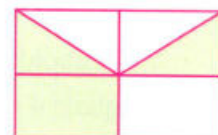
(c)



(d)

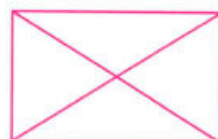
- 5** The area of the shaded part from the area of the figure =

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$



- 6** The great number of triangles in the opposite figure =

(a) 4 (b) 6 (c) 8 (d) 10

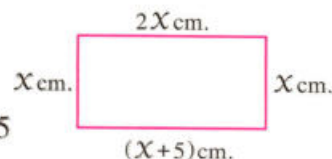


- 7** If X is an angle , then $m(\angle X) + m(\text{reflex } \angle X) = \dots\dots\dots$

(a) two right angles. (b) three right angles.
 (c) five right angles. (d) four right angles.

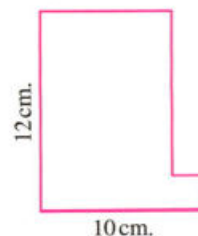
- 8** The area of the rectangle in the opposite figure = cm^2

(a) 50 (b) 30 (c) 20 (d) 15



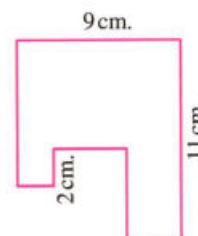
- 9 The perimeter of the opposite figure = cm.

(a) 22 (b) 24 (c) 44 (d) 120



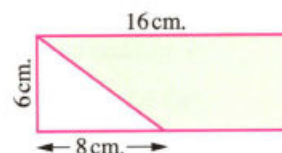
- 10 The perimeter of the opposite figure = cm.

(a) 99 (b) 44 (c) 22 (d) 20

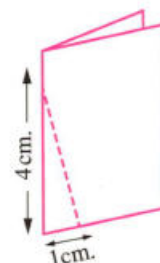


- 11 The area of the shaded part in the opposite figure = cm^2 .

(a) 24 (b) 44 (c) 48 (d) 72

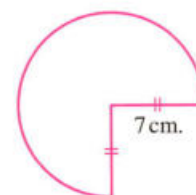


- 12 A rectangular piece of paper is folded as in the opposite figure and a part of it is cut off with aligning the dashed line, then at opening the cut small part, it will be in the form of
- (a) an equilateral triangle. (b) an isosceles triangle.
(c) a right-angled triangle. (d) two isosceles triangles.



2 Complete the following :

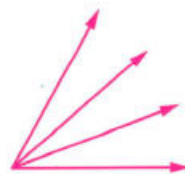
- 1 A cube, the area of one of its faces is 25 cm^2 , then its volume = cm^3 .
- 2 A cuboid, its volume = 48 cm^3 , if the length of its base equals 6 cm. and its width equals 4 cm. , then its height = cm.
- 3 The angle whose measure is 89° , is angle.
- 4 If $m(\angle A) = 2m(\angle B)$, $\angle A$ complements $\angle B$, then $m(\angle A) = \dots\dots\dots^\circ$.
- 5 The area of the opposite figure = cm^2 ($\pi = \frac{22}{7}$)



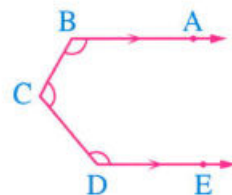
- 6 The perimeter of the opposite figure = cm. ($\pi = \frac{22}{7}$)



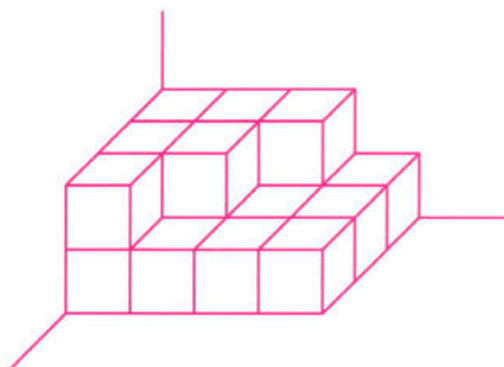
- 7 The number of acute angles in the opposite figure is



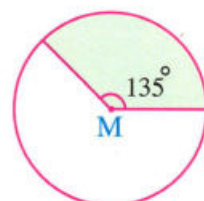
- 8 In the opposite figure :
 $m(\angle B) + m(\angle C) + m(\angle D) = \dots\dots\dots^\circ$



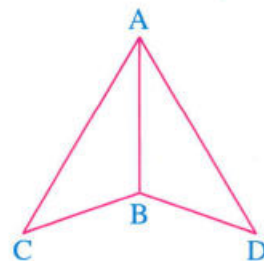
- 9 The volume of the opposite figure = volume units.



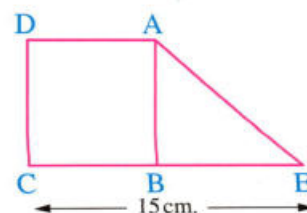
- 10 The percentage of the area of the shaded part to the area of the circle is



- 11 In the opposite figure :
 If $\triangle ABC \cong \triangle ABD$,
 the perimeter of the figure ABCD = 20 cm.
 and $AB = 6$ cm.
 , then the perimeter of $\triangle ABC = \dots\dots\dots$ cm.



- 12 In the opposite figure :
 ABCD is a square of area 49 cm^2
 , if $EC = 15 \text{ cm}$, then the area
 of $\triangle ABE = \dots\dots\dots \text{ cm}^2$





By a group of supervisors

NOTEBOOK

- Accumulative Tests
- Monthly Tests
- Important Questions
- Final Revision
- Final Examinations

1st PREP.
2024
SECOND TERM

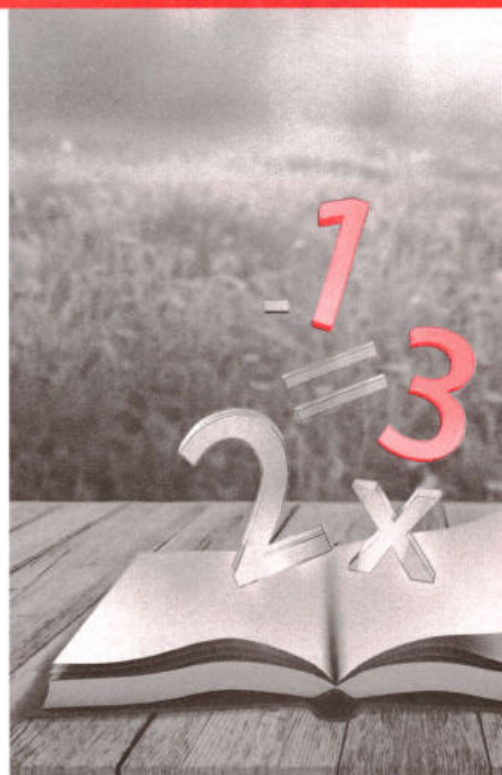


Maths

CONTENTS

First Algebra and Statistics

- 9 Accumulative tests
- Monthly tests
- Important questions
- Final revision
- Final examinations :
 - School book examinations (2 models + model for the merge students)
 - 12 schools examinations



Second Geometry and Measurement

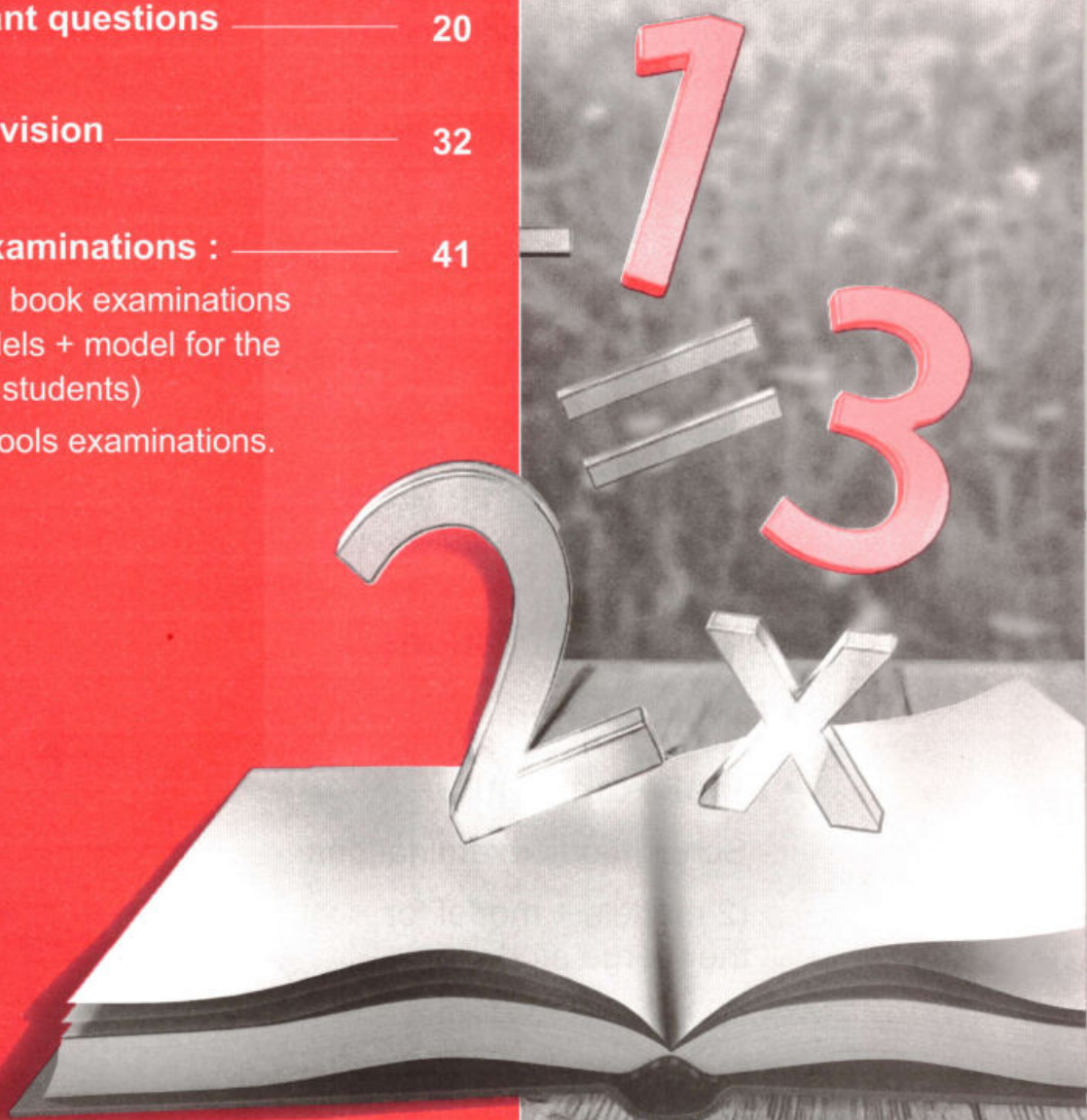
- 12 Accumulative tests
- Monthly tests
- Important questions
- Final revision
- Final examinations :
 - School book examinations (2 models + model for the merge students)
 - 12 schools examinations



First

Algebra and Statistics

- 9 Accumulative tests 5
- Monthly tests 15
- Important questions 20
- Final revision 32
- Final examinations : 41
 - School book examinations
(2 models + model for the
merge students)
 - 12 schools examinations.



Accumulative Tests

on Algebra and Statistics





Accumulative test

1

on lesson 1 – unit 1

1 Choose the correct answer from those given :

1 The additive inverse of the number $\left(\frac{-2}{3}\right)^{\text{zero}}$ is

(a) zero

(b) 1

(c) -1

(d) $\frac{2}{3}$

2 $64\% = \left(\frac{4}{5}\right)^{\dots\dots}$

(a) 1

(b) 2

(c) 3

(d) 4

3 $(2y)^3 = \dots\dots\dots$

(a) $2y^3$

(b) $8y^3$

(c) $8y$

(d) $32y$

4 $(-1)^{13} \dots\dots\dots (-1)^2$

(a) >

(b) <

(c) =

(d) \geq

2 Complete each of the following :

1 The multiplicative inverse of the number $(-1)^2$ is

2 $\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right)^3 = \dots\dots\dots$

3 If $X = y$, then $\left(\frac{3}{4}\right)^{X-y} = \dots\dots\dots$

4 If $X = \frac{1}{3}$, $y = 3$, then $X^{15}y^{16} = \dots\dots\dots$

3 Find the value of the following in the simplest form : $\left(\frac{-2}{3}\right)^3 \times \left(\frac{1}{3}\right)^3 \div \left(\frac{-2}{9}\right)^2$

4 If $X = \frac{-1}{2}$, $y = \frac{3}{4}$, $z = \frac{3}{8}$

Find in the simplest form the numerical value of : $(X + y)^3 \div z$

Accumulative test

2

till lesson 2 – unit 1

1 Choose the correct answer from those given :

1 $2^3 \times 2^5 = \dots\dots\dots$

(a) 2^2

(b) 2^8

(c) 2^{15}

(d) 2

2 $3^x + 3^x + 3^x = \dots\dots\dots$

(a) 9^x

(b) 27^x

(c) $3x^3$

(d) 3^{x+1}

3 $2^7 \times 3^7 = \dots\dots\dots$

(a) 5^7

(b) 6^7

(c) 6^{14}

(d) 6^{49}

4 The quarter of the number 2^{16} equals $\dots\dots\dots$

(a) 2^4

(b) 2^{12}

(c) 2^{15}

(d) 2^{14}

2 Complete each of the following :

1 If $0.027 = \left(\frac{3}{10}\right)^n$, then $n = \dots\dots\dots$

2 If $a^x = 2$, $a^y = 3$, then $a^{x+y} = \dots\dots\dots$

3 $2^{10} + 2^{10} = \dots\dots\dots$

4 The additive inverse of the number $\left(\frac{-1}{3}\right)^2$ is $\dots\dots\dots$

3 [a] Find the value of : $\frac{(-4)^5 \times (-3)^7}{(-4)^3 \times (-3)^5}$

[b] Simplify to the simplest form : $\frac{(-4a^3b^4)^2}{(-2ab^2)^4}$ such that $a \neq 0$ and $b \neq 0$

, then find the numerical value of the result if $a = 2$, $b = 1$

4 If $a = \frac{-1}{2}$, $b = 2$, $c = \frac{3}{4}$

Find the numerical value of the expression : $a^3b^2 + b^2c - 8abc$

Accumulative test

3

till lesson 3 – unit 1

1 Choose the correct answer from those given :

1 If $2^{-7} + 2^{-7} = 2^m$, then $m = \dots\dots\dots$

(a) -7

(b) -6

(c) -8

(d) zero

2 Double the number 2^{10} is $\dots\dots\dots$

(a) 4^{10}

(b) 2^{20}

(c) 2^{11}

(d) 4^{20}

3 $3^{10} + 3^{10} + 3^{10} = \dots\dots\dots$

(a) 3^{10}

(b) 3^{30}

(c) 3^{11}

(d) 9^{10}

4 $(2)^{-3} \dots\dots\dots (-2)^3$

(a) $>$

(b) $<$

(c) $=$

(d) \leq

2 Complete each of the following :

1 $\left(\frac{2}{3}\right)^{-2} = \dots\dots\dots$

2 $\frac{x^{-5}}{y^{-5}} = \left(\frac{\dots\dots\dots}{\dots\dots\dots}\right)^5$ (Where $x, y \neq \text{zero}$)

3 The multiplicative inverse of the number 5^{-2} is $\dots\dots\dots$

4 If $\frac{x}{y} = \frac{-2}{3}$, then $\left(\frac{x}{y}\right)^2 = \dots\dots\dots$

3 [a] Calculate the value of : $\frac{(10)^2 \times (0.01)^3}{10^{-3}}$

[b] If $x = \frac{1}{2}$, $y = \frac{3}{4}$ Find the value of : $\left(\frac{x^2}{y}\right)^2$

4 Simplify to the simplest form : $\frac{x^5 \times x^{-3}}{x^{-2} \times x^6}$ (Where $x \neq 0$), then find the numerical value of the result at $x = \frac{1}{2}$

Accumulative test

4

till lesson 4 – unit 1

1 Choose the correct answer from those given :

1 $13400000 = 1.34 \times \dots\dots\dots$

(a) 10^7

(b) 10^{-7}

(c) 10^6

(d) 10^{-6}

2 The standard form of the number : 750×10^{-6} is

(a) 7.5×10^{-8}

(b) 7.5×10^{-7}

(c) 7.5×10^{-4}

(d) 7.5×10^4

3 $2^{-1} + 4^{-1} = \dots\dots\dots$

(a) 0.20

(b) 0.40

(c) 0.60

(d) 0.75

4 Which of the following is the greatest ?

(a) 2.3×10^4

(b) 2.3×10^5

(c) 3.2×10^4

(d) 3.2×10^5

2 Complete each of the following :

1 The quarter of the number $4^{11} = \dots\dots\dots$

2 If $0.00037 = 3.7 \times 10^n$, then $n = \dots\dots\dots$

3 If $a = -3$, $b = -5$, then $\left(\frac{a}{b}\right)^2 = \dots\dots\dots$

4 The standard form of the number : 0.6×0.005 is

3 [a] Find the result of : 60000×5000 in the standard form.[b] Find in the standard form the result of : $(18 \times 10^9) \div (3 \times 10^4)$

4 [a] Calculate the value of : $\left(\frac{9^3 \times 9}{9^5}\right)^{-3}$

[b] Write the result in the standard form : $(5.8 \times 10^3) + (3.2 \times 10^2)$

Accumulative test

5

till lesson 5 – unit 1

1 Choose the correct answer from those given :

1 $40 - 4 \times 3^2 = \dots\dots\dots$

(a) -4

(b) 1

(c) -1

(d) 4

2 Half the number 4^5 is $\dots\dots\dots$

(a) 2^5

(b) 2^9

(c) 2^4

(d) 4^2

3 $4 + 4 \times 4 \div 4 + 2^2 = \dots\dots\dots$

(a) 4

(b) 12

(c) 16

(d) 8

4 $\left(-1\frac{1}{4}\right)^3 = \dots\dots\dots$

(a) $\frac{125}{64}$

(b) $-\frac{125}{64}$

(c) $\frac{25}{16}$

(d) $-\frac{1}{64}$

2 Complete each of the following :

1 $3 \times 6 + 4 \div 2 = \dots\dots\dots$

2 If $0.00025 = 2.5 \times 10^n$, then the value of $n = \dots\dots\dots$

3 $2 \times 6 - 4 \div (2)^2 = \dots\dots\dots$

4 $x^5 \div x^{-3} = \dots\dots\dots$ (Where $x \neq 0$)

3 **[a]** Calculate the value of : $2[(5^2 + 1) - (4^2 - 1)]$ **[b]** Find the value of : $\frac{2^7 \times 2^3}{2^2 \times 2^6}$ «Show steps»**4** **[a]** If $a = \frac{3}{4}$, $b = \frac{-3}{2}$ Find the numerical value of : $\left(\frac{a^2}{b^3}\right)^2$ **[b]** Find the value of : $5^2 + [3 \times 8 \div 2^2 - 2 \times 3]$

Accumulative test

6

till lesson 6 – unit 1

1 Choose the correct answer from those given :

1 If $x = 0.0009$, then $\sqrt{x} = \dots\dots\dots$

(a) 0.0003

(b) 0.0081

(c) 0.003

(d) 0.03

2 $2 \times 6 - 4 \div 2 = \dots\dots\dots$

(a) 4

(b) 8

(c) 10

(d) 2

3 The side length of the square whose area is $9x^2 \text{ cm}^2$ is $\dots\dots\dots$

(a) $|3x|$

(b) $3x^2$

(c) $9x$

(d) $9x^2$

4 The multiplicative inverse of the number $\sqrt{2\frac{1}{4}}$ is $\dots\dots\dots$

(a) $\frac{9}{4}$

(b) $\frac{3}{2}$

(c) $\frac{2}{3}$

(d) $\frac{4}{9}$

2 Complete each of the following :

1 $\sqrt{(-6)^2 + (-8)^2} = \dots\dots\dots$

2 Third the number 3^9 is $\dots\dots\dots$

3 $\sqrt{100} = \sqrt{36} + \sqrt{\dots\dots\dots}$

4 The sum of the two square roots of the number $6\frac{1}{4}$ is $\dots\dots\dots$

3 [a] Simplify to the simplest form : $\left(\frac{-3}{7}\right)^{\text{zero}} \times \left(-\frac{5}{2}\right)^{-2} \times \sqrt{6\frac{1}{4}}$

[b] Simplify to the simplest form : $\frac{2^8 \times 2^{-7}}{2^{-2} \times 2^3}$

4 In $\triangle ABC$ If $(AB)^2 = 16 \text{ cm}^2$, $(BC)^2 = 25 \text{ cm}^2$, then find : $AB + BC$

Accumulative test

7

till lesson 7 – unit 1

1 Choose the correct answer from those given :**1** If $x + 3 = 7$, then $5x = \dots\dots\dots$

(a) 5

(b) 9

(c) 20

(d) 50

2 The additive inverse of the number $(-2)^3$ equals $\dots\dots\dots$

(a) 8

(b) -8 (c) $-\frac{1}{8}$ (d) $\frac{1}{6}$ **3** $(0.2)^{-1} = \dots\dots\dots$ (a) $\frac{1}{5}$ (b) $\frac{1}{10}$

(c) 5

(d) $\frac{1}{2}$ **4** If the length of a rectangle is double its width and its width = x cm., then its perimeter is $\dots\dots\dots$ cm.(a) $3x$ (b) $2x^2$ (c) $5x$ (d) $6x$ **2** Complete each of the following :**1** The S.S. of the equation : $x + 7 = 2$ in \mathbb{N} is $\dots\dots\dots$ **2** The S.S. of the equation : $x + 3 = 3$ in \mathbb{N} is $\dots\dots\dots$ **3** $12 \times 2^2 \div 24 + 3^2 = \dots\dots\dots$ **4** If the age of a student now is x years, then his age 4 years ago is $\dots\dots\dots$ years.**3** [a] Find the S.S. of the following equation in \mathbb{Q} : $5x - 3 = 2(x - 1)$ [b] Simplify to the simplest form : $\frac{(7)^3 \times (-7)^4}{(7)^5}$ **4** The sum of three consecutive integers is zero find them.

Accumulative test

8

till lesson 8 – unit 1

1 Choose the correct answer from those given :**1** If $-X < 3$, then :

(a) $X > 3$

(b) $X < 3$

(c) $X > -3$

(d) $X < -3$

2 The number which satisfies the inequality : $X - 2 > 1$ is

(a) 1

(b) 2

(c) 3

(d) 4

3 If $0.00032 = 3.2 \times 10^n$, then $n =$

(a) -3

(b) 3

(c) -4

(d) 4

4 The S.S. of the inequality : $X < 2$ in the counting numbers (C) is

(a) $\{0\}$

(b) $\{1\}$

(c) $\{0, 1\}$

(d) \emptyset

2 Complete each of the following :**1** The S.S. of the inequality : $X \geq 1$ in \mathbb{N} is**2** The S.S. of the inequality : $1 < X < 2$ in \mathbb{N} is**3** Half the number $4^{20} =$ **4** If $7 - 2X = 3$, then $X =$ where $X \in \mathbb{Q}$ **3** [a] Find in \mathbb{Z} the S.S. of the inequality : $3X + 7 \geq 19$, then represent it on the number line.**[b]** Find in \mathbb{Q} the S.S. of the inequality : $4X - 2(X - 1) \geq 0$ **4** [a] Simplify to the simplest form : $\left(\frac{3}{2}\right)^{-2} \times \sqrt{\frac{81}{16}} \times \left(\frac{3}{4}\right)^{\text{zero}}$ **[b]** Find the S.S. in \mathbb{Q} of the equation : $3X + 5X + 6 = 30$

Accumulative test

9

till lesson 2 – unit 2

1 Choose the correct answer from those given :

- 1** If a fair coin tossed 160 times , then the nearest expected number of appearance of a head equals
- (a) 60 (b) 78 (c) 90 (d) 159
- 2** Which of the following may be the probability of occurrence of an event ?
- (a) $\frac{7}{5}$ (b) 1.3 (c) 76 % (d) - 6 %
- 3** $\left(\frac{1}{2}\right)^5 \div \left(\frac{1}{2}\right)^3 = \dots\dots\dots$
- (a) $\frac{1}{8}$ (b) $\frac{1}{32}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$
- 4** The sum of the probabilities for all possible outcomes of a random experiment
- (a) = zero (b) = 1 (c) < 1 (d) > 1

2 Complete each of the following :

- 1** If the probability of the success of a student in a math exam is 0.8 , then the probability of his failure is
- 2** When a fair die is tossed once , then the probability of getting an even number is
- 3** The probability of the certain event is
- 4** If $0.000054 = 5.4 \times 10^n$, then n =

3 [a] Simplify to the simplest form : $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^{\text{zero}}$

[b] Find in Q the S.S. of the following equation : $6x - 7 = 3x + 8$

- 4** A class contains 40 students , 30 of them succeeded in maths , 24 succeeded in science and 20 succeeded in both. A student is chosen randomly. **Find the probability that this student :**
- 1** Succeeded in maths. **2** Succeeded in science.
- 3** Failed in science. **4** Failed in both maths and science.

Monthly Tests

on Algebra and Statistics

October contents

* Unit One : Numbers and algebra :

- Repeated multiplication in \mathbb{Q}
- Non-negative integer powers.
- Negative integer powers.
- Scientific notation of the rational number.
- Order of mathematical operations.

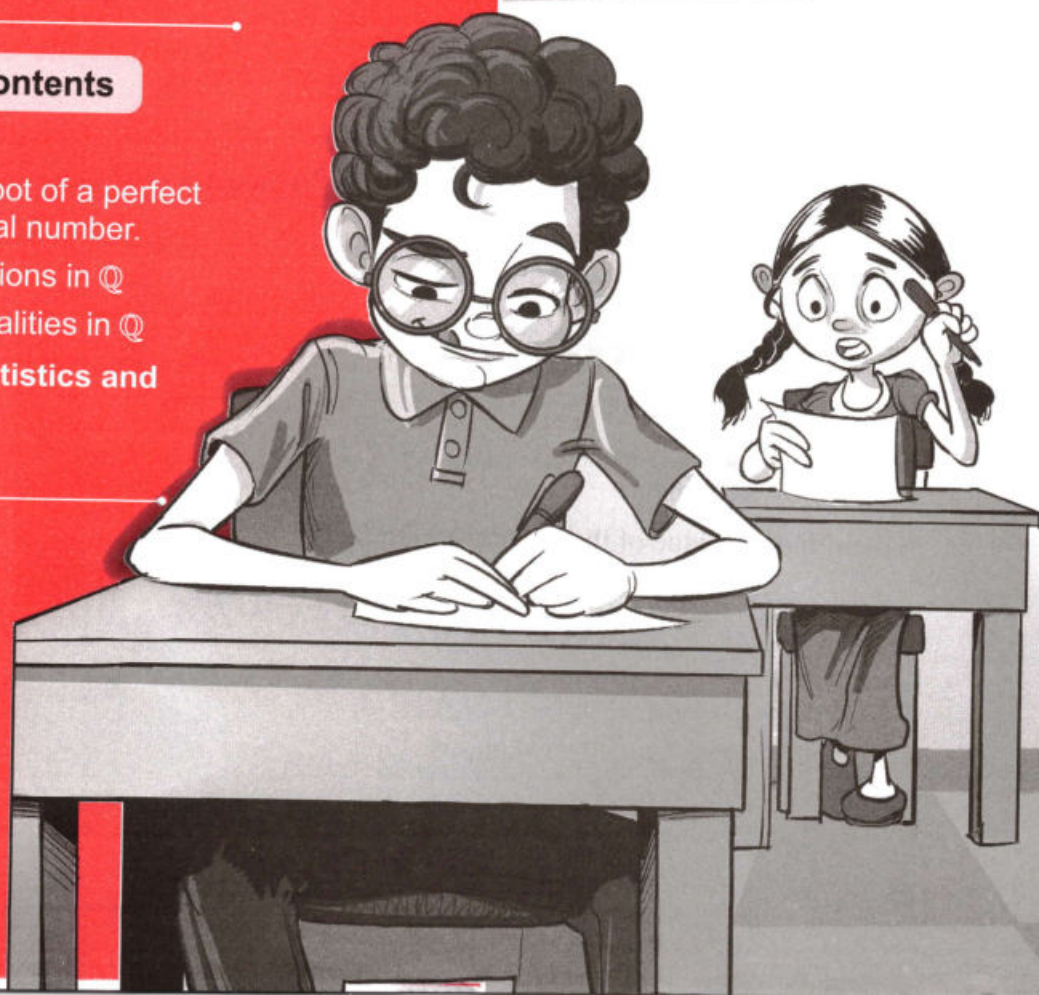
November contents

* Unit One :

- The square root of a perfect square rational number.
- Solving equations in \mathbb{Q}
- Solving inequalities in \mathbb{Q}

* Unit Two : Statistics and probability :

- Samples.





Test

1

Total mark

10

Answer the following questions :

1 Choose the correct answer from the given ones :

(3 Marks)

1 The multiplicative inverse of $\left(\frac{-3}{5}\right)^2$ is

(a) $-\left(\frac{5}{3}\right)^2$

(b) $\frac{-9}{25}$

(c) $\frac{25}{9}$

(d) $\left(\frac{3}{5}\right)^2$

2 $\frac{x^{-5}}{y^{-5}} = (\dots\dots\dots)^5$, $y \neq 0$, $x \neq 0$

(a) xy

(b) $\frac{y}{x}$

(c) $x - y$

(d) $\frac{x}{y}$

3 If $2^{10} + 2^{10} = 2^k$, then $k = \dots\dots\dots$

(a) 4

(b) 20

(c) 100

(d) 11

2 Complete :

(3 Marks)

1 $6^2 + 6 \times 6 \div 6 - 6 = \dots\dots\dots$

2 $\frac{-27}{125} = \left(\frac{-3}{5}\right)^{\dots\dots\dots}$

3 If the standrad form of -0.0002 is -2×10^n , then $n = \dots\dots\dots$

3 If $x = 0.4$, $y = \frac{1}{2}$, $z = -2$

(2 Marks)

Find the value of : $2xy + z^2$

4 Simplify : $\frac{b^3 \times b^{-5}}{b^{-2} \times b^6}$ (where $b \neq 0$)

(2 Marks)

, then find the value of the result when $b = 2$

Test 2

Total mark

10

Answer the following questions :

1 Choose the correct answer from the given ones :

(3 Marks)

1 If $2^{-5} \times 3^{-5} = 6^k$, then $k = \dots\dots\dots$

(a) 6

(b) -10

(c) 25

(d) -5

2 If $0.0028 = 2.8 \times a$, then $a = \dots\dots\dots$

(a) 3

(b) -3

(c) 10^3 (d) 10^{-3}

3 $4x^{-1}y^{-2} = \frac{4}{\dots\dots\dots}$ (where $x \neq 0$, $y \neq 0$)

(a) y^2x^{-1} (b) xy^{-2} (c) xy^2 (d) yx^2

2 Complete :

(3 Marks)

1 The additive inverse of $(-1)^3$ is $\dots\dots\dots$

2 $[4 - (5 - 2)] - 1 = \dots\dots\dots$

3 If $\left(\frac{x-3}{5}\right)^0 = 1$, then $x \neq \dots\dots\dots$

3 Find the following in the standard form :

(2 Marks)

$(18 \times 10^9) \div (3 \times 10^4)$

4 Simplify to the simplest form : $\frac{4^{n+1} \times 3^{n-1}}{12^n}$

(2 Marks)

Test

1

Total mark

10

Answer the following questions :

1 Choose the correct answer from the given ones :

(3 Marks)

1 If $-x > 3$, then

(a) $x < 3$

(b) $x < -3$

(c) $x > -3$

(d) $x > 3$

2 If the age of a man now is x years , then his age 3 years ago is years.

(a) $3x$

(b) $x - 3$

(c) $3 + x$

(d) $\frac{x}{3}$

3 The sum of the two square roots of 25 is

(a) 5

(b) ± 5

(c) zero

(d) 10

2 Complete :

(3 Marks)

1 $a = \frac{1}{3}$, $b = 1\frac{1}{3}$, then $\sqrt{ab} = \dots\dots\dots$

2 $\sqrt{1} + \sqrt{4} + \sqrt{9} + \sqrt{36} = \dots\dots\dots$

3 If $b > a$, then $b + 3 \dots\dots\dots a + 3$

3 Find in \mathbb{Q} the S.S. of :

(2 Marks)

$$2 - 3x \leq 7$$

4 Simplify to the simplest form :

(2 Marks)

$$\left(\frac{2}{5}\right)^{-2} \times \sqrt{\frac{4}{25}} \times 2$$

Test 2

Total mark

10

Answer the following questions :

1 Choose the correct answer from the given ones :

(3 Marks)

1 The negative square root of 49 is

(a) 7

(b) -7

(c) ± 7 (d) $|-7|$

2 If $3 \times y = 21$, then $7 \times y = \dots\dots\dots$

(a) 21

(b) 147

(c) 49

(d) 10

3 The side length of the square whose area $36 \times^2 \text{ cm}^2$ is cm.

(a) $18 \times$ (b) $|6 \times|$ (c) $9 \times$ (d) $6 \times^2$

2 Complete :

(3 Marks)

1 The S.S. of : $\times > -3$ in \mathbb{N} is

2 $\sqrt{\sqrt{25} + 2^2} = \dots\dots\dots$

3 Two integers their sum is 6, if one of them is \times , then the other integer is

3 Find in \mathbb{Q} the S.S. of : $2 + 3 \times = 4$

(2 Marks)

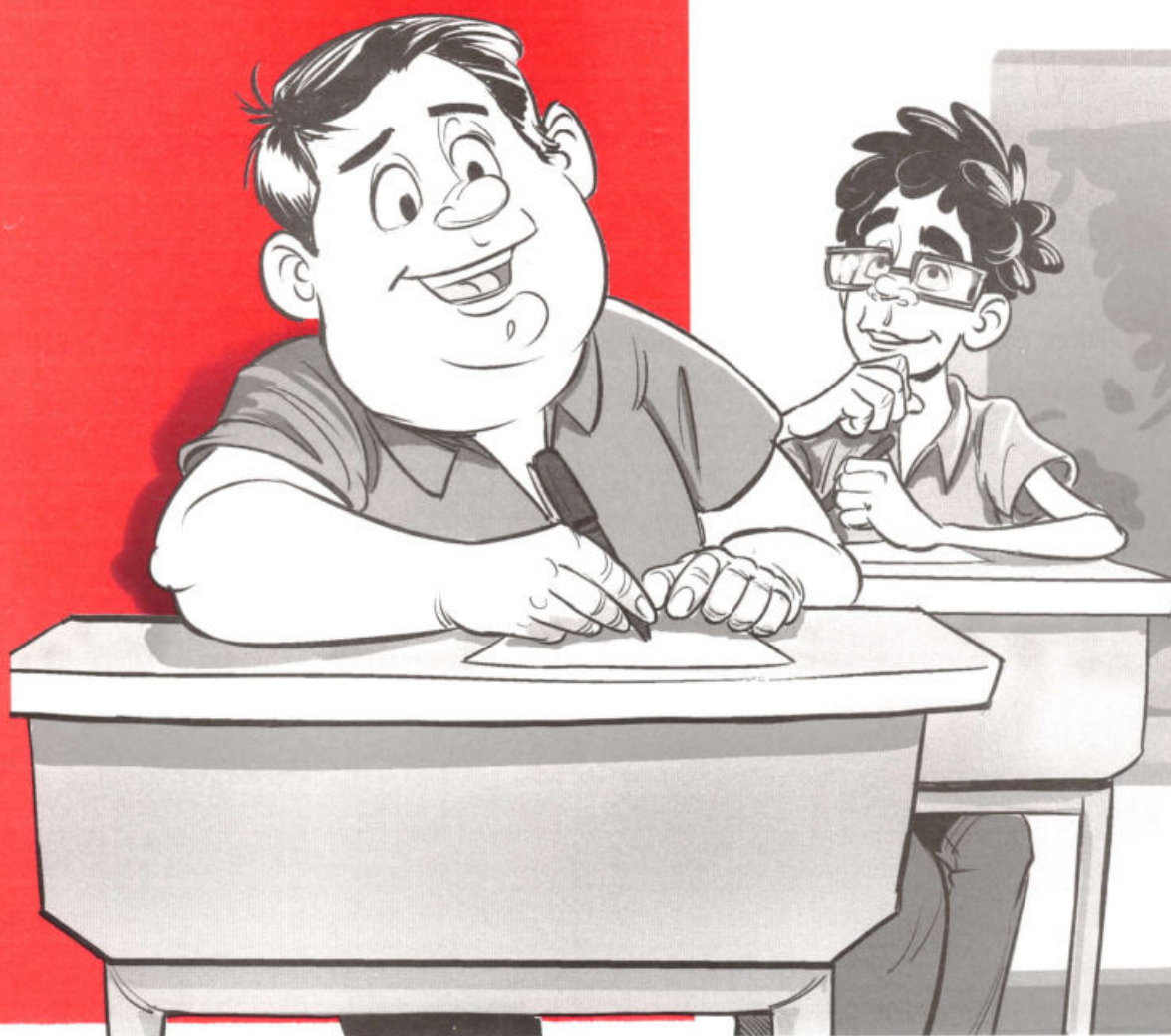
4 Three consecutive integers, their sum is 42

(2 Marks)

Find the numbers.

Important Questions

on Algebra and Statistics



Important questions on Unit One ?

Algebra and Statistics

First Multiple choice questions

1 $\left(-1\frac{1}{4}\right)^3 = \dots\dots\dots$

(a) $\frac{125}{64}$

(b) $-\frac{125}{64}$

(c) $\frac{25}{16}$

(d) $-\frac{1}{64}$

2 $\left(-\frac{2}{3}\right)^{-3} = \dots\dots\dots$

(a) $-\frac{8}{27}$

(b) $-\frac{27}{8}$

(c) $\frac{8}{27}$

(d) $\frac{27}{8}$

3 If $(X)^{-1} = 2$, then $X = \dots\dots\dots$

(a) -2

(b) ± 2

(c) $-\frac{1}{2}$

(d) $\frac{1}{2}$

4 The greatest value of $\left(\frac{1}{5}\right)^X$ when $X = \dots\dots\dots$

(a) zero

(b) 1

(c) 2

(d) 3

5 If $a = -3$, $b = 5$, then $\left(\frac{a}{b}\right)^2 = \dots\dots\dots$

(a) $-\frac{9}{25}$

(b) $-\frac{25}{9}$

(c) $\frac{9}{25}$

(d) $\frac{25}{9}$

6 The multiplicative inverse of the number $(-3)^{\text{zero}}$ is $\dots\dots\dots$

(a) 3

(b) 3

(c) -1

(d) 1

7 The multiplicative inverse of the number $\sqrt{6\frac{1}{4}}$ is $\dots\dots\dots$

(a) $-\frac{5}{2}$

(b) $\frac{5}{2}$

(c) $\frac{2}{5}$

(d) $-\frac{2}{5}$

8 The additive inverse of the number $\left(-\frac{2}{3}\right)^4$ is $\dots\dots\dots$

(a) $\frac{2}{3}$

(b) $-\frac{16}{81}$

(c) $\frac{81}{16}$

(d) $-\frac{81}{16}$

9 $\left(\frac{a}{b}\right)^3 \times \frac{b^3}{a^3} = \dots\dots\dots$ (Where $a \neq \text{zero}$, $b \neq \text{zero}$)

(a) ab

(b) $\left(\frac{a}{b}\right)^6$

(c) $(ab)^{\text{zero}}$

(d) $\frac{a}{b}$

10 If $2^X = 5$, then $2^{X+1} = \dots\dots\dots$

(a) 6

(b) 7

(c) 10

(d) 64

11 $a^{-4} \div a^{-6} = \dots\dots\dots$ (Where $a \neq \text{zero}$)

(a) a^{-10}

(b) a^{-2}

(c) a^2

(d) a^{10}

12 $3^{10} + 3^{10} + 3^{10} = \dots\dots\dots$

(a) 3^{11}

(b) 3^{30}

(c) 9^{10}

(d) 9^{30}

13 Half the number $2^{20} = \dots\dots\dots$

(a) 2^{18}

(b) 2^{19}

(c) 2^4

(d) 2^5

14 The number which is in standard form from the following is $\dots\dots\dots$

(a) 11×10^8

(b) 9.7×10^{-5}

(c) 10.2×10^{-2}

(d) 0.87×10^8

15 If $0.00049 = 4.9 \times 10^n$, then $n = \dots\dots\dots$

(a) 4

(b) -4

(c) 5

(d) -5

16 If $6300000 = 6.3 \times 10^n$, then $n = \dots\dots\dots$

(a) 6

(b) -6

(c) 5

(d) -5

17 The standard form of the number 53700 is $5.37 \times \dots\dots\dots$

(a) 10^3

(b) 10^4

(c) 10^{-4}

(d) 10^{-3}

18 Which of the following is the smallest ?

(a) 314×10^3

(b) 3.14×10^4

(c) 31.4×10^5

(d) 0.314×10^6

19 Which of the following is the greatest ?

(a) 2.3×10^7

(b) 3.2×10^7

(c) 7.6×10^6

(d) 6.7×10^6

20 If the thickness of a paper is 0.012 cm., which of the following is the height of a ream of 400 sheets ?

(a) (48×10^{-3}) cm.

(b) (48×10^{-2}) cm.

(c) $(4.8 \times 10^{\text{zero}})$ cm.

(d) 48 cm.

21 The standard form of the number 750×10^{-6} is $\dots\dots\dots$

(a) 7.5×10^{-8}

(b) 7.5×10^{-7}

(c) 7.5×10^{-4}

(d) 7.5×10^4

22 Which of the following equals $\frac{1}{4}$ million ?

(a) 25×10^5

(b) 0.25×10^5

(c) 0.25×10^6

(d) 0.25×10^7

23 If $400000 = 4 \times 10^n$, then $n = \dots\dots\dots$

(a) 6

(b) 5

(c) -5

(d) -6

24 $2.73 \times 10^{-4} = \dots\dots\dots$

(a) 0.00237

(b) 0.000237

(c) 23700

(d) 0.0000237

25 $40 - 4 \times 3^2 = \dots\dots\dots$

(a) -4

(b) 1

(c) -1

(d) 4

26 $8 \div 4 (3 - 2) = \dots\dots\dots$

(a) 1

(b) 2

(c) 12

(d) 24

27 $4 \times 2^3 - 20 = \dots\dots\dots$

(a) 32

(b) 48

(c) 12

(d) -12

28 $8 + 3^2 \div 9 - 7 = \dots\dots\dots$

(a) 2

(b) 3

(c) 4

(d) 5

29 $\frac{1}{3 \times 2^{-3}} = \dots\dots\dots$

(a) 6^3 (b) $\frac{3}{24}$ (c) $\frac{8}{3}$ (d) $\frac{1}{6^3}$

30 If $\sqrt[3]{x} = 9$, then $x = \dots\dots\dots$

(a) 3

(b) -3

(c) 81

(d) -81

31 $\sqrt{(10)^2 - (6)^2} = \dots\dots\dots$

(a) 4

(b) ± 4

(c) 8

(d) ± 8

32 The multiplicative inverse of the number $\sqrt{\frac{9}{16}}$ is $\dots\dots\dots$

(a) $-\frac{3}{4}$ (b) $-\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

33 If $x = 0.0009$, then $\sqrt{x} = \dots\dots\dots$

(a) 0.0003

(b) 0.0081

(c) 0.003

(d) 0.03

34 $\sqrt{(-5)^2} = \dots\dots\dots$

(a) -5

(b) 5

(c) 25

(d) ± 5

35 If $x + 9 = 11$, then $7x = \dots\dots\dots$

(a) 7

(b) 9

(c) 14

(d) 2

36 If $5x = 35$, then $2x + 1 = \dots\dots\dots$

(a) 7

(b) 8

(c) 15

(d) 71

- 37** The side length of a square whose area is $9x^2 \text{ cm}^2$ is cm.
 (a) $|3x|$ (b) $3x^2$ (c) $9x$ (d) $9x^2$
-
- 38** If $3x + 1 = 10$, then $x =$
 (a) 3 (b) 4 (c) 6 (d) 9
-
- 39** If three times of a number equals 27, then $\frac{1}{9}$ of this number is
 (a) 1 (b) 3 (c) 9 (d) 27
-
- 40** If $x = 4$, $y = -1$, then which of the following will be negative ?
 (a) $x + y^2$ (b) $\frac{y}{x}$ (c) $x - y$ (d) $x + y$
-
- 41** The S.S. of the equation : $x + 6 = 5$ in \mathbb{N} is
 (a) $\{2\}$ (b) $\{-1\}$ (c) $\{6\}$ (d) \emptyset
-
- 42** Twice of the number x subtracted from 3 equals
 (a) $2x + 3$ (b) $3 - 2x$ (c) $2x - 3$ (d) $3x - 2$
-
- 43** If $-x > 4$, then
 (a) $x < 4$ (b) $x > 4$ (c) $x < -4$ (d) $x > -4$
-
- 44** The S.S. of the inequality : $-x > -1$ where $x \in \mathbb{N}$ is
 (a) \emptyset (b) $\{1, 2\}$ (c) $\{-1, -2\}$ (d) $\{0\}$
-
- 45** The S.S. of the inequality : $-5x < \text{zero}$ in \mathbb{Q} is
 (a) \mathbb{Q}^+ (b) \mathbb{Q}^- (c) \mathbb{Z}^+ (d) \emptyset

Second Complete questions

- 1** $\left(\frac{3}{2}\right)^{\text{zero}} =$
-
- 2** $\left(\frac{2}{3}\right)^{-2} =$
-
- 3** $2\frac{1}{4} = \left(\frac{2}{3}\right)^{\text{.....}}$
-
- 4** The additive inverse of the number 3^{-1} is
-
- 5** The multiplicative inverse of the number $\left(\frac{-2}{5}\right)^{-2}$ equals
-
- 6** $(9x)^{-3} = \frac{9}{\dots}$ (where $x \neq 0$)

- 7 $3^8 \times 3^{-8} = 2^{\dots\dots\dots}$
- 8 $(b^{-1})^{-3} = b^{\dots\dots\dots}$ (Where $b \neq 0$)
- 9 If $(x^6)^2 = (x^3)^m$, then $m = \dots\dots\dots$
- 10 If $a = b$, then $2^{a-b} = \dots\dots\dots$
- 11 $\frac{x^{-5}}{y^{-5}} = (\dots\dots\dots)^5$ (where $x \neq 0, y \neq 0$)
- 12 $a^{-5} + 1 = a^{-5} (\dots\dots\dots + \dots\dots\dots)$ where $a \neq 0$
- 13 $\frac{2}{3^{-2}} = \dots\dots\dots$
- 14 $3^6 + 3^6 + 3^6 = 3^{\dots\dots\dots}$
- 15 $\left(\frac{3}{2}\right)^{-2} \times \frac{9}{4} \times \left(\frac{3}{4}\right)^{\text{zero}} = \dots\dots\dots$
- 16 The standard form of the number 0.00000721 is $\dots\dots\dots$
- 17 The standard form of seven millions is $\dots\dots\dots$
- 18 The number 534600 in the standard form is $\dots\dots\dots$
- 19 The standard form of the number 37×10^5 is $\dots\dots\dots$
- 20 If $0.00037 = x \times 10^{-4}$, then $x = \dots\dots\dots$
- 21 The standard form of the number 0.09×0.0005 is $\dots\dots\dots$
- 22 $3 + 4 \times 5 - 20 = \dots\dots\dots$
- 23 If $x = 3$, $y = -1$, then $2x + 2(3 - y) = \dots\dots\dots$
- 24 $3 \times 4 \div 6 - 2^3 \div (-2) = \dots\dots\dots$
- 25 The sum of the two square roots of the rational numbers 81 equals $\dots\dots\dots$
- 26 The additive inverse of the number $\sqrt{\frac{9}{16}}$ is $\dots\dots\dots$
- 27 The multiplicative inverse of the number $\sqrt{\frac{25}{36}}$ is $\dots\dots\dots$

28 $\sqrt{100 - 64} = 10 - \dots\dots\dots$

29 $\sqrt{(-8)^2 + (-6)^2} = \dots\dots\dots$

30 $\sqrt{0.36} = \dots\dots\dots$

31 If $a + b = 25$, then $2a + 2b = \dots\dots\dots$

32 If $5a = 40$, then $3a = \dots\dots\dots$

33 The S.S. of the equation : $X + 5 = 2$ in \mathbb{N} is $\dots\dots\dots$

34 If $3X = 5y = 15$, then $Xy = \dots\dots\dots$

35 If $z > y$, $y > X$, then $z > \dots\dots\dots$

36 The S.S. of the inequality : $X + 3 < 3$ in \mathbb{N} is $\dots\dots\dots$

37 The S.S. of the inequality : $2 < X \leq 4$ in \mathbb{N} is $\dots\dots\dots$

38 If the age of Ahmed now is X years, then his age 3 years ago is $\dots\dots\dots$ years.

Third Essay questions

1 Find the value of the following in the simplest form : $\frac{5^6 \times 5^2}{5^7}$

2 Find the value of the following in the simplest form : $\frac{a^5 \times a^8}{a^3 \times a^2 \times a^4}$ (where $a \neq \text{zero}$)

3 Calculate : $\left(\frac{3^4 \times 7^2}{7^3 \times 3^2}\right)^{-1}$

4 Calculate : $\frac{(10)^2 \times (0.01)^3}{(10)^{-3}}$

5 Find the value of the following in the simplest form : $\left(-\frac{3}{5}\right)^3 \times \left(\frac{-25}{27}\right)$

6 Put the expression : $\left(\frac{1}{2}\right)^2 \times \left(\frac{-1}{2}\right)^3$ in its simplest form.

7 If $X = \frac{1}{2}$, $y = \frac{1}{3}$, find the numerical value of : $(X + y)^{-2}$

8 If $a = \frac{3}{4}$, $b = \frac{-1}{2}$, find the numerical value of the expression : $\left(\frac{a}{b^2}\right)^2$

9 Simplify to the simplest form : $(X^2)^{-3} \div (X^{-1})^2$ where $X \neq 0$

- 10 If $x = \frac{-3}{2}$, $y = \frac{1}{2}$, $z = \frac{-4}{3}$, find the numerical value of the expression : $x^2 y^2 z^2$
- 11 Calculate the value of : $\left(\frac{-2}{5}\right)^x + \left(\frac{2}{5}\right)^y$ If $x = 4$, $y = 3$
- 12 Which is greater $(-2)^{82}$ or $(-2)^{83}$?
- 13 If $\frac{x}{y}$ is a rational number , $\left(\frac{x}{y}\right)^2 = 36$ Find : $\left(\frac{x}{y}\right)^3$
- 14 Find the result in the standard form : $(2.3 \times 10^3) + (6.3 \times 10^4)$
- 15 Write the result of : $(4.4 \times 10^3) \times (2 \times 10^5)$ in the standard form.
- 16 Calculate the value of the following in the standard form : $(3.6 \times 10^8) \div (1.8 \times 10^3)$
- 17 Write the following number in the standard form : 581 200 000 000
- 18 Put in the standard form : 0.000014×10^2
- 19 Find the value of the expression : $(5.4 \times 10^4) + (3.7 \times 10^4)$ in the form of $a \times 10^n$ where n is an integer.
- 20 Write in the standard form : 0.7×10^{-7}
- 21 Find in the simplest form the value of : $(12 - 4) \div 8 + 5$
- 22 Find the value of the expression : $12 \times 2^2 \div 24 + 3^2$
- 23 Calculate the following and put the result in the simplest form : $\left(\frac{1}{2}\right)^3 \times \left(\frac{3}{2}\right)^2 \div \left(\frac{-3}{4}\right)^2$
- 24 Calculate the value of the following : $3 + [5 + 2(8 \div 4)]$
- 25 Find the value of the expression : $12 \times 2^2 \div 24 \times \frac{1}{2}$
- 26 If $x = \frac{-2}{5}$, $y = \frac{3}{5}$, then find the value of : $x^2 + y^2$ in the simplest form.
- 27 If $x = 9$, $y = 7$, find the value of : $\sqrt{2x + y}$
- 28 Simplify to the simplest form : $\sqrt{11 \frac{5}{4}} \times \left(\frac{2}{7}\right)^{\text{zero}} \times \left(\frac{-2}{7}\right)^2$

29 Simplify to the simplest form : $\sqrt{6 \frac{1}{4}} + \frac{1}{5} \sqrt{16+9}$

30 Find the value of the following in the simplest form : $\left(\frac{-3}{7}\right)^{\text{zero}} \times \left(\frac{-2}{5}\right)^2 \times \sqrt{\frac{25}{4}}$

31 Find in the simplest form : $\left(\frac{-1}{5}\right)^2 + \sqrt{\left(\frac{-24}{25}\right)^2} - \left(\frac{3}{15}\right)^{-1}$

32 Find in the simplest form : $\sqrt{\frac{25 x^2 y^2}{36}}$

33 Find the number if added to its three times the result will be 28

34 Two numbers , the smaller number is x and the another number exceeds it by 4 , if the sum of the two numbers is 26 , then find the two numbers.

35 Find the S.S. of the equation : $2x - 3 = 5$ where $x \in \mathbb{Q}$

36 Find the S.S. in \mathbb{Q} of the equation : $2(x - 5) = 12$

37 Find the S.S. in \mathbb{Q} of the equation : $(3x + 2) - 5 = 12$

38 Find in \mathbb{Q} the S.S. of the equation : $5x - 4 = 2x + 11$

39 Find the S.S. of the inequality : $5x - 8 \geq 7$ where $x \in \mathbb{Q}$

40 Find in \mathbb{Q} the S.S. of the inequality : $3x - 1 > 5$

41 Find in \mathbb{Q} the S.S. of the inequality : $3 - 2x \leq 7$

42 Find in \mathbb{Q} the S.S. of the inequality : $9x + 1 \leq 4\left(2x + \frac{1}{4}\right)$

Important questions on Unit Two



Algebra and Statistics

First Multiple choice questions

- 1 When a fair die is tossed once, then the probability of getting an odd number is
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$
- 2 A fair die is rolled once, then the probability of getting a number less than 1 is
(a) $\frac{1}{6}$ (b) \emptyset (c) $\frac{1}{2}$ (d) zero.
- 3 The probability of the certain event equals
(a) 1 (b) zero. (c) -1 (d) 2
- 4 As flipping a fair coin once, the probability of appearing a head is
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$
- 5 Which of the following may be the probability of an event ?
(a) -0.35 (b) 85 % (c) 1.03 (d) -1
- 6 The sum of the probabilities for all possible outcomes of a random experiment
(a) = zero (b) = 1 (c) > 1 (d) < 1
- 7 If the probability of success of a student is 0.7, then the probability of his failure is
(a) 0.3 (b) 3 % (c) zero. (d) 1
- 8 When a fair die is tossed once, then the probability of getting a prime number equals
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

Second Complete questions

- 1 In the experiment of throwing a fair die once and observing the upper face, the probability of appearing an even number equals
- 2 As throwing a fair die once, the probability of appearance of an even prime number equals
- 3 If the probability of failure of a student is 20 %, then the probability of his success is

- 4 If the probability of success of a pupil is $\frac{5}{8}$, then the probability of his failure is
- 5 A class has 21 boys and 15 girls. A pupil of them is selected randomly, then the probability that the pupil is a girl =
- 6 The probability of occurring of the impossible event equals
- 7 As throwing a fair die once and observing the upper face, the probability of appearance of the number 4 =
- 8 The probability of any event is not less than and not more than

Third Essay questions

- 1 A box contains 4 white, 5 red and 6 blue balls, a ball is drawn randomly from the box. Find the probability of getting.
 - 1 A red ball.
 - 2 A white or red ball.
- 2 A fair dice is thrown once and the upper face is observed, find the probability of appearing :
 - 1 A prime number.
 - 2 A multiple of the number 3
 - 3 A number greater than 7
- 3 A card is selected at random from 8 cards numbered from 1 to 8, write the sample space, then find the probability of each of the following events :
 - 1 Getting an odd number.
 - 2 Getting a number divisible by 3
 - 9 Getting a number less than 9
- 4 A card is drawn randomly from a bag of 25 cards numbered from 1 to 25. Calculate the probability that the drawn card carries :
 - 1 A number divisible by 5
 - 2 A number ≥ 20
 - 3 A perfect square number
 - 4 An odd number greater than 13 and less than 25
- 5 A bag has a number of similar balls, 2 of them are green, 4 are blue and the rest are red. If the probability of drawing a green ball is $\frac{1}{6}$, find the number of red balls.

- 6 A card is drawn randomly from ten cards numbered from 1 to 10 , what is the probability that the chosen card shows :

- 1 An even number. 2 An odd number greater than 3

-
- 7 A fair die is rolled once and the number of dots on the upper face is observed.

Find the following :

- 1 The sample space.
2 The probability of getting number 7
3 The probability of getting an odd number
4 The probability of getting a prime number
5 The probability of getting a number less than 3

-
- 8 In the experiment of throwing a fair die once , **find :**

- 1 The sample space.
2 The probability of getting a number greater than 6
3 The probability of getting a number satisfying the inequality : $2 < X < 4$

Final Revision

of Algebra and Statistics





Remember

The repeated multiplication and the laws of powers

• If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, m and n are two integers, then :

① $\left(\frac{a}{b}\right)^n \times \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n+m}$ "when multiplying the like base, add their powers (indices)"

② $\left(\frac{a}{b}\right)^n \div \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n-m}$ where $\frac{a}{b} \neq 0$

when dividing like bases, we subtract their powers (indices)

③ $\left(\left(\frac{a}{b}\right)^n\right)^m = \left(\frac{a}{b}\right)^{n \times m}$

④ $\left(\frac{a}{b} \times \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \times \left(\frac{c}{d}\right)^n$

⑤ $\left(\frac{a}{b} \div \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \div \left(\frac{c}{d}\right)^n$ where $\frac{c}{d} \neq 0$

• If $\frac{a}{b}$ is a rational number, then $\left(\frac{a}{b}\right)^0 = 1$ where $a \neq 0$

• If a is a rational number, $a \neq 0$, n is a positive integer, then : $a^{-n} = \frac{1}{a^n}$, $a^n = \frac{1}{a^{-n}}$

• If $\frac{a}{b}$ is a rational number, where $\frac{a}{b} \neq 0$, n is a positive integer, then : $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Example

Simplify each of the following to the simplest form :

① $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3$

② $\left(\frac{2}{9}\right)^7 \div \left(\frac{2}{9}\right)^5$

③ $\left(-\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$

④ $\frac{6^{-3} \times 6^5}{6^2}$

⑤ $\left(\frac{5^3 \times 5^4}{5^5}\right)^{-2}$

⑥ $\frac{(-4x^3y^4)^2}{(-2xy^2)^4}$

Solution

① $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$

② $\left(\frac{2}{9}\right)^7 \div \left(\frac{2}{9}\right)^5 = \left(\frac{2}{9}\right)^2 = \frac{2^2}{9^2} = \frac{4}{81}$

③ $\left(-\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 = -\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 = -\left(\frac{1}{2}\right)^5 = -\frac{1}{32}$

④ $\frac{6^{-3} \times 6^5}{6^2} = 6^{-3+5-2} = 6^0 = 1$

⑤ $\left(\frac{5^3 \times 5^4}{5^5}\right)^{-2} = (5^{3+4-5})^{-2} = (5^2)^{-2} = 5^{-4} = \frac{1}{5^4} = \frac{1}{625}$

⑥ $\frac{(-4x^3y^4)^2}{(-2xy^2)^4} = \frac{(-4)^2 \times x^6 \times y^8}{(-2)^4 \times x^4 \times y^8} = \frac{16x^6y^8}{16x^4y^8} = x^2$ where $x, y \neq 0$

Remember**The standard scientific notation of the rational number**

The number is written in the standard form as : $a \times 10^n$ where $1 \leq |a| < 10$ and $n \in \mathbb{Z}$

For example : Each of the following two numbers is written in its standard form :

• 4.6×10^8

• 5.27×10^{-6}

• -9.6×10^{10}

• 1×10^{-13}

Each of the previous numbers is the product of two numbers :

- The first number could be positive or negative and its absolute value must be greater than or equal 1 and less than 10
- The second number expresses the powers of the number 10 (These powers could be positive or negative)

Example 1

Put each of the following numbers in the standard form :

① 8 200 000 000

② 0.000 000 135

③ 45×10^8

④ 706.4×10^5

⑤ -0.0015×10^{-9}

Solution

① $8\,200\,000\,000 = 8.2 \times 10^9$

② $0.000\,000\,135 = 1.35 \times 10^{-7}$

③ $45 \times 10^8 = 4.5 \times 10 \times 10^8 = 4.5 \times 10^9$

④ $706.4 \times 10^5 = 7.064 \times 10^2 \times 10^5 = 7.064 \times 10^7$

⑤ $-0.0015 \times 10^{-9} = -1.5 \times 10^{-3} \times 10^{-9} = -1.5 \times 10^{-12}$

Notice that

To move the decimal point 9 places towards left , we multiplied by 10^9

Notice that

To move the decimal point 7 places towards right , we multiplied by 10^{-7}

Example 2

Find the value of n in each of the following :

① $500\,000 = 5 \times 10^n$

② $0.00\,052 = 5.2 \times 10^n$

③ $7\,293 = n \times 10^3$

Solution

① $\because 500\,000 = 5 \times 10^5$

$\therefore n = 5$

② $\because 0.00\,052 = 5.2 \times 10^{-4}$

$\therefore n = -4$

③ $\because 7\,293 = 7.293 \times 10^3$

$\therefore n = 7.293$

Remember The order of mathematical operations

The order of performing the mathematical operations is as the following :

- ① Perform the operations within parentheses (interior parentheses then exterior ones).
- ② Evaluate the powers.
- ③ Perform multiplications and divisions in order from left to right.
- ④ Perform additions and subtractions in order from left to right.

Notice that

In the problems containing fractions, we should perform the operations in the numerator and denominator before divisions.

Example

Calculate the value of each of the following :

① $4 - 3 [4 - 2 (6 - 3)] \div 2$

② $8 \times 2^2 - 7 \times (4 + 1)$

③ $\frac{11 - (5 - 4)}{5^2 - 10 \times 2}$

Solution

$$\begin{aligned}
 \textcircled{1} \quad 4 - 3 [4 - 2 (6 - 3)] \div 2 &= 4 - 3 [4 - 2 \times 3] \div 2 && \text{(the interior parentheses)} \\
 &= 4 - 3 [4 - 6] \div 2 && \text{(multiplication inside parentheses)} \\
 &= 4 - 3 [-2] \div 2 && \text{(subtraction inside parentheses)} \\
 &= 4 + 6 \div 2 && \text{(multiplication by parentheses)} \\
 &= 4 + 3 && \text{(division)} \\
 &= 7 && \text{(addition)}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad 8 \times 2^2 - 7 \times (4 + 1) &= 8 \times 2^2 - 7 \times 5 && \text{(addition inside parentheses)} \\
 &= 8 \times 4 - 7 \times 5 && \text{(powers)} \\
 &= 32 - 35 && \text{(multiplication)} \\
 &= -3 && \text{(subtraction)}
 \end{aligned}$$

$$\textcircled{3} \quad \frac{11 - (5 - 4)}{5^2 - 10 \times 2} = \frac{11 - 1}{25 - 20} = \frac{10}{5} = 2$$

Remember The square root of a perfect square rational number

- The square root of the perfect square rational number "a" is the number whose square equals "a"

For example : $\sqrt{16} = 4$, $-\sqrt{16} = -4$, $\pm \sqrt{16} = \pm 4$

- It is meaningless to find \sqrt{a} if a is a negative rational number.

- $\sqrt{a^2} = |a|$

For example : $\sqrt{(-3)^2} = |-3| = 3$

- When there is an addition or a subtraction operation under the square root , it must be performed first before finding the square root.

Example 1

Find each of the following in the simplest form :

① $\sqrt{36}$

② $-\sqrt{\frac{16}{25}}$

③ $\pm \sqrt{2\frac{1}{4}}$

④ $\sqrt{\left(-\frac{2}{7}\right)^2}$

⑤ $-\sqrt{0.25}$

⑥ $\sqrt{16+9}$

⑦ $\sqrt{100-36}$

⑧ $\sqrt{\frac{36a^8}{49d^4}}$

Solution

① $\sqrt{36} = 6$ because $6^2 = 36$

② $-\sqrt{\frac{16}{25}} = -\frac{4}{5}$ because $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$

③ $\pm \sqrt{2\frac{1}{4}} = \pm \sqrt{\frac{9}{4}} = \pm \frac{3}{2}$

④ $\sqrt{\left(-\frac{2}{7}\right)^2} = \left|-\frac{2}{7}\right| = \frac{2}{7}$

⑤ $-\sqrt{0.25} = -\sqrt{\frac{25}{100}} = -\frac{5}{10} = -\frac{1}{2}$

⑥ $\sqrt{16+9} = \sqrt{25} = 5$

⑦ $\sqrt{100-36} = \sqrt{64} = 8$

⑧ $\sqrt{\frac{36a^8}{49d^4}} = \frac{6a^4}{7d^2}$

Example 2

Simplify each of the following to the simplest form :

① $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2$

② $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0$

Solution

① $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2 = -\frac{2}{7} \times \frac{7}{2} \times \frac{4}{49} = -\frac{4}{49}$

② $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$

Remember The properties of the equality relation for solving the equation

- We can add any rational number to both sides of the equation.

For example : If $x - 1 = 5$, then $x - 1 + 1 = 5 + 1$ *i.e.* $x = 6$

- We can subtract any rational number from both sides of the equation.

For example : If $x + 3 = 2$, then $x + 3 - 3 = 2 - 3$ *i.e.* $x = -1$

- We can multiply both sides of the equation by the same rational number.

For example : If $\frac{1}{5}x = 2$, then $\frac{1}{5}x \times 5 = 2 \times 5$ *i.e.* $x = 10$

- We can divide both sides of the equation by the same rational number not equal to zero.

For example : If $7x = 14$, then $\frac{7x}{7} = \frac{14}{7}$ *i.e.* $x = 2$

Example

Find in **Q** the solution set of each of the following equations :

① $2x - 5 = 13$

② $3x + 4 = 2(x + 1)$

Solution

① $\because 2x - 5 = 13$ “Adding 5 to both sides”

$\therefore 2x - 5 + 5 = 13 + 5$

$\therefore 2x = 18$ “Dividing both sides by 2”

$\therefore \frac{2x}{2} = \frac{18}{2}$

$\therefore x = 9$

\therefore The S.S. = $\{9\}$

② Notice that the variable (x) exists in the two sides , then we try to collect it in one side (say the left side)

$\therefore 3x + 4 = 2(x + 1)$

Using the distribution property

$\therefore 3x + 4 = 2x + 2$

Subtracting $2x$ from both sides

$\therefore 3x - 2x + 4 = 2x - 2x + 2$

$\therefore x + 4 = 2$

Subtracting 4 from both sides

$\therefore x + 4 - 4 = 2 - 4$

$\therefore x = -2$

\therefore The S.S. = $\{-2\}$

Remarks for solving word problems

- If a number = X , then its twice = $2X$ and its three times = $3X$,
- If a number = X and another number exceeds it by 5, then the other number = $X + 5$
- If a number = X and another number decreases than it by 5, then the other number = $X - 5$
- If the age of a man now = X years, then :
 - * His age after 3 years = $(X + 3)$ years. * His age 3 years ago = $(X - 3)$ years.
- Three consecutive integers are : X , $X + 1$ and $X + 2$
- Three consecutive natural (even or odd) numbers are X , $X + 2$ and $X + 4$
- The perimeter of a rectangle = $2(\text{length} + \text{width})$
- The perimeter of a square = side length $\times 4$
- The perimeter of the triangle = the sum of its sides lengths.
- The area of the triangle = $\frac{1}{2}$ the base length \times the height.
- The sum of measures of the interior angles of the triangle = 180°

Example 1

Two natural numbers, one of them is thrice of the other, if the sum of them is 16. Find the two numbers.

Solution

Let one of the two numbers be X

- | | |
|---|--------------------------------------|
| \therefore The other number is thrice of this number | \therefore The other number = $3X$ |
| \therefore The sum of the two numbers = 16 | $\therefore X + 3X = 16$ |
| $\therefore 4X = 16$ "Dividing by 4" | $\therefore X = 4$ |
| \therefore One of the two numbers = 4, the other number = $3 \times 4 = 12$ | |

Example 2

A rectangle with length equals twice its width and its perimeter = 18 cm. Find the dimensions of the rectangle.

Solution

- | | |
|---|------------------------------------|
| Let the width of the rectangle be X cm. | \therefore Its length = $2X$ cm. |
| \therefore The perimeter of the rectangle = $2(\text{length} + \text{width})$ | |
| $\therefore 18 = 2(2X + X)$ | $\therefore 18 = 2 \times 3X$ |
| $\therefore 18 = 6X$ | $\therefore X = 3$ |
| \therefore The width of the rectangle = 3 cm. and its length = 6 cm. | |

Remember**The properties of inequality for solving the inequalities**

- We can add any rational number to both sides of the inequality without change in the inequality relation.

For example : If $X - 1 > 5$, then $X - 1 + 1 > 5 + 1$ *i.e.* $X > 6$

- We can subtract any rational number from the two sides of the inequality without change in the inequality relation.

For example : If $X + 3 < 2$, then $X + 3 - 3 < 2 - 3$ *i.e.* $X < -1$

- We can multiply any positive rational number by both sides of the inequality without change in the inequality relation.

For example : If $\frac{1}{3} X < 2$, then $\frac{1}{3} X \times 3 < 2 \times 3$ *i.e.* $X < 6$

- We can multiply any negative rational number by both sides of the inequality with change in the inequality relation.

For example : If $-\frac{1}{3} X < 2$, then $-\frac{1}{3} X \times -3 > 2 \times -3$ *i.e.* $X > -6$

- We can divide any positive rational number by both sides of the inequality without change in the inequality.

For example : If $7X > 14$, then $\frac{7X}{7} > \frac{14}{7}$ *i.e.* $X > 2$

- We can divide any negative rational number by both sides of the inequality with change in the inequality.

For example : If $-7X > 14$, then $\frac{-7X}{-7} < \frac{14}{-7}$ *i.e.* $X < -2$

Example

Find in \mathbb{Q} the solution set of each of the following inequalities :

① $2X - 5 > 5$

② $4 - 2X \leq 2$

Solution

① $\because 2X - 5 > 5$ “Adding 5 to both sides”

$\therefore 2X > 10$ “Multiplying both sides by $\frac{1}{2}$ ”

$\therefore X > 5$

$\therefore 2X - 5 + 5 > 5 + 5$

$\therefore \frac{1}{2} \times 2X > 10 \times \frac{1}{2}$

\therefore The S.S. = $\{X : X \in \mathbb{Q}, X > 5\}$

② $\because 4 - 2X \leq 2$ “Adding -4 to both sides”

$\therefore -2X \leq -2$ “Dividing both sides by (-2) ”

Notice that : The change of inequality sign

$\therefore X \geq 1$

$\therefore -4 + 4 - 2X \leq -4 + 2$

$\therefore \frac{-2X}{-2} \geq \frac{-2}{-2}$

\therefore The S.S. = $\{X : X \in \mathbb{Q}, X \geq 1\}$

Remember The probability

- The probability of any event occurrence $A \subset S$ is denoted by $P(A)$ and it is given by using the relation : $P(A) = \frac{\text{The number of elements of the event « A »}}{\text{The number of elements of sample space « S »}} = \frac{n(A)}{n(S)}$
- The probability of the impossible event = 0 • The probability of the certain event = 1
- The value of probability of any event is not less than zero and not more than one
i.e. $0 \leq \text{The probability of an event occurrence} \leq 1$
- It is meaningless that the probability of the occurrence of an event is 140% or -0.2

Example 1

A fair die is rolled once and we observe the apparent number on the upper face , what is the probability of getting :

- ① the number 4 ② an even number. ③ a number greater than or equal to 5
④ a prime number. ⑤ a number greater than 6 ⑥ a number smaller than 10

Solution

- ① The probability of getting the number 4 = $\frac{1}{6}$
 ② The probability of getting an even number = $\frac{3}{6} = \frac{1}{2}$
 ③ The probability of getting a number greater than or equal to 5 = $\frac{2}{6} = \frac{1}{3}$
 ④ The probability of getting a prime number = $\frac{3}{6} = \frac{1}{2}$
 ⑤ The probability of getting a number greater than 6 = $\frac{0}{6} = 0$
 ⑥ The probability of getting a number smaller than 10 = $\frac{6}{6} = 1$

Example 2

A bag contains 4 red balls , 6 green balls and 5 black balls , if a ball is drawn randomly from it , calculate :

- ① The probability that the drawn ball is green.
 ② The probability that the drawn ball is black.
 ③ The probability that the drawn ball is not red.

Solution

- ① The probability that the drawn ball is green = $\frac{\text{The number of green balls}}{\text{The total number of balls}} = \frac{6}{15} = \frac{2}{5}$
 ② The probability that the drawn ball is black = $\frac{\text{The number of black balls}}{\text{The total number of balls}} = \frac{5}{15} = \frac{1}{3}$
 ③ The probability that the drawn ball is not red = $\frac{6+5}{15} = \frac{11}{15}$

Final Examinations

on Algebra and Statistics

- School book examinations
- Schools examinations





Model

1

Answer the following questions :

1 Complete :

1 $\frac{81}{625} = \left(\frac{25}{9}\right)^{\dots\dots\dots}$

2 If $7 - 2x = 3$, then $x = \dots\dots\dots$ where $x \in \mathbb{N}$

3 $3^{-1} + 4^{-1} = \dots\dots\dots$

4 The standard form of the number $0.7 \times 0.005 = \dots\dots\dots$

5 The probability of the certain event = $\dots\dots\dots$

2 Choose the correct answer :

1 The sum of the probabilities for all possible outcomes of a randomly experiment is $\dots\dots\dots$

(a) zero

(b) 1

(c) > 1

(d) < 1

2 If $3a = \sqrt{4}b$, then $\frac{a}{b} = \dots\dots\dots$

(a) $2:3$

(b) $3:2$

(c) $3:4$

(d) $4:3$

3 $\left(\frac{-2}{3}\right)^{-3}$ equals $\dots\dots\dots$

(a) $\frac{-27}{8}$

(b) $\frac{-8}{27}$

(c) $\frac{8}{27}$

(d) $\frac{27}{8}$

4 There are 21 boys and 15 girls in a classroom, one pupil is chosen randomly, the probability that the chosen pupil is a girl = $\dots\dots\dots$

(a) $\frac{5}{12}$

(b) $\frac{7}{12}$

(c) $\frac{4}{7}$

(d) $\frac{5}{6}$

5 $\sqrt{(-8)^2 + (-6)^2} = \dots\dots\dots$

(a) $|-10|$

(b) ± 10

(c) 14

(d) -14

6 10 % of L.E. $2\frac{1}{2}$ = L.E. $\dots\dots\dots$

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) 1

(d) 25

3 [a] Simplify to the simplest form : $\left(-\frac{3}{7}\right)^0 \times \left(\frac{-2}{5}\right)^2 \times \sqrt{6\frac{1}{4}}$

[b] Find the numerical value of the expression :

$3ab + 8a \div (4b)$ when $a = 4$, $b = -2$

4 [a] Find in \mathbb{Q} the S.S. of : $3x + 1 = 25$

[b] Find the value of : $\frac{8 \times 8^{-3}}{8^{-4}}$

5 [a] A factory of a tire record the distance that traveled by a certain type of them before damage for 800 units of this type as following.

The distance in thousand (km.)	Less than 50	50 to 100	More than 100 till 150	More than 150
The number of damage tire	80	120	280	320

If you bought a tyre of this type , what is the probability of change it :

- [1] Before traveled 50 thousand km.
 [2] After traveled more than 100 thousand km.

[b] Find in \mathbb{Q} the S.S. of : $2x + 5 < 16$

Model

2

Answer the following questions :

1 Complete :

[1] $\left(\frac{-2}{3}\right)^0 = \dots\dots\dots$

[2] $\sqrt{\frac{16}{49}} = \dots\dots\dots$

[3] The probability of the impossible event = $\dots\dots\dots$

[4] 1 , 2 , 3 , 5 , 8 , $\dots\dots\dots$, $\dots\dots\dots$ (In the same pattern)

[5] If the probability that the student is absent in a school is 0.15 , if the number of students of this school is 600 , then the number of the present students that day is $\dots\dots\dots$

2 Choose the correct answer :

[1] $2^3 \times 2^3 = \dots\dots\dots$

(a) 2^6

(b) 2^8

(c) 2^{15}

(d) 2^{53}

[2] Which of the following is the greatest ?

(a) 2.3×10^4

(b) 2.3×10^5

(c) 3.2×10^4

(d) 3.2×10^5

3 $(x^2)^{-3} \times x^6 = \dots\dots\dots$

(a) x^{12}

(b) x^{-12}

(c) x

(d) 1

4 Which of the following may be probability of an event ?

(a) -0.35

(b) 87 %

(c) 1.05

(d) 130 %

5 If $-x > 4$, then

(a) $x > -4$

(b) $x > 4$

(c) $x < -4$

(d) $x < 4$

6 Area of a rectangle of length 120 cm. and width 80 cm. equals m^2

(a) 9600

(b) 400

(c) 9.6

(d) 0.96

3 [a] Two integers numbers, the smaller one is $2x$ and the greater is $5x$, if the difference between them is 30 Find the two numbers.

[b] Find the value of : $\frac{5^{-4} \times 5^7}{5^3}$ in the simplest form.

4 [a] Find in \mathbb{Q} the S.S. of each of the following :

1 $(3x + 2) + 5 = 13$

2 $2x + 15 < 19$

[b] Find the value of the expression in the simplest form :

$$\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$$

5 [a] If a regular die is thrown once and observed the number on upper face ,
find the probability of each of the following :

1 Getting a prime even number.

2 Getting an odd number less than 4

[b] If $x = -\frac{1}{2}$, $y = -\frac{3}{4}$, find in the simplest form : $\left(\frac{y}{x^2}\right)^{-2}$

Model examination for the merge students

Answer the following questions :

1 Choose the correct answer :

1 $\left(\frac{-2}{3}\right)^2 = \dots\dots\dots$

(a) $\frac{4}{9}$

(b) $\frac{-4}{9}$

(c) $\frac{4}{6}$

(d) $\frac{-4}{6}$

2 $\left(\frac{4}{7}\right)^0 = \dots\dots\dots$

(a) 0

(b) 1

(c) $\frac{4}{7}$

(d) -1

3 $2 \times 6 - 4 \times 2 = \dots\dots\dots$

(a) 4

(b) 8

(c) 10

(d) 2

4 $(7)^{-2} = \dots\dots\dots$

(a) 49

(b) $\frac{1}{49}$

(c) 14

(d) -14

5 $\sqrt{9+16} = \dots\dots\dots$

(a) 7

(b) 5

(c) 25

(d) -7

2 Complete each of the following :

1 If $X + 2 = 6$, then $X = \dots\dots\dots$

2 When tossing a coin once, then the probability of the appearance of a tail = $\dots\dots\dots$

3 The probability of the impossible event = $\dots\dots\dots$

4 $\sqrt{\left(\frac{2}{5}\right)^2} = \dots\dots\dots$

5 $7(6^2 - 5 \times 6) = \dots\dots\dots$

3 Complete the solution to find the result :

1 $12 \times 2^2 \div 24 + 3^2 = 12 \times \dots\dots\dots \div 24 + \dots\dots\dots$

$= \dots\dots\dots \div 24 + \dots\dots\dots = \dots\dots\dots + \dots\dots\dots = \dots\dots\dots$

2 $\frac{8+20-4}{8-4} = \frac{\dots\dots\dots-4}{\dots\dots\dots} = \frac{\dots\dots\dots}{\dots\dots\dots} = \dots\dots\dots$

4 Put (✓) or (X) :

1 If $2x + 3 = 7$, then $x = 2$ ()

2 $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)^6$ ()

3 $(x^2)^3 = x^6$ ()

4 $\left(\frac{3}{2}\right)^2 = -\frac{9}{4}$ ()

5 $\sqrt{100 - 64} = 2$ ()

5 A card is drawn randomly from 8 cards numbered from 1 to 8

, join from column (A) to column (B) :

Column (A)	Column (B)
1 The event of getting an even number equals	• $\frac{1}{2}$
2 The probability of getting an even number equals	• $\{8, 6, 4, 2\}$
3 The event of getting a number > 6 equals	• 1
4 The probability of getting a number < 9 equals	• $\frac{1}{8}$
5 The probability of getting a number 8 equals	• $\{8, 7\}$

Some Schools Examinations



on Algebra and Statistics

1

Cairo Governorate



Rod El-Farag Educational Directorate
St. Mary's School

Answer the following questions :

1 Choose the correct answer :

1 $\sqrt{8^2 + 6^2} = \dots\dots\dots$

(a) ± 10

(b) 10

(c) 14

(d) -14

2 $(a^2)^4 = \dots\dots\dots$

(a) a^8

(b) a^4

(c) a^2

(d) a^6

3 Half of the number $2^{20} = \dots\dots\dots$

(a) 2^{10}

(b) 2^{11}

(c) 2^{19}

(d) 2^{40}

4 The probability of the certain event is $\dots\dots\dots$

(a) zero

(b) 1

(c) $\frac{1}{2}$

(d) $\frac{1}{3}$

5 If $\frac{x}{2} < 7$, then $\dots\dots\dots$

(a) $x > \frac{7}{2}$

(b) $x > \frac{2}{7}$

(c) $x > 14$

(d) $x < 14$

6 If $4x = 20$, then $3x - 1 = \dots\dots\dots$

(a) 14

(b) 15

(c) 16

(d) 17

2 Complete :

1 $3 \times 2 - 16 \div 8 = \dots\dots\dots$

2 $3^{10} + 3^{10} + 3^{10} = 3 \dots\dots\dots$

3 The multiplicative inverse of $(3)^{-2}$ is $\dots\dots\dots$

4 If $\frac{9}{16} = \left(\frac{4}{3}\right)^x$, then $x = \dots\dots\dots$

5 If $x = 7^{-3}$, $y = 7^3$, then $xy = \dots\dots\dots$

6 If $0.0000056 = 5.6 \times 10^x$, then $x = \dots\dots\dots$

3 [a] Simplify : $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

[b] Find the solution set of each of the following, $x \in \mathbb{Q}$:

1 $3x + 2 = 11$

2 $2x + 15 < 19$

4 [a] The sum of three consecutive numbers is 63, find the numbers.

[b] Simplify : $\frac{5^{-4} \times 5^7}{5^2}$

5 [a] If $3^x = 2$, find the value of : 9^x

[b] A box contains 6 red balls and 4 blue balls. A ball is drawn randomly.

Find the probability that :

1 The drawn ball is white.

2 The drawn ball is not blue.

2

Cairo Governorate



Waili Administration
Modern Future Language School

Answer the following questions :

1 Choose the correct answer from the given ones :

1 Half of the number $2^{20} = \dots\dots\dots$

(a) 2^{10}

(b) 2^{19}

(c) 2^{29}

(d) 2^{40}

2 If $\frac{x}{2} < 7$, then $\dots\dots\dots$

(a) $x > \frac{7}{2}$

(b) $x > \frac{2}{7}$

(c) $x > 14$

(d) $x < 14$

3 $\sqrt{8^2 + 6^2} = \dots\dots\dots$

(a) 10

(b) ± 10

(c) 14

(d) -14

4 $2^3 \times 2^4 = \dots\dots\dots$

(a) 2^6

(b) 2^{34}

(c) 2^7

(d) 2^{12}

5 The probability of the impossible event equals $\dots\dots\dots$

(a) 0

(b) 1

(c) -1

(d) 2

6 $(2)^3 \dots\dots\dots (2)^{-5}$

(a) <

(b) =

(c) >

(d) \leq

2 Complete each of the following :

1 The standard form of the number $0.7 \times 0.005 = \dots\dots\dots$

2 $|-2| + |5| = \dots\dots\dots$ (in the simplest form)

3 The multiplicative inverse of $\sqrt{\frac{16}{49}}$ is $\dots\dots\dots$

4 If the probability of success of a student is 0.8, then the probability of his failure is $\dots\dots\dots$

5 $\left(\frac{1}{2}\right)^{-1} = \dots\dots\dots$

6 The degree of the algebraic term $5x^2$ is $\dots\dots\dots$

3 [a] Find the S.S. in \mathbb{Q} : $2x + 4 < 16$

[b] Find in the simplest form : $\frac{5^{-4} \times 5^7}{5^2}$

- 4 [a] Simplify to the simplest form : $\left(-\frac{2}{5}\right)^0 \times \left(-\frac{2}{5}\right)^2 \times \sqrt{6 \frac{1}{4}}$
 [b] If $x = \frac{2}{3}$, $y = \frac{3}{4}$, find : $x^2 y^2$

- 5 [a] A box contains 6 red balls and 4 blue balls. A ball is drawn randomly from the box.

Find the probability that :

- 1 The drawn ball is white. 2 The drawn ball is not blue.

- [b] Find the solution set in \mathbb{Z} : $3x + 1 = 25$

3

Giza Governorate

6th October Educational Directorate

Answer the following questions :

- 1 Choose the correct answer :

- 1 $3^3 \times 3^4 = \dots\dots\dots$
 (a) 3^{12} (b) 3 (c) 3^7 (d) 3^{-1}
 2 $3x^{-1} = \dots\dots\dots$, $x \neq 0$
 (a) $-3x$ (b) $\frac{3}{x}$ (c) $3x$ (d) $\frac{1}{3x}$
 3 The multiplicative inverse of the number $\sqrt{\frac{4}{9}}$ is $\dots\dots\dots$
 (a) $-\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{2}$
 4 If $-x < 2$, then $\dots\dots\dots$
 (a) $x > -2$ (b) $x > 2$ (c) $x < -2$ (d) $x < 2$
 5 $6000 \times 50 = \dots\dots\dots$
 (a) 300×10^2 (b) 30×10^5 (c) -3×10^3 (d) 3×10^5
 6 $\sqrt{16+9} = 4 + \dots\dots\dots$
 (a) 3 (b) 1 (c) 7 (d) 4

- 2 Complete :

- 1 The additive inverse of $\left(-\frac{2}{3}\right)^2$ is $\dots\dots\dots$
 2 The probability of the impossible event equals $\dots\dots\dots$
 3 $\frac{6a^2x^4}{2a^3x^3} = \dots\dots\dots$ where $a \neq 0$, $x \neq 0$
 4 If $2x + 1 = 11$, then $6x - 1 = \dots\dots\dots$
 5 The multiplicative inverse of the number 2^{-3} is $\dots\dots\dots$
 6 $2^3 \times 3^3 = \dots\dots\dots^3$

3 [a] Find the solution set in \mathbb{Q} :

1 $8x + 4 = 12$

2 $3x - 1 \leq 3$

[b] * Find the value of : $12 + (9 - 2) \times 3^2$

4 [a] If $x = \frac{1}{2}$, $y = \frac{2}{3}$, $z = \frac{-3}{2}$, find : $(xyz)^2$

[b] Simplify : $\frac{x^3 \times x^{-2}}{x^{-5} \times x}$ where $x \neq 0$, then find the value when : $x = -2$

5 A fair die is rolled once. Calculate the probability of appearance on the upper face :

1 An even prime number.

2 Number 5

3 A number ≤ 6

4

Giza Governorate



Awseem Educational Directorate

Answer the following questions :

1 Complete the following :

1 If $\frac{x}{y} = \frac{8}{5}$, then $\frac{5x}{8y} = \dots\dots\dots$

2 In an experiment of throwing a regular die once , the probability of appearance of a number greater than 6 is $\dots\dots\dots$

3 If $x + 7 = 10$, then the value of $4x = \dots\dots\dots$

4 $\sqrt{16 + 9} = 4 + \dots\dots\dots$

5 If $\left(\frac{4}{9}\right)^x = \left(\frac{9}{4}\right)^4$, then $x = \dots\dots\dots$

6 If $0.00037 = 3.7 \times 10^n$, then the value of $n = \dots\dots\dots$

2 Choose the correct answer :

1 The S.S. of : $-2x + 1 = -3$ in \mathbb{Z} is $\dots\dots\dots$

(a) $\{2\}$

(b) $\{-2\}$

(c) $\{-4\}$

(d) \emptyset

2 $\left(\frac{1}{3}\right)^{-2} = \dots\dots\dots$

(a) $\frac{1}{9}$

(b) -9

(c) $-\frac{1}{9}$

(d) 9

3 Quarter of the number $4^{20} = \dots\dots\dots$

(a) 4^5

(b) 4^{10}

(c) 4^{19}

(d) 2^{10}

4 The multiplicative inverse of $\left(\frac{-3}{5}\right)^2$ is $\dots\dots\dots$

(a) $\frac{9}{25}$

(b) $\frac{-5}{3}$

(c) $\frac{25}{9}$

(d) 1

5 If $x = y$, then $5^{y-x} = \dots\dots\dots$

(a) zero

(b) 9

(c) 5

(d) 1

6 The age of Ahmed now is X years , then his age 5 years ago is years.

- (a) $5X$ (b) $5 + X$ (c) $5 - X$ (d) $X - 5$

3 [a] Find the S.S. of the inequality : $3X - 2 > X + 4$, where $X \in \mathbb{Q}$

[b] If $a = \frac{-1}{2}$, $b = 2$ and $c = \frac{3}{4}$

, then find in the simplest form the numerical value of : $a^2 b^3 + b^2 c$

4 Find each of the following :

1 $\left(\frac{4}{5}\right)^2 \times \sqrt{\frac{25}{16}} \times \left(\frac{1}{4}\right)^0$

2 $\frac{(-3)^6 \times (3)^{-3}}{(3)^5 \times (3)^{-4}}$

5 [a] Find in \mathbb{Q} the S.S. of the equation : $2(X + 4) = 15$

[b] A card is drawn randomly from 10 cards numbered from 1 to 10 , then calculate the probability of drawing :

- 1 A card carrying an odd number greater than 10
2 A card carrying an even number less than 10
3 A card carrying a prime number.

5

Alexandria Governorate



2nd El-Montaza Educational Zone
Math's Supervision

Answer the following questions :

1 Choose the correct answer :

1 The degree of the algebraic term $4Xy$ is

- (a) first. (b) second. (c) third. (d) fourth.

2 If $0.00043 = 4.3 \times 10^n$, then $n =$

- (a) -5 (b) -4 (c) 4 (d) 5

3 When tossing a coin once , then the probability of the appearance of a tail is

- (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$

4 $(2y)^3 =$

- (a) $2y^3$ (b) $8y^3$ (c) $8y$ (d) $32y$

5 If $2^X = 2$, $2^Y = 3$, then $2^{X+Y} =$

- (a) 1 (b) -1 (c) $\frac{2}{3}$ (d) 6

6 $(-1)^3$ $(-1)^2$

- (a) $<$ (b) $>$ (c) $=$ (d) \geq

6 $\left(\frac{1}{2}\right)^5 \div \left(\frac{1}{2}\right)^3 = \dots\dots\dots$

(a) $\frac{1}{32}$

(b) $\frac{1}{16}$

(c) $\frac{1}{8}$

(d) $\frac{1}{4}$

2 Complete the following :

1 If $2x = 6$, then $5x = \dots\dots\dots$

2 If the probability of success of Seif is $\frac{7}{8}$, then the probability of his failure is $\dots\dots\dots$

3 If $x = 2^5 + 2^5$, $y = 2^6 + 2^6 + 2^6$, then $x + y = 2^{\dots\dots\dots}$

4 If $0.00049 = 4.9 \times 10^n$, then $n = \dots\dots\dots$

5 The multiplicative inverse of 7^{-1} is $\dots\dots\dots$

6 If the solution set of the inequality : $x < k$ in \mathbb{N} is $\{0, 1\}$, then $k = \dots\dots\dots$

3 [a] Find in \mathbb{Q} the solution set of the equation : $3x - 5 = x + 7$

[b] Simplify the following to the simplest form : $1\frac{1}{3} \times \sqrt{\frac{81}{16}} \times \left(-\frac{2}{3}\right)^0$

4 [a] Simplify the following to the simplest form : $\frac{3^{-1} \times 3^7}{(-3)^5}$

[b] Find in \mathbb{Q} the solution set of the inequality : $4x - 7 \geq 5$

5 [a] If $x = \frac{-3}{2}$ and $y = \frac{1}{2}$, find the value of : $\frac{x^2}{y^2}$

[b] A box contains 5 white balls , 4 red balls and 6 black balls. If a ball is drawn randomly , find the probability that the drawn ball is :

1 Black.

2 Not red.

7

El-Sharkia Governorate



Math Inspection

Answer the following questions :

1 Choose the correct answer :

1 The multiplicative inverse of the number 3^{-2} is $\dots\dots\dots$

(a) 9

(b) -9

(c) $\frac{1}{9}$

(d) $-\frac{1}{9}$

2 Half of the number $2^8 = \dots\dots\dots$

(a) 2^2

(b) 2^4

(c) 2^6

(d) 2^7

3 If $3x = -36$, then $x^2 = \dots\dots\dots$

(a) -12

(b) 144

(c) -24

(d) 24

- 4 The probability of the certain event equals
 (a) zero (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1
- 5 Which of the following may be the probability of an event ?
 (a) -0.35 (b) 87 % (c) 1.05 (d) 130 %
- 6 $\sqrt{(-8)^2 + (-6)^2} = \dots\dots\dots$
 (a) 10 (b) ± 10 (c) 14 (d) -14

2 Complete :

- 1 If $-x < 3$, then $x \dots\dots\dots -3$
- 2 If $a = b$, then $\left(\frac{3}{4}\right)^{a-b} = \dots\dots\dots$
- 3 If the probability of success of a student is 0.8 , then the probability of his failure is
- 4 If $a = 2$, $b = -4$, then $a^2 + b = \dots\dots\dots$
- 5 The number 0.000053 in the scientific notation is
- 6 $\frac{9}{16} = \left(\frac{4}{3}\right)^{\dots\dots\dots}$

3 Find the solution set in \mathbb{Q} :

- 1 $3x + 2 = 8$ 2 $3 - 2x \leq 7$

- 4 [a] Simplify : $\left(\frac{-3}{7}\right)^0 \times \frac{-2}{5} \times \sqrt{6\frac{1}{4}}$
 [b] Simplify to the simplest form : $\frac{5^{-4} \times 5^7}{5^3}$

- 5 [a] A box contains 4 white , 5 red and 6 blue balls. A ball is drawn randomly from the box. Calculate the probability of getting :

- 1 A blue ball. 2 A white or red ball.
 3 A green ball.
- [b] If $x = \frac{3}{4}$, $y = \frac{4}{3}$, find the numerical value of : $x^2 y^2$

8

El-Gharbia Governorate



**The Central Maths Supervision
Official Language Schools**

Answer the following questions :

- 1 Choose the correct answer from those given :**

- 1 $\sqrt{10^2 - 6^2} = \dots\dots\dots$
 (a) 4 (b) 8 (c) ± 8 (d) ± 4

2 Half of $2^{20} = \dots\dots\dots$

- (a) 2^5 (b) 2^{19} (c) 2^{18} (d) 4^5

3 $3^{10} + 3^{10} + 3^{10} = \dots\dots\dots$

- (a) 3^{10} (b) 3^{30} (c) 9^{10} (d) 3^{11}

4 10 % of L.E. $2\frac{1}{2} = \text{L.E.} \dots\dots\dots$

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) 25

5 If $-X < 5$, then $X \dots\dots\dots$

- (a) > -5 (b) > 5 (c) < -5 (d) < 5

6 $\frac{6a^2X^4}{2a^3X^3} = \dots\dots\dots$ where $a \neq 0$, $X \neq 0$

- (a) $3aX^2$ (b) $3a^5X^7$ (c) $\frac{3X}{a}$ (d) $\frac{3}{aX}$

2 Complete each of the following :

1 The sum of probabilities of all outcomes of a random experiment equals $\dots\dots\dots$

2 The additive inverse of $\left(-\frac{2}{5}\right)^2$ is $\dots\dots\dots$

3 If $0.00025 = 2.5 \times 10^n$, then $n = \dots\dots\dots$

4 $|-5| + |3| = \dots\dots\dots$ (in the simplest form)

5 If the probability that the student is absent in a school is 0.15, and the number of students of this school is 600, then the number of the present students that day is $\dots\dots\dots$ students.

6 $2\frac{1}{4} = \left(\frac{3}{2}\right)^{\dots\dots\dots}$

3 [a] If $X = -\frac{1}{2}$, $y = \frac{3}{4}$, find in the simplest form : $\left(\frac{y^2}{X}\right)^2$

[b] Find in the simplest form : $\left(-\frac{3}{7}\right)^0 \times \left(-\frac{2}{5}\right)^2 \times \sqrt{6\frac{1}{4}}$

4 [a] Find the value of : $\left(\frac{9^3 \times 9}{9^5}\right)^{-3}$

[b] Two natural numbers, one of them is twice the other, and their sum is 108
Find the two numbers.

5 [a] Find in \mathbb{Q} the S.S. of :

1 $2X + 15 < 19$ 2 $3X + 1 = 25$

[b] A box contains 5 white, 4 black and 7 red balls. A ball is drawn randomly from the box. Calculate the probabilities of the following events :

1 The drawn ball is white. 2 The drawn ball is red.

3 The drawn ball is not white.



Answer the following questions :

1 Complete :

1 $\left(-\frac{3}{5}\right)^0 = \dots\dots\dots$

2 $\sqrt{\frac{25}{49}} = \dots\dots\dots$

3 The probability of the impossible event equals

4 1 , 2 , 3 , 5 , 8 , , (in the same pattern)

5 $*(8 - 6 \div 2)^2 + 3 \times 4 = \dots\dots\dots$

6 The standard form of the number 0.8×0.006 is

2 Choose the correct answer :

1 The multiplicative inverse of the number $(-1)^5$ is

(a) $(-1)^3$

(b) $(-1)^2$

(c) 3

(d) $(-3)^0$

2 $\left(\frac{5}{3}\right)^2 \times \left(\frac{3}{5}\right)^0 = \dots\dots\dots$

(a) $\frac{5}{3}$

(b) $\frac{25}{9}$

(c) 0

(d) 1

3 $2^3 \times 2^3 = \dots\dots\dots$

(a) 2^6

(b) 2^8

(c) 2^{15}

(d) 2^{53}

4 If $-X > -4$, then

(a) $X > -4$

(b) $X > 4$

(c) $X < -4$

(d) $X < 4$

5 The area of a rectangle of length 120 cm. and width 80 cm. equals m^2

(a) 9600

(b) 400

(c) 9.6

(d) 0.96

6 The sum of the probabilities of all possible outcomes of a random experiment is

(a) zero

(b) 1

(c) > 1

(d) < 1

3 [a] Find in \mathbb{Q} the solution set of : $X + 4 > 1$

[b] Find the value of : $\frac{8 \times 8^{-3}}{8^{-4}}$

4 [a] Simplify to the simplest form : $\left(-\frac{3}{5}\right)^0 \times \left(\frac{-2}{5}\right)^2 \times \sqrt{6 \frac{1}{4}}$

[b] Write the following number in the standard form : 581200000000

- 5 [a] A card is chosen randomly from ten cards numbered from 1 to 10

What is the probability that the chosen card shows :

1 An odd number.

2 A prime number.

- [b] If $x = -\frac{1}{2}$, $y = -\frac{3}{4}$, find the value of : $x^2 - y$

10

El-Beheira Governorate



Kafr El-Dawar Educational Zone
Private Education Administration

Answer the following questions :

- 1 Choose the correct answer :

- 1 $x^6 \div x^{-2} = \dots\dots\dots$ (where $x \neq 0$)

(a) x^3

(b) x^4

(c) x^{12}

(d) x^8

- 2 If $x + 9 = 11$, then $7x = \dots\dots\dots$

(a) 2

(b) 4

(c) 14

(d) 7

- 3 If $-2x \geq 1$, then $x \dots\dots\dots$

(a) $\geq -\frac{1}{2}$

(b) $\leq -\frac{1}{2}$

(c) ≥ -2

(d) ≥ -1

- 4 If the probability that a pupil succeeds is 0.7 , then the probability of his failure is $\dots\dots\dots$

(a) 0.6

(b) 0.5

(c) 0.3

(d) 0.4

- 5 $(x^{-2})^3 = \dots\dots\dots$, $x \neq 0$

(a) $\frac{1}{x^5}$

(b) x^{-5}

(c) x^6

(d) $\frac{1}{x^6}$

- 6 $2^x + 2^x = \dots\dots\dots$

(a) 2^{x-2}

(b) 2^{x-1}

(c) 2^{x+1}

(d) 2^x

- 2 Complete the following :

- 1 If $0.00085 = 8.5 \times 10^k$, then $k = \dots\dots\dots$

- 2 The degree of the algebraic term $-5x^3y$ is $\dots\dots\dots$

- 3 The sum of the two square roots of the number 16 is $\dots\dots\dots$

- 4 A fair die is thrown once , the probability of appearance of a prime odd number is $\dots\dots\dots$

- 5 $\sqrt{100 - 64} = 10 - \dots\dots\dots$

- 6 The multiplicative inverse of $\left(-\frac{4}{5}\right)^2$ is $\dots\dots\dots$

- 3 [a] If $x \in \mathbb{Q}$, find the S.S. of the equation : $5x - 1 = 3x + 13$

- [b] If $a = \frac{2}{3}$, $b = -\frac{1}{2}$, then find the value of :

1 $a^2 b^2$

2 $(a - b)^{-1}$

4 [a] Find the result in the simplest form : $\left(\frac{-1}{2}\right)^2 \times \sqrt{\frac{81}{25}} \times \frac{4}{3}$

[b] Simplify : $\frac{X^5 \times X^3}{X^7 \times X^4}$, then find the numerical value of the result when : $X = 4$

5 [a] Find the S.S. of the following inequality in \mathbb{Q} : $4X + 3 < 27$

[b] A box contains 4 white , 6 blue and 5 red balls. A ball is drawn randomly from the box.

Find the probabilities of the following events :

1 The drawn ball is red.

2 The drawn ball is not blue.

3 The drawn ball is white or blue.

4 The drawn ball is black.

11

El-Menia Governorate



Bani Mazar Administration
Math Department

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer :

1 $\left\{\frac{3}{4}\right\} \dots \mathbb{Q}$

(a) \in

(b) \notin

(c) \subset

(d) $\not\subset$

2 $2^5 + 2^5 = \dots$

(a) 2^6

(b) 2^{10}

(c) 4^{10}

(d) 4^5

3 $\sqrt{2\frac{7}{9}} = \dots$

(a) $2\frac{7}{9}$

(b) $1\frac{2}{3}$

(c) $\frac{25}{9}$

(d) $\frac{5}{9}$

4 $\left(-\frac{2}{5}\right)^{-1} = \dots$

(a) 2.5

(b) $\frac{2}{5}$

(c) $-\frac{2}{5}$

(d) -2.5

5 $0.0000073 = 7.3 \times \dots$

(a) 10^{-6}

(b) 10^5

(c) 10^{-5}

(d) 10^6

6 The probability of the impossible event is

(a) \emptyset

(b) 1

(c) 0 %

(d) 0.5

2 Complete each of the following :

1 $12 \div 3 \times 5 = \dots$

2 $\left(\frac{-21X^5}{67Y^9}\right)^{\text{zero}} = \dots, X \neq 0, Y \neq 0$

3 $\frac{3}{4} = \dots \%$

4 The solution set of the equation : $3x = 5$ in \mathbb{Z} is

5 The probability of the sure event equals

6 If $\left(-\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^m$, then $m =$

3 [a] Simplify to the simplest form : $\left(-\frac{3}{7}\right)^0 \times \left(-\frac{2}{5}\right)^{-2} \times \sqrt{\left(-\frac{4}{25}\right)^2}$

[b] Find the value of : $\frac{5^{-4} \times 5^{-3}}{5^{-7}}$

4 [a] If $x = 3$, $y = -4$, then find the value of : $\sqrt{x^2 + y^2}$

[b] A fair dice is rolled once, and the upper face is observed.

1 Write the sample space of this random experiment.

2 Find the probability of appearing of an odd number.

3 Find the probability of appearing of an even prime number.

5 Find the solution set of each of the following in \mathbb{Q} :

1 $5x + 6 = -9$

2 $2x - 5 < 3$

12 South Sinai Governorate



The Educational Directorate
Tur Sinai Educational Zone

Answer the following questions :

1 Choose the correct answer :

1 $\left(\frac{2}{3}\right)^{-2} =$

(a) $-\frac{9}{4}$

(b) $-\frac{4}{9}$

(c) $\frac{4}{9}$

(d) $\frac{9}{4}$

2 $x^5 \times x^2 =$

(a) x^{10}

(b) x^7

(c) x^4

(d) x^3

3 If $x + 9 = 11$, then $7x =$

(a) 14

(b) 2

(c) 7

(d) 9

4 $\sqrt{\frac{9}{16}} =$

(a) $\frac{3}{4}$

(b) $-\frac{3}{4}$

(c) $\frac{4}{3}$

(d) $-\frac{4}{3}$

5 The solution set of the inequality : $x < 2$ in \mathbb{N} is

(a) \emptyset

(b) $\{2\}$

(c) $\{0, 1\}$

(d) $\{1\}$

6 Half of the number $2^8 = \dots\dots\dots$

(a) 2^2

(b) 2^4

(c) 2^6

(d) 2^7

2 Complete the following :

1 The probability of the impossible event equals $\dots\dots\dots$

2 $(a^2)^4 = \dots\dots\dots$

3 $\sqrt{9+16} = \dots\dots\dots$

4 The standard form of the number 33000 is $\dots\dots\dots$

5 If $X = y$, then $\left(\frac{2}{3}\right)^{X-y} = \dots\dots\dots$

6 The solution set of the equation : $X + 3 = 3$ in \mathbb{N} is $\dots\dots\dots$

3 Find in \mathbb{Q} the solution set of each of the following :

1 $5X - 2 = 13$

2 $3X + 2 \leq 8$

4 Find in the simplest form :

1 $\frac{2^3 \times 2^6}{2^7}$

2 $\left(-\frac{3}{7}\right)^0 \times \left(-\frac{2}{5}\right)^2 \times \sqrt{\frac{25}{4}}$

5 [a] If $X = -\frac{3}{2}$, $y = \frac{1}{2}$, $z = -\frac{4}{3}$, find the numerical value of : $(X y z)^2$

[b] If a regular die is thrown once and the number on the upper face is observed, then find the probability of getting :

1 An even number.

2 An odd number less than 4

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1

Cairo Governorate



Hel. Educ. Administration
St. Joseph's School

Answer the following questions :

1 Choose the correct answer from the given ones :

1 $79000 = 7.9 \times \dots\dots\dots$

(a) 10^3

(b) 10^5

(c) 10^4

(d) 10^2

2 If the probability of success of a student is $\frac{3}{4}$, then the probability of his failure is

(a) 75 %

(b) 25 %

(c) 30 %

(d) 40 %

3 Which of the following is the probability of occurrence of an event ?

(a) 1.1

(b) -0.5

(c) 0

(d) 101 %

4 If $\frac{x}{2} < 7$, then

(a) $x > \frac{7}{2}$

(b) $x > \frac{2}{7}$

(c) $x > 14$

(d) $x < 14$

5 If $x^{34} + x^{43} = 0$, then $x = \dots\dots\dots$

(a) 2

(b) 1

(c) -1

(d) -2

2 Complete :

1 If $2x = 100$, then $x = \dots\dots\dots$

2 If $a = 2$ and $b = 6$, then the numerical value of $\frac{b-a}{a^3} = \dots\dots\dots$

3 The multiplicative inverse of 9^{-1} is

4 $\left(\frac{x^{-3}}{x^2}\right)^2$ in the simplest form is where $x \neq 0$

5 If $a b^{-1} = \frac{1}{3}$, then $\frac{b}{a} = \dots\dots\dots$

3 [a] A class has 50 students, the probability that the student is a boy is $\frac{2}{5}$, find the number of girls.

[b] Find the S.S. of each of the following inequalities :

1 $9 \leq 4x + 1 \leq 17, x \in \mathbb{Z}$

2 $9 - 5x < 14, x \in \mathbb{Q}$

4 [a] Solve in \mathbb{Q} : $4(x-1) - (x+3) = 0$

[b] If $\frac{x}{y}$ is a rational number and $\frac{x^2}{y^2} = 0.16$, then find the value of : $\left(\frac{x}{y}\right)^3$

5 [a] 1 If $x = 3$, $y = \frac{2}{3}$, evaluate : $x^2 y^2$

2 If $3^x = 2$, find the value of : 9^x

[b] If x = the area of a square whose side length is 10 units and $y = \left(\frac{2}{3}\right)^{-2} \times \frac{4}{9}$, evaluate : $x + xy$

2

Cairo Governorate



Rod El-Farag Educational Zone

Answer the following questions :

1 Choose the correct answer :

1 $2 \times 6 - 4 \div 2 = \dots\dots\dots$

(a) 10

(b) 4

(c) 2

(d) 1

2 $\sqrt{8^2 + 6^2} = \dots\dots\dots$

(a) 10

(b) ± 10

(c) 14

(d) -14

3 $(a^2)^4 = \dots\dots\dots$

(a) a^8 (b) a^4 (c) a^2 (d) a^6

4 If $4x = 20$, then $3x - 1 = \dots\dots\dots$

(a) 14

(b) 15

(c) 16

(d) 17

5 Half the number $2^{20} = \dots\dots\dots$

(a) 2^{10} (b) 2^{11} (c) 2^{19} (d) 2^{40}

2 Complete :

1 $5^{\text{zero}} = \dots\dots\dots$

2 The probability of the certain event is $\dots\dots\dots$

3 0.75×10^4 in the standard form is $\dots\dots\dots$

4 The S.S. of the equation : $2x + 2 = 10$ in \mathbb{N} is $\dots\dots\dots$

5 If the probability of success of a student is 0.8 , then the probability of his failure is $\dots\dots\dots$

3 [a] Find the S.S. in \mathbb{Q} : $2x + 15 < 19$

[b] Find the value of : $\frac{8^3 \times 8}{8^4}$

4 Simplify to the simplest form :

1 $\left(\frac{3}{7}\right)^0 \times \left(\frac{2}{5}\right)^2 \times \sqrt{6 \frac{1}{4}}$

2 $10 \times 4 - (2 \times 6 \div 4 + 3)$

- 5 [a] A box contains 6 red balls and 4 blue balls. A ball is drawn randomly from the box.

Find the probability that :

- [1] The drawn ball is white. [2] The drawn ball is not blue.

- [b] What is the number if we add it to its three times , the result will be 24 ?

3

Giza Governorate



Al-Haram Zone
Al-Jazeera Language School

Answer the following questions :

- 1 Choose the correct answer :

- [1] A class has 25 boys and 20 girls. One pupil of them is chosen randomly , then the probability that the chosen pupil is a girl is

- (a) $\frac{1}{25}$ (b) $\frac{1}{20}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

- [2] The probability of the certain event is

- (a) zero (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

- [3] If $3x + 1 = 16$, then the value of $5x =$

- (a) 10 (b) 15 (c) 25 (d) 26

- [4] $2^3 \times 2^4 =$

- (a) 2^6 (b) 2^{34} (c) 2^7 (d) 2^{12}

- [5] The multiplicative inverse of the number $\sqrt{\frac{9}{16}}$ is

- (a) $-\frac{4}{3}$ (b) $-\frac{3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

- 2 Complete each of the following :

- [1] The number 542×10^4 in the standard form is

- [2] If $x = \frac{1}{2}$, $y = \frac{3}{4}$, then $y \div x =$

- [3] $\left(\frac{3}{4}\right)^4 \div \left(\frac{3}{4}\right)^3 =$

- [4] $\sqrt{\left(\frac{2}{5}\right)^2} =$

- [5] $\left(\frac{3}{4}\right)^{-2} =$

- 3 [a] Find the S.S. in Q :

[1] $3x + 1 = 25$

[2] $2x + 5 < 16$

- [b] Find in the simplest form : $\frac{5^{-2} \times 5^5}{5^3}$

4 [a] Simplify to the simplest form : $\left(-\frac{3}{7}\right)^0 \times \left(-\frac{2}{5}\right)^2 \times \sqrt{6 \frac{1}{4}}$

[b] If $x = -\frac{1}{2}$, $y = \frac{3}{4}$, $z = -\frac{3}{2}$, find the numerical value of : $x^3 \div y z^2$

5 [a] The sum of three consecutive even numbers is 60 , find these numbers.

[b] One card is selected randomly from 10 cards numbered from 1 to 10

Find the probability of each of the following events :

1 Getting an even number.

2 Getting a number divisible by 5

4

Giza Governorate



South Giza Educational Administration
Math Inspection

Answer the following questions :

1 Choose the correct answer :

1 $2^3 \times 2^3 = \dots\dots\dots$

(a) 2^3

(b) 2^9

(c) 2^6

(d) 4^6

2 The probability of success of a student is 0.8 , then the probability of his failure is

(a) 1

(b) - 1

(c) 0.2

(d) 0.8

3 $6000 \times 50 = \dots\dots\dots$

(a) 300×10^2

(b) 3×10^5

(c) 30×10^5

(d) $- 3 \times 10^5$

4 $3^{10} \times 3^{10} \times 3^{10} = \dots\dots\dots$

(a) 9^{10}

(b) 27^2

(c) 3^{11}

(d) 3^{30}

5 The S.S. of the inequality : $x < 3$, $x \in \mathbb{N}$ is

(a) $\{0\}$

(b) $\{0 , 1 , 2\}$

(c) $\{1 , 2\}$

(d) \emptyset

2 Complete the following :

1 The multiplicative inverse of the number 7 is

2 $\sqrt{16 + 9} = 4 + \dots\dots\dots$

3 If $a = b$, then $\left(\frac{3}{4}\right)^{a-b} = \dots\dots\dots$

4 $(2\sqrt{3})^2 = \dots\dots\dots$

5 $2 \times 6 - 4 \div 2 = \dots\dots\dots$

3 Find in the simplest form :

1 $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

2 $\frac{7^{-2} \times 7^7}{7^3}$

4 [a] Find in \mathbb{Q} the S.S. of the equation : $2x + 1 = 9$

[b] If $x = \frac{2}{3}$, $y = \frac{3}{4}$, find : $x^2 y^2$

5 [a] Find in \mathbb{Q} the S.S. of the inequality : $3x - 1 < 8$

[b] A bag contains 6 red balls and 4 blue balls. A ball is drawn randomly from the bag.
Calculate the probability of getting :

1 A red ball.

2 A green ball.

5

Alexandria Governorate



El-Montazah Educational Zone
Math's Supervision (B)

Answer the following questions :

1 Choose the correct answer :

1 $x^4 \div x^{-2} = x^{\dots\dots\dots}$ (where $x \neq 0$)

(a) 2

(b) -2

(c) 6

(d) -6

2 The additive inverse of the number 9^{-1} is

(a) $\frac{-1}{9}$

(b) $\frac{1}{9}$

(c) 9

(d) -9

3 Which of the following is the probability of an event ?

(a) 1.6

(b) -0.6

(c) $\frac{9}{5}$

(d) 85 %

4 If $\frac{24}{x} + 1 = 13$, then $x = \dots\dots\dots$

(a) 10

(b) 2

(c) 37

(d) 12

5 If $-x > 3$, then

(a) $x > -3$

(b) $x > 3$

(c) $x < -3$

(d) $x < 3$

2 Complete :

1 The probability of the certain event is

2 If $0.0009 = 9 \times 10^x$, then $x = \dots\dots\dots$

3 $\sqrt{(10)^2 - (6)^2} = 4 + \dots\dots\dots$

4 $4 \times 2^3 - 20 = \dots\dots\dots$

5 $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots\dots$

3 [a] Simplify to the simplest form : $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^{\text{zero}}$

[b] If $x = \frac{-1}{2}$, $y = \frac{3}{4}$, find the numerical value of : $\left(\frac{x^2}{y}\right)^2$

4 Find in \mathbb{Q} the S.S. of :

1 $3x + 1 = x + 2$

2 $2x - 5 < 6$

5 [a] Simplify : $\frac{a^4 \times a^{-7}}{a^{-8}}, a \neq 0$

[b] A card is selected at random from ten cards numbered from 1 to 10**Find the probability that the selected card shows :****1** An odd number greater than 3**2** A prime number less than 8**6 El-Kalyoubia Governorate**

Math Supervision

Answer the following questions :**1 Choose the correct answer :**

1 If $0.00052 = 5.2 \times 10^m$, then $m = \dots\dots\dots$

(a) 5

(b) 4

(c) -4

(d) -5

2 $2^3 + 2^3 = \dots\dots\dots$

(a) 2^6

(b) 2^9

(c) 2^4

(d) 1

3 If $x + 3 = 8$, then $3x = \dots\dots\dots$

(a) 20

(b) 15

(c) 12

(d) 9

4 $\sqrt{9+16} = 3 + \dots\dots\dots$

(a) 25

(b) 22

(c) 4

(d) 2

5 $\left(\frac{1}{2}\right)^{-1} = \dots\dots\dots$

(a) -2

(b) $\frac{1}{2}$

(c) 2

(d) $-\frac{1}{2}$

2 Complete each of the following :

1 The additive inverse of the number $\frac{5}{6}$ is $\dots\dots\dots$

2 If $a = -2$, $b = -4$, then $a^2 + b = \dots\dots\dots$

3 The probability of the impossible event is $\dots\dots\dots$

4 If $-x < 3$, then $x \dots\dots\dots -3$

5 If x is an odd number, then the next odd number is $\dots\dots\dots$

3 [a] Simplify : $\left(-\frac{3}{5}\right)^0 \times \sqrt{\frac{81}{16}} \times \left(-\frac{2}{3}\right)^2$

[b] Find the solution set of the inequality in \mathbb{Q} : $2x + 3 \leq 11$

4 [a] Find in the simplest form : $\frac{5^{-4} \times 5^7}{5^2}$

[b] Find the solution set of the equation in \mathbb{Q} : $5x - 4 = 11$

5 [a] If $a = \frac{-1}{2}$ and $b = \frac{2}{3}$, find the value of : $a^2 b^2$

[b] A fair dice is thrown once and the upper face is observed.

Find the probability of appearing :

1 An even number.

2 An odd number less than 4

7

El-Sharkia Governorate



Diarb Negm Educational Zone
El-Sweedy Official Lang. Schools

Answer the following questions :

1 Choose the correct answer :

1 If $x + 9 = 11$, then the value of $7x = \dots\dots\dots$

(a) 9

(b) 14

(c) 2

(d) 13

2 $\left(\frac{-2}{3}\right)^{-3} = \dots\dots\dots$

(a) $-\frac{27}{8}$

(b) $-\frac{8}{27}$

(c) $\frac{8}{27}$

(d) $\frac{27}{8}$

3 $2^3 \div 2^{-3} = \dots\dots\dots$

(a) 2^6

(b) 2^4

(c) 2^9

(d) 1

4 If $-x > 4$, then $\dots\dots\dots$

(a) $x > -4$

(b) $x > 4$

(c) $x < -4$

(d) $x < 4$

5 $x^2 + x^2 = \dots\dots\dots$

(a) x^4

(b) x^2

(c) $2x^2$

(d) $2x^4$

2 Complete the following :

1 The probability of the certain event equals $\dots\dots\dots$

2 The number 0.000053 in the scientific notation is $\dots\dots\dots$

3 The multiplicative inverse of $\sqrt{\frac{9}{25}}$ is $\dots\dots\dots$

4 A class has 36 pupils, 20 of them are boys. If a pupil is chosen randomly, then the probability that the pupil is a girl is $\dots\dots\dots$

5 1, 2, 3, 5, 8, $\dots\dots\dots$ (in the same pattern).

3 [a] Find the value of the expression in the simplest form : $\left(\frac{-5}{3}\right)^2 \times \left(\frac{-4}{9}\right)^0 \times \sqrt{3\frac{6}{25}}$

[b] Find the solution set of each of the following where $x \in \mathbb{Q}$:

[1] $3x + 1 > 25$

[2] $5x - 8 = 7$

4 [a] Simplify to the simplest form : $\frac{5^{-4} \times 5^7}{5^3}$

[b] The sum of three consecutive integers is 30 , find them.

5 [a] If $x = 3$ and $y = -4$, find the value of : $\sqrt{x^2 + y^2}$

[b] A box contains 4 white , 5 red and 6 blue balls. A ball is drawn randomly from the box.

Calculate the probability of getting :

[1] A blue ball.

[2] A white or red ball.

[3] A green ball.

8

El-Monofia Governorate



Shebin El-Koum Zone
Math Supervision

Answer the following questions :

1 Choose the correct answer :

[1] The probability of the certain event is

(a) zero

(b) S

(c) \emptyset

(d) 1

[2] $(-4c)^{\text{zero}} = \dots\dots\dots$ ($c \neq \text{zero}$)

(a) 1

(b) -1

(c) 4

(d) -4

[3] Four fifths the number 40 is

(a) 32

(b) 40

(c) 50

(d) 80

[4] Half the number $2^8 = \dots\dots\dots$

(a) 2^2

(b) 2^4

(c) 2^6

(d) 2^7

[5] The multiplicative inverse of the number $(3)^{-2}$ is

(a) 9

(b) -9

(c) $\frac{1}{9}$

(d) $-\frac{1}{9}$

2 Complete :

[1] 2 , 5 , 11 , 20 , (in the same pattern).

[2] A fair die is rolled once , then the probability of appearing the number 4 on the upper face is

[3] $\frac{9}{16} = \left(\frac{4}{3}\right)^{\dots\dots\dots}$

[4] If $(XY)^2 = 144 \text{ cm}^2$ and Z is the midpoint of \overline{XY} , then the length of $\overline{XZ} = \dots\dots\dots \text{ cm}$.

[5] If $3x = 8$, then $3x - 1 = \dots\dots\dots$

3 [a] Find the solution set in \mathbb{Q} :

1 $3x + 2 = 8$

2 $3 - 2x \leq 7$

[b] The sum of two consecutive even numbers is 42 , find the two numbers.

4 [a] Find in the simplest form the value of : $\frac{4^7 \times 4^{-2}}{4^5}$

[b] Find in the simplest form the value of : $\left(\frac{-2}{5}\right)^2 \times \sqrt{6\frac{1}{4}} + \frac{3}{5}$

5 [a] 1 Put : 0.000023 in the standard form.

2 Find the result of : $2 + 6 \times 2^2 \div 8 - 3$ (Show steps)

[b] A card is drawn randomly from 20 cards numbered from 1 to 20 , write the sample space , then find :

1 The probability of getting a number divisible by 3

2 The probability of getting a perfect square number.

9

El-Gharbia Governorate

Central Mathematics Supervision
Official Language Schools

Answer the following questions :

1 Choose the correct answer from those given :

1 If $-x > 4$, then

(a) $x > -4$

(b) $x > 4$

(c) $x < -4$

(d) $x < 4$

2 $7(6^2 - 5 \times 6) = \dots\dots\dots$

(a) 14

(b) 42

(c) 24

(d) 41

3 10 % of $2\frac{1}{2}$ L.E. = L.E.

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) 1

(d) 25

4 $\sqrt{144 + \dots\dots\dots} = 12 + 5$

(a) 40

(b) 25

(c) 16

(d) 145

5 $\frac{6a^2x^4}{2a^3x^3} = \dots\dots\dots$ where $a \neq 0$

(a) $3ax^2$

(b) $3a^5x^7$

(c) $\frac{3x}{a}$

(d) $\frac{3}{ax}$

2 Complete each of the following :

1 $(x - 5)^0 = 1$, if $x \neq \dots\dots\dots$

2 $\frac{81}{625} = \left(\frac{25}{9}\right)^{\dots\dots\dots}$

- 3 The standard form of the number $0.7 \times 0.005 = \dots\dots\dots$
- 4 1 , 2 , 3 , 5 , 8 , $\dots\dots\dots$, $\dots\dots\dots$ (in the same pattern).
- 5 The probability that the student is absent at a day in a school is 0.15 , if the number of students in this school is 600 , then the number of the present students that day is $\dots\dots\dots$

3 [a] If $x = -\frac{1}{2}$, $y = -\frac{3}{4}$, find in the simplest form : $\left(\frac{y}{x^2}\right)^{-2}$

[b] Find in the simplest form : $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

4 [a] Simplify to the simplest form : $\frac{x^3 \times x^{-2}}{x^{-5} \times x}$, then find the value when $x = -2$

- [b] The length of a rectangle is twice its width. If the length decreases by 5 cm. and the width increases by 6 cm. , then the rectangle becomes a square.
Find the area of the rectangle.

5 [a] Find in Q the S.S. of : $15 + 2x < 1$

- [b] The set $\{2, 3, 5\}$ is used in writing a 2-digit number.

Find the probability of each of the following events :

- 1 The tens digit is odd. 2 The units digit is even.

10 El-Dakahlia Governorate



Maths Supervision

Answer the following questions :

- 1 Choose the correct answer from those given :

1 0.000053 = $\dots\dots\dots$ (in the standard form)

- (a) 53×10^{-5} (b) 5.3×10^{-2} (c) 0.53×10^{-5} (d) 5.3×10^{-5}

2 $(x-5)^0 = 1$, if $x \neq \dots\dots\dots$

- (a) 1 (b) 5 (c) -5 (d) -1

3 If $3x = -36$, then $x^2 = \dots\dots\dots$

- (a) -12 (b) 144 (c) -24 (d) 24

4 $3^x + 3^x + 3^x = \dots\dots\dots$

- (a) $3^3 x$ (b) 9^x (c) 3^{x+1} (d) 3^x

5 The probability of the certain event is $\dots\dots\dots$

- (a) zero (b) 1 (c) $\frac{1}{2}$ (d) -1

2 Complete each of the following :

1 $1, 4, 9, 16, \dots$ (in the same pattern)

2 $1 - \frac{4}{5} = \dots\%$

3 $\sqrt{16+9} = 4 + \dots$

4 $\frac{81}{625} = \left(\frac{25}{9}\right)^{\dots}$

5 If the probability of success of a student is 0.8 , then the probability of his failure is

3 [a] Find the value of :

1 $\frac{8 \times 8^{-3}}{8^{-4}}$

2 $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

[b] Two natural numbers , one of them is twice the other , and their sum is 108
Find the two numbers (Show steps)**4 [a] Find in \mathbb{Q} the solution set of : $3x + 2 = 11$** [b] If $x = \frac{1}{2}$, $y = \frac{2}{3}$, find the value of : $(xy)^{-2}$ **5 [a] Find in \mathbb{Q} the solution set of the inequality : $7 + 2x < 17$** [b] A ball is chosen randomly from a bag containing 3 red balls , 5 blue balls and 2 black balls
Find the probability of :

1 Getting a red ball.

2 Getting a blue or black ball.

11**Ismailia Governorate****Directorate of Education
Math's Supervision****Answer the following questions :****1 Choose the correct answer :**1 If $x = 7^{-3}$, $y = 7^3$, then $xy = \dots$

(a) 4

(b) 7

(c) 1

(d) zero

2 Half the number 4^{20} is(a) 4^{19} (b) 4^{10} (c) 2^{20} (d) 2^{39} 3 $x^9 \div x^6 = \dots$ at $x \neq 0$ (a) x^{-3} (b) x^3 (c) x^6 (d) x^{15} 4 If $x^{-1} = \frac{2}{3}$, then $x = \dots$

(a) 1

(b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

5 If we tossed a coin once , then the probability of appearance of a head is

(a) 50 %

(b) 25 %

(c) 5 %

(d) 5

2 Complete :

1 The probability of the certain event is

2 If $X + 3 = 8$, then $3X =$

3 If $0.00074 = 7.4 \times 10^n$, then $n =$

4 If $\frac{X}{y} = \frac{7}{2}$, then $\frac{7X}{2y} =$

5 If the degree of the algebraic term $5X^m y^2$ is fifth , then $m =$

3 [a] Simplify : $\left(\frac{-3}{7}\right)^0 \times \frac{-2}{5} \times \sqrt{6\frac{1}{4}}$

[b] Find the S.S. in \mathbb{N} :

1 $3X + 2 = 8$

2 $2X + 3 \geq 7$

4 [a] Simplify : $\frac{X^2 \times X^5}{X^3}$ where $X \neq 0$

[b] If $X = \frac{3}{4}$, $y = \frac{4}{3}$, find the numerical value of : $X^2 y^2$

5 [a] Calculate the value of : $2 \times 4 - 4 \div 2^2$

[b] In the experiment of tossing a fair die once , then find the probability of :

1 Appearance of an even number on the upper face.

2 Appearance of an odd number less than 4

12

Kafr El-Sheikh Governorate



Central Math Supervision

Answer the following questions :

1 Complete :

1 The probability of the impossible event equals

2 If $0.0000056 = 5.6 \times 10^X$, then $X =$

3 $2 + 4 \times 6 \div 3 =$

4 $\sqrt{25 - 9} = 5 -$

5 If $2X - 3 = 5$, then $\frac{1}{2}X =$

2 Choose the correct answer :**1** Third the number $3^9 = \dots\dots\dots$

- (a) 1^3 (b) 1^9 (c) 3^3 (d) 3^8

2 If $-X < 5$, then $X \dots\dots\dots$

- (a) > 5 (b) < 5 (c) > -5 (d) < -5

3 A fair die is thrown once, the probability of appearance of number 5 is $\dots\dots\dots$

- (a) 5 (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{5}$

4 The age of Omar now is X years, then his age 5 years ago is $\dots\dots\dots$ years.

- (a) $5X$ (b) $X + 5$ (c) $X - 5$ (d) $5 - X$

5 The degree of the algebraic term Xy^3 is $\dots\dots\dots$

- (a) first. (b) second. (c) third. (d) fourth.

3 [a] Find the S.S. of the equation : $4X + 5 = 17$ (where $X \in \mathbb{Q}$)**[b]** Find the S.S. of the inequality : $3X - 1 \geq -7$ (where $X \in \mathbb{Q}$)

[c] If the length of a rectangle exceeds its width by 3 cm. and its perimeter is 26 cm.
Calculate its area.

4 Find each of the following (With steps) :

1 $\left(\frac{-1}{3}\right)^2 \times \sqrt{\frac{81}{64}} \times \left(\frac{3}{7}\right)^0$ **2** $\frac{5^{-3} \times 5^4 \times 5^2}{5^3}$

3 $4^2 \div 2 \times 3 - 3$

5 [a] If $X = \frac{2}{3}$, $y = \frac{3}{4}$, then find : X^3y

[b] A box contains 10 balls numbered from 1 to 10, if a ball is drawn randomly,
then find :

- 1** The probability of getting a number divisible by 7
2 The probability of getting an even number.
3 The probability of getting an odd number less than 7

13 Beni Suef Governorate


 Directorate of Official Language School
Education Administration

Answer the following questions :

1 Choose the correct answer :

- 1 The multiplicative inverse of the number 5^{-1} is
- (a) -5 (b) $-\frac{1}{5}$ (c) $\frac{1}{5}$ (d) 5
- 2 If $-3x > 6$, then
- (a) $x < 2$ (b) $x > 2$ (c) $x < -2$ (d) $x > -2$
- 3 The number which is in the standard form from the following is
- (a) 11×10^8 (b) 9.7×10^{-5} (c) 10.2×10^{-2} (d) 0.87×10^8
- 4 A class has 25 boys and 20 girls. A pupil of them is selected randomly , then the probability that the pupil is a girl equals
- (a) $\frac{1}{25}$ (b) $\frac{1}{20}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$
- 5 The probability of the certain event equals
- (a) zero (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

2 Complete each of the following :

- 1 $\left(-\frac{4}{7}\right)^0 = \dots\dots\dots$
- 2 $2\frac{1}{4} = \left(\frac{3}{2}\right)^{\dots\dots\dots}$
- 3 The standard form of the number $750 \times 10^{-9} = \dots\dots\dots$
- 4 $|-5| + |3| = \dots\dots\dots$ (in the simplest form)
- 5 $1, 1, 2, 3, 5, 8, \dots\dots\dots$, (in the same pattern)

- 3 [a] If $x = \frac{1}{4}$ and $y = \frac{-3}{4}$, find in the simplest form the numerical value of : $\left(\frac{x}{y^2}\right)^3$
- [b] Find the value of : $\frac{5 \times 5^{-2}}{5^{-3}}$ in the simplest form.

- 4 [a] 1 Simplify to the simplest form : $\sqrt{\left(-\frac{5}{6}\right)^2}$

- 2 Find the value of : $7(6^2 - 5 \times 6)$

- [b] Find in @ the solution set of the equation : $2(x + 5) = 12$

5 [a] Find in \mathbb{Q} the solution set of the inequality : $6x - 9 > -15$

[b] A fair die is rolled once and the number of dots on the upper face is observed.

Write down the sample space , **then find the probability of each of the following events :**

- 1** Getting a number greater than 6
- 2** Getting an even number.

14

Souhag Governorate



Jouhina Administration
Jouhina Governmental Language School

Answer the following questions :

1 Choose the correct answer :

1 If $-x > 4$, then

- (a) $x < 4$ (b) $x > 4$ (c) $x < -4$ (d) $x > -4$

2 Which of the following may be the probability of an event ?

- (a) -0.35 (b) 87% (c) 1.05 (d) 130%

3 The multiplicative inverse of $\sqrt{\frac{100}{36}}$ is

- (a) $\pm \frac{10}{6}$ (b) $\pm \frac{6}{10}$ (c) $\frac{10}{6}$ (d) $\frac{6}{10}$

4 The probability of the certain event is

- (a) zero (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

5 $3^{10} + 3^{10} + 3^{10} =$

- (a) 3^{30} (b) 3^{11} (c) 9^{10} (d) 9^{11}

2 Complete :

1 If $7 - 2x = 3$, then $x =$

2 If $3a = \sqrt{4}b$, then $\frac{a}{b} =$

3 1 , 2 , 3 , 5 , 8 , (in the same pattern)

4 $\sqrt{100 - 36} = 10 -$

5 The standard form of the number $0.7 \times 0.005 =$

3 [a] Find the S.S. of the equation in \mathbb{Q} : $2x + 1 = 9$

[b] Find the value of : $\frac{8^{-4} \times 8^7}{8^3}$

4 [a] Find in \mathbb{Q} the S.S. of the inequality : $3x + 6 > 3$

[b] Simplify : $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{2}{5}\right)^0$

5 [a] If $x = \frac{1}{2}$, $y = -\frac{3}{4}$, find in the simplest form : $\left(\frac{y}{x^2}\right)^{-2}$

[b] A regular die is thrown once and the number on the upper face is observed.

Find the probability of each of the following :

- 1 Getting a prime even number.
- 2 Getting an odd number less than 4

15 Aswan Governorate



Kom Ombo Educational Directorate
Al Qahmury Language School

Answer the following questions :

1 Choose the correct answer :

1 $5a^{\text{zero}} = \dots\dots\dots$, $a \neq 0$

- (a) 1 (b) 5 (c) 0 (d) $5a$

2 Which of the following may be the probability of an event ?

- (a) 1.5 (b) -0.4 (c) 75 % (d) 120 %

3 $2^5 \times 2^3 = \dots\dots\dots$

- (a) 2^2 (b) 2^8 (c) 2^{15} (d) 2^{53}

4 $5 \times 4 \times 3 \times 2 \times 1 \times 0 = \dots\dots\dots$

- (a) 0 (b) 50 (c) 60 (d) 15

5 $3 \times 2 - 16 \div 8 = \dots\dots\dots$

- (a) 2 (b) 4 (c) 8 (d) 10

2 Complete the following :

1 As throwing a fair die once , then the probability of getting a number greater than 5 is

2 If $m^a = 3$, $b = 2$, then $m^{ab} = \dots\dots\dots$

3 The S.S. of the equation : $x + 5 = 5$ in \mathbb{N} is

4 If $0.00037 = 3.7 \times 10^n$, then $n = \dots\dots\dots$

5 $5a + 2b + 3a - 2b = \dots\dots\dots$

3 [a] Find in \mathbb{Q} the S.S. of the equation : $3(2x - 5) = 9$

[b] Simplify : $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{2}{7}\right)^{\text{zero}}$

4 [a] Find the solution set of the inequality : $8x + 3 > 19$, $x \in \mathbb{Q}$

[b] Find the value of the expression : $\frac{3^2 \times 3^7}{3^3 \times 3^2}$

5 [a] If $a = \frac{3}{4}$, $b = \frac{-2}{5}$

, find the numerical value of : $a^2 b^2$

[b] A box contains 5 white , 4 black and 7 red balls. A ball is drawn randomly from the box.

Calculate the probabilities of the following events :

1 The ball is red.

2 The ball is not white.

Second | Geometry and Measurement

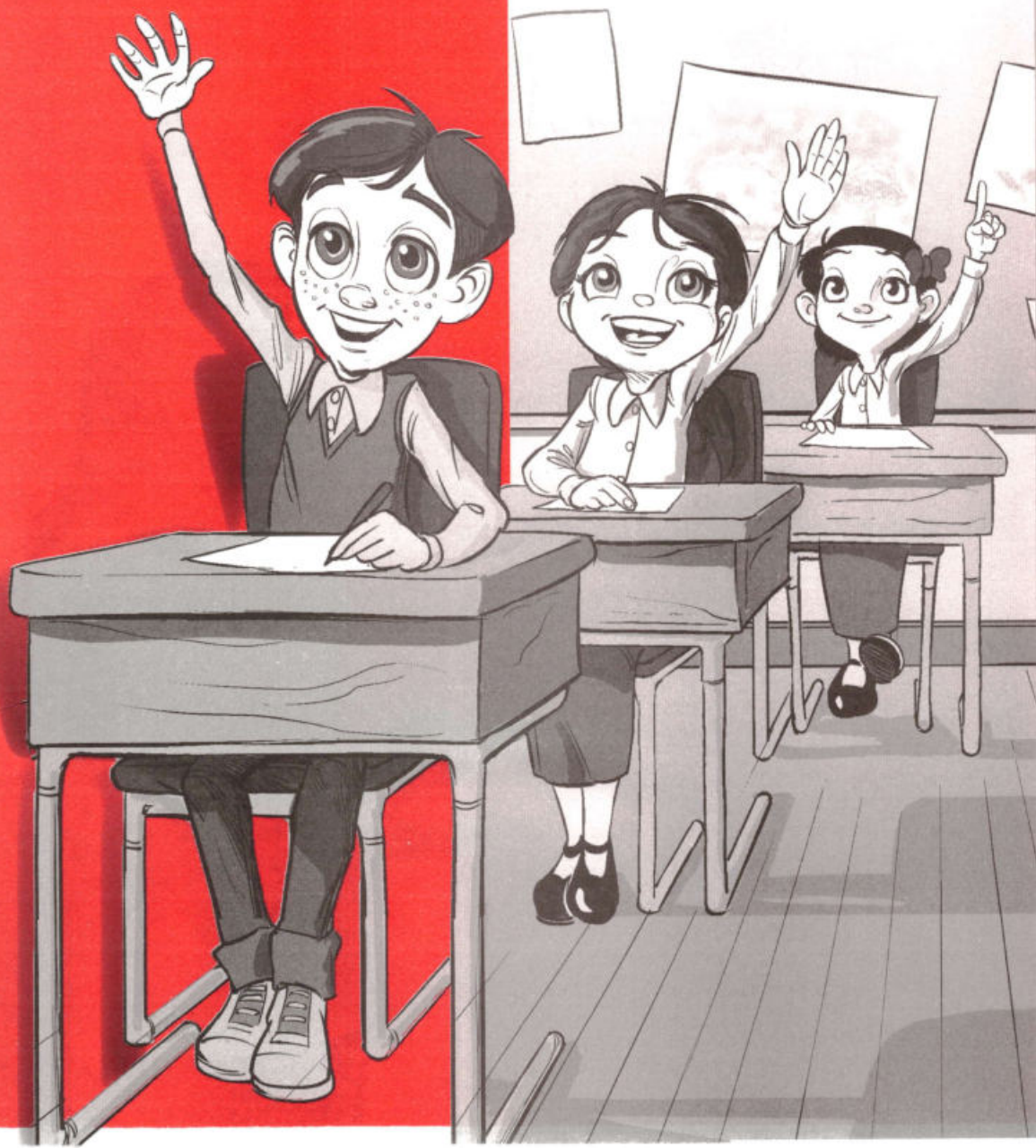
• 12 Accumulative tests	62
• Monthly tests	76
• Important questions	81
• Final revision	96
• Final examinations :	104

- School book examinations
(2 models + model for the merge students)
- 12 schools examinations.



Accumulative Tests

on Geometry and
Measurement



Accumulative test

1

on lesson 1 – unit 3

1 [a] Choose the correct answer from those given :

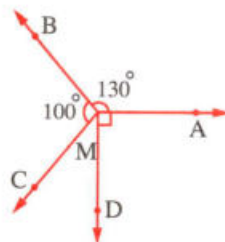
- 1** The sum of the measures of the accumulative angles at a point is equal to
- (a) 90° (b) 180° (c) 270° (d) 360°
- 2** If two straight lines intersect , then each two vertically opposite angles are
- (a) supplementary (b) complementary
(c) corresponding (d) equal in measure

[b] In the opposite figure :

$$m(\angle AMB) = 130^\circ, m(\angle BMC) = 100^\circ$$

$$, m(\angle AMD) = 90^\circ$$

$$\text{Find : } m(\angle CMD)$$



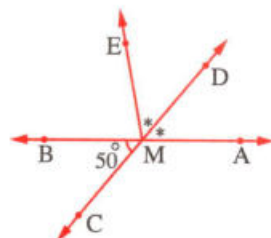
2 In the opposite figure :

$$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$$

, \overleftrightarrow{MD} bisects $\angle AME$

$$, m(\angle CMB) = 50^\circ$$

Find by proof : $m(\angle EMB)$



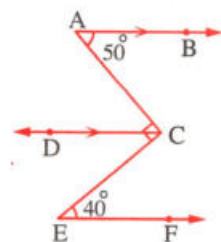
3 In the opposite figure :

$$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}, m(\angle A) = 50^\circ$$

$$, m(\angle ACE) = 90^\circ$$

$$, m(\angle E) = 40^\circ$$

Proof that : $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

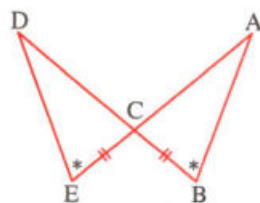


4 In the opposite figure :

$$\overleftrightarrow{AE} \cap \overleftrightarrow{BD} = \{C\}, BC = CE$$

$$, m(\angle B) = m(\angle E)$$

Proof that : $AB = DE$



Accumulative test

2

till lesson 2 – unit 3

1 Choose the correct answer from those given :

- 1 The measure of an interior angle of a regular octagon equals
 (a) 120° (b) 108° (c) 135° (d) 140°
- 2 The polygon in which the sum of measures of its exterior angles equals the sum of measure of its interior angles is called
 (a) trianlge. (b) quadrilateral. (c) pentagon. (d) hexagon.
- 3 The sum of measures of the exterior angles of any convex polygon equals
 (a) 720° (b) 360° (c) 180° (d) 270°
- 4 The concave polygon should have at least angle.
 (a) an acute (b) a right (c) an obtuse (d) a reflex

2 Complete each of the following :

- 1 The number of diagonals of a hexagon equals
- 2 The measure of the interior angle of the regular pentagon = $^\circ$
- 3 If the measure of an exterior angle of a regular polygon is 45° , then the number of its sides is
- 4 The sum of measures of the interior angles of the hexagon equals

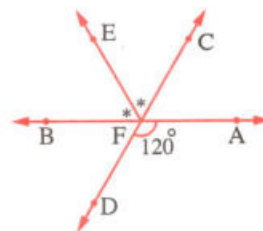
3 In the opposite figure :

$$m(\angle CFE) = m(\angle EFB)$$

$$, m(\angle AFD) = 120^\circ$$

$$, \overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{F\}$$

Find : $m(\angle CFE)$



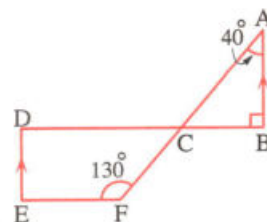
4 In the opposite figure :

$$\overline{BD} \cap \overline{AF} = \{C\}, \overline{AB} \parallel \overline{DE}$$

$$, m(\angle A) = 40^\circ, m(\angle B) = 90^\circ$$

$$, m(\angle F) = 130^\circ$$

Find by proof : $m(\angle E)$



Accumulative test

3

till lesson 3 – unit 3

1 Choose the correct answer from those given :

1 In a parallelogram , every two opposite angles are

- (a) equal in measure. (b) complementary angles.
(c) supplementary angles. (d) vertically opposite angles.

2 The sum of measures of the interior angles of a polygon of n sides equals

- (a) $n \times 180^\circ$ (b) $(n - 2) \times 180^\circ$ (c) $\frac{(n - 2) \times 180^\circ}{n}$ (d) $\frac{(n - 2) \times 180^\circ}{2n}$

3 ABCD is a parallelogram in which $m(\angle A) = \frac{1}{2} m(\angle B)$
 , then $m(\angle B) = \dots\dots\dots$

- (a) 30° (b) 60° (c) 90° (d) 120°

4 ABCD is a parallelogram in which $m(\angle B) = 2 m(\angle C)$, then $m(\angle D) = \dots\dots\dots$

- (a) 30° (b) 60° (c) 90° (d) 120°

2 Complete each of the following :

1 ABCD is a parallelogram in which $m(\angle A) = 100^\circ$, then $m(\angle B) + m(\angle D) = \dots\dots\dots^\circ$

2 The angle whose measure is 70° is vertically opposite an angle of measure

3 The measure of the interior angle of a regular hexagon equals

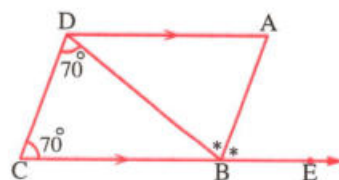
4 If ABCD is a parallelogram in which $AB = 5$ cm. , $BC = 3$ cm. , then its perimeter equals cm.

3 In the opposite figure :

$\overrightarrow{DA} \parallel \overrightarrow{CB}$, $E \in \overrightarrow{CB}$, $m(\angle EBA) = m(\angle ABD)$

, $m(\angle BDC) = m(\angle C) = 70^\circ$

Proof that : The figure ABCD is a parallelogram

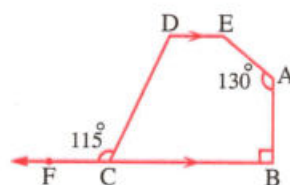


4 In the opposite figure :

$F \in \overrightarrow{BC}$, $\overrightarrow{ED} \parallel \overrightarrow{BC}$, $m(\angle DCF) = 115^\circ$, $m(\angle A) = 130^\circ$

, $m(\angle B) = 90^\circ$

Find by proof : $m(\angle E)$



Accumulative test

4

till lesson 4 – unit 3

1 Choose the correct answer from those given :

- 1 The parallelogram whose two diagonals are perpendicular is called
 (a) square (b) rectangle
 (c) rhombus (d) otherwise
- 2 If the measure of an interior angle of a regular polygon is 144° , then the number of its sides is
 (a) 4 sides. (b) 6 sides. (c) 8 sides. (d) 10 sides.
- 3 The polygon in which the number of sides = the number of diagonals is called
 (a) triangle. (b) quadrilateral. (c) pentagon. (d) hexagon.
- 4 The square is one of its angles is right.
 (a) rectangle. (b) parallelogram. (c) rhombus. (d) trapezium.

2 Complete each of the following :

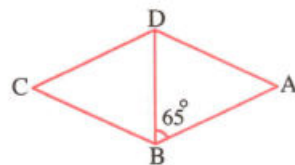
- 1 The rectangle is a parallelogram in which the two diagonals are
- 2 The perimeter of a rhombus is 24 cm. , then its side length is cm.
- 3 The quadrilateral in which only two opposite sides are parallel and are not equal in length is
- 4 ABCD is a parallelogram in which $m(\angle A) = 60^\circ$, then $m(\angle B) = \dots\dots\dots^\circ$

3 In the opposite figure :

ABCD is a rhombus in which

$$m(\angle ABD) = 65^\circ$$

Find : $m(\angle A)$

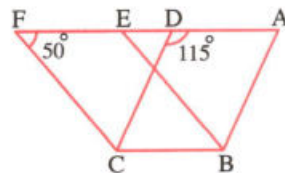


4 In the opposite figure :

ABCD , EBCF are two parallelograms

$$, m(\angle F) = 50^\circ , m(\angle ADC) = 115^\circ$$

Calculate : $m(\angle ABE)$



Accumulative test

5

till lesson 5 – unit 3

1 Choose the correct answer from those given :

- 1 The least number of acute angles in any triangle equals
 (a) zero (b) 1 (c) 2 (d) 3
- 2 ABC is a triangle in which $m(\angle B) = m(\angle C) = 45^\circ$, then $m(\angle A) =$
 (a) 45° (b) 180° (c) 90° (d) 135°
- 3 It is possible to draw a triangle each of its interior angles is of measure
 (a) 50° (b) 60° (c) 70° (d) 180°
- 4 ABC is a triangle in which $m(\angle A) = 3X^\circ$, $m(\angle C) = 4X^\circ$, $m(\angle B) = 7X^\circ$, then $\angle B$ is angle.
 (a) an acute. (b) an obtuse. (c) a right. (d) a reflex.

2 Complete each of the following :

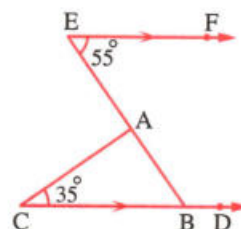
- 1 The sum of measures of any two consecutive angles in the parallelogram equals
- 2 The measure of the interior angle of the regular pentagon =°
- 3 The measure of the exterior angle of the equilateral triangle equals
- 4 The rectangle is with a right angle.

3 In the opposite figure :

$\overrightarrow{EF} \parallel \overrightarrow{CD}$, $m(\angle E) = 55^\circ$

, $m(\angle C) = 35^\circ$

Find each of : $m(\angle BAC)$, $m(\angle ABD)$



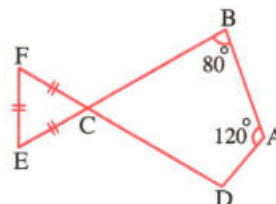
4 In the opposite figure :

ABCD is a quadrilateral, $m(\angle A) = 120^\circ$

, $m(\angle B) = 80^\circ$

, $\triangle CEF$ is an equilateral triangle.

Find by proof : $m(\angle D)$



Accumulative test

6

till lesson 6 – unit 3

1 Choose the correct answer from those given :

- 1 The measure of an interior angle of a regular polygon = 108° , then the number of its sides is sides.
 (a) 4 (b) 5 (c) 6 (d) 7
- 2 If ABCD is a square , then $m(\angle CAB) = \dots\dots\dots$
 (a) 30° (b) 45° (c) 60° (d) 90°
- 3 $\triangle ABC$ is an equilateral triangle whose perimeter = 12 cm. , if X , Y and Z are the midpoints of its sides , then the perimeter of $\triangle XYZ = \dots\dots\dots$ cm.
 (a) 12 (b) 6 (c) 4 (d) 3
- 4 The ratio between the length of the line segment joining the midpoints of two sides of a triangle and the length of the third side equals
 (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 2 : 3

2 Complete each of the following :

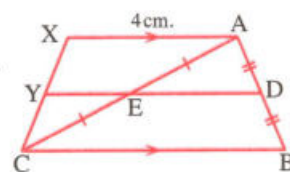
- 1 The line segment joining the midpoints of two sides of a triangle is to the third side.
- 2 XYZ is a triangle in which : D and E are the midpoints of \overline{XY} , \overline{YZ} respectively , $DE = 6$ cm. , then $XZ = \dots\dots\dots$ cm.
- 3 If ABCD is a parallelogram in which $m(\angle A) = 60^\circ$, then $m(\angle C) = \dots\dots\dots^\circ$
- 4 The ray drawn from the midpoint of a side of a triangle parallel to another side the third side.

3 In the opposite figure :

$AD = DB$, $AE = EC$, $AX = 4$ cm. , $\overline{AX} \parallel \overline{BC}$, $\overline{DE} \cap \overline{XC} = \{Y\}$

1 **Proof that :** Y is the midpoint of \overline{XC}

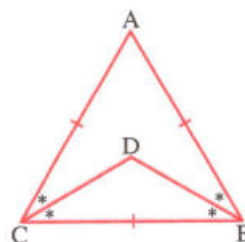
2 **Find :** The length of \overline{EY}



4 In the opposite figure :

ABC is a triangle in which : $AB = AC = BC$
 \overline{BD} bisects $\angle ABC$, \overline{CD} bisects $\angle ACB$

Find by proof : $m(\angle BDC)$



Accumulative test

7

till lesson 7 – unit 3

1 Choose the correct answer from those given :

- 1 If the lengths of the two sides of a right-angled triangle are 6 cm. , 8 cm. , then the length of its hypotenuse = cm.
 (a) 10 (b) 14 (c) 2 (d) 48
- 2 ΔZYX is a right-angled triangle at Y in which $YX = 12$ cm. , $ZX = 13$ cm. then $ZY =$ cm.
 (a) 3 (b) 4 (c) 5 (d) 6
- 3 ΔABC in which X , Y are midpoints of \overline{AB} , \overline{AC} and $BC = 14$ cm. then $XY =$
 (a) 7 cm. (b) 4 cm. (c) 6 cm. (d) 14 cm.
- 4 The two diagonals are perpendicular and equal in length in
 (a) square. (b) rectangle. (c) parallelogram. (d) rhombus.

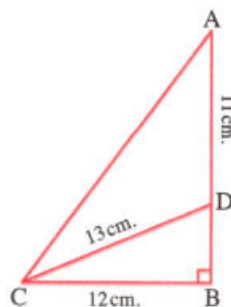
2 Complete each of the following :

- 1 If ABC is a right-angled triangle at B , then $(AB)^2 + (BC)^2 =$
- 2 In a right-angled triangle the area of the square on the hypotenuse equals the sum of areas
- 3 A rectangle whose width is 3 cm. and its diagonal length equals 5 cm. , then its area equals
- 4 ABCD is a parallelogram in which $m(\angle A) + m(\angle C) = 120^\circ$, then $m(\angle B) =$ °

3 In the opposite figure :

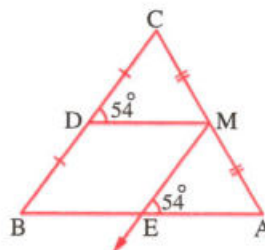
$CD = 13$ cm.
 , $AD = 11$ cm.
 , $BC = 12$ cm.
 , $m(\angle B) = 90^\circ$

Find : The length of each of \overline{BD} , \overline{AC}



4 In the opposite figure :

ABC is a triangle in which M is the midpoint of \overline{AC}
 , D is the midpoint of \overline{BC}
 $\overrightarrow{ME} \cap \overline{AB} = \{E\}$, $m(\angle MDC) = 54^\circ$, $m(\angle AEM) = 54^\circ$
Proof that : BEMD is a parallelogram



Accumulative test

8

till lesson 8 – unit 3

1 Choose the correct answer from those given :

- 1** The image of the point $(4, 3)$ by the transformation $(X, y) \longrightarrow (-X, y - 1)$ is
 (a) $(4, 2)$ (b) $(-4, 2)$ (c) $(-4, -4)$ (d) $(-4, -2)$
- 2** The image of the point $(4, 6)$ by the transformation $(X, y) \longrightarrow (-X, y - 7)$ is
 (a) $(-4, -1)$ (b) $(4, 6)$ (c) $(4, -6)$ (d) $(-4, 1)$
- 3** The length of the line segment joining the midpoints of two sides of a triangle equals the length of the third side.
 (a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$
- 4** If the two diagonals in a rectangle are perpendicular, then it will be
 (a) rhombus. (b) trapezium. (c) square. (d) rectangle.

2 Complete each of the following :

- 1** If the measures of two angles in a triangle are 30° and 40° , then the triangle is -angled.
- 2** If two straight lines intersect, then the measures of each two vertically opposite angles
- 3** If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is sides.
- 4** If ABCD is a rhombus in which $m(\angle ABD) = 35^\circ$, then $m(\angle BAC) = \dots\dots\dots^\circ$

3 Write the image of each of the following two points by transformation

$(X, y) \longrightarrow (X + 3, y - 2)$ where :

- 1** $(-3, 5)$ **2** $(4, 4)$

4 [a] In the opposite figure :

ABCD is a rectangle

, $AX = YC$

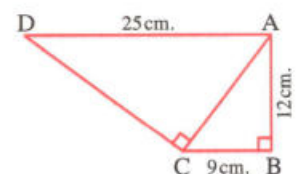
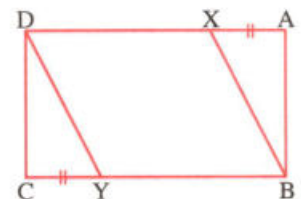
Proof that : XBYD is a parallelogram.

[b] In the opposite figure :

$m(\angle B) = m(\angle ACD) = 90^\circ$

, $AB = 12$ cm. , $BC = 9$ cm. , $AD = 25$ cm.

Find : The length of \overline{DC}



Accumulative test

9

till lesson 9 – unit 3

1 Choose the correct answer from those given :

- 1 The image of the point $(-2, 3)$ by reflection in the y -axis is
- (a) $(-2, 3)$ (b) $(-2, -3)$ (c) $(2, -3)$ (d) $(2, 3)$
- 2 ABCD is a parallelogram in which $m(\angle A) = 2m(\angle B)$, then $m(\angle C) = \dots\dots\dots$
- (a) 60° (b) 120° (c) 180° (d) 90°
- 3 ABC is a triangle in which $m(\angle A) = m(\angle C) - m(\angle B)$, then $m(\angle C) = \dots\dots\dots$
- (a) 45° (b) 60° (c) 90° (d) 180°
- 4 If the point $(1, k)$ is the image of the point $(m, -2)$ by reflection in y -axis, then $k - m = \dots\dots\dots$
- (a) 1 (b) -1 (c) 3 (d) -3

2 Complete each of the following :

- 1 ABC is a right-angled triangle at B in which $AB = 9$ cm. , $AC = 15$ cm. , then $BC = \dots\dots\dots$ cm.
- 2 If the image of the point $(8, m - 4)$ by reflection in X -axis is itself , then $m = \dots\dots\dots$
- 3 The image of the point $(3, 4)$ by reflection in X -axis is
- 4 The number of axes of symmetry of the rectangle is

- 3 [a] Draw $\triangle ABC$ which is a right-angled triangle at B , $AB = 4$ cm. , $BC = 3$ cm. , then draw its image by reflection at \overleftrightarrow{AB}

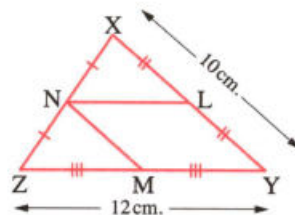
- [b] On the square lattice , draw $\triangle ABC$ where $A(1, 0)$, $B(0, 2)$, $C(-3, 1)$, then draw its image by reflection in X -axis

4 In the opposite figure :

L, M, N are the midpoints of \overline{XY} , \overline{YZ} , \overline{XZ} respectively
 $YZ = 12$ cm. , $XY = 10$ cm.

Find with proof :

The perimeter of the figure LYMN and what is the name of this figure ?



Accumulative test 10 till lesson 10 – unit 3

1 Choose the correct answer from those given :

- 1 The image of the triangle by reflection in the origin point is
 (a) triangle. (b) square. (c) point. (d) straight line.
- 2 If ABC is a triangle in which $m(\angle A) = 4x^\circ$, $m(\angle B) = 2x^\circ$, $m(\angle C) = 3x^\circ$, then $\angle A$ is angle.
 (a) an acute (b) a right (c) an obtuse (d) a reflex
- 3 The point $(2, -3)$ is the image of the point by reflection in the origin point.
 (a) $(-2, -3)$ (b) $(-2, 3)$ (c) $(2, 3)$ (d) $(3, 2)$
- 4 If the image of the point (a, b) by reflection in the origin point is the point (X, y) and $a > b$, then X y
 (a) $>$ (b) $=$ (c) \geq (d) $<$

2 Complete each of the following :

- 1 The image of the point $(3, -2)$ by reflection in y-axis is
- 2 The image of the point $(3, -4)$ by reflection in the origin point is
- 3 The square is a rhombus with two diagonals
- 4 The polygon that contains a reflex angle is called

3 On a square lattice draw the triangle ABC where : $A(-2, 4)$, $B(5, 0)$, $C(3, -3)$, then draw :

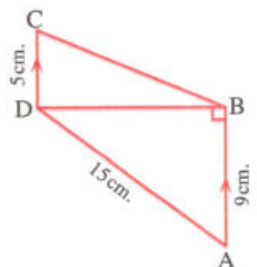
- 1 The image of $\triangle ABC$ by reflection in X-axis.
- 2 The image of $\triangle ABC$ by reflection in origin point.

4 In the opposite figure :

$\overline{AB} \parallel \overline{DC}$, $m(\angle ABD) = 90^\circ$

, $AB = 9 \text{ cm.}$, $AD = 15 \text{ cm.}$, $CD = 5 \text{ cm.}$

Find : The length of each of \overline{BD} and \overline{BC}



Accumulative test 11 till lesson 11 – unit 3

1 Choose the correct answer from those given :

- 1 The image of the point $(5, 3)$ by translation $(X, y) \longrightarrow (X + 2, y + 1)$ is
 (a) $(3, 2)$ (b) $(7, 4)$ (c) $(7, 2)$ (d) $(5, 3)$
- 2 The image of the point $(3, 4)$ by translation of magnitude of four units in the negative direction of the y-axis is
 (a) $(3, 8)$ (b) $(-3, 0)$ (c) $(0, 4)$ (d) $(3, 0)$
- 3 If $\hat{A}(3, -3)$ is the image of the point A by translation $(X, y) \longrightarrow (X - 1, y - 4)$, then the point A is
 (a) $(2, -7)$ (b) $(4, 1)$ (c) $(-4, -1)$ (d) $(2, 1)$
- 4 The image of E $(2, -1)$ is $\hat{E}(5, 2)$ by translation
 (a) $(3, 3)$ (b) $(7, 1)$ (c) $(-3, -3)$ (d) $(5, 2)$

2 Complete each of the following :

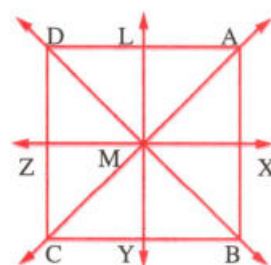
- 1 The measure of an interior angle of a regular polygon $= 120^\circ$, then the number of its sides is
- 2 The image of the point $(2, -3)$ by translation $(2, -1)$ is
- 3 The image of the point $(0, -4)$ by reflection in y-axis is
- 4 XYZ is a right-angled triangle at Y in which $XZ = 25$ cm., $YZ = 24$ cm., then $XY =$

3 [a] In the opposite figure :

ABCD is a square, X, Y, Z, L are midpoints of its sides.

Find : 1 The image of $\triangle AXM$ by reflection in \overleftrightarrow{XZ}

2 The image of $\triangle AXM$ by translation \overrightarrow{AM} in the direction of \overrightarrow{AM}



[b] On lattice, draw $\triangle ABC$ where : A $(1, 1)$, B $(3, 4)$, C $(5, 2)$, then draw its image :

1 By reflection in the origin point.

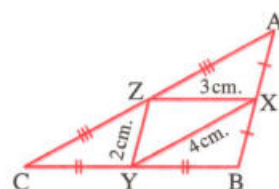
2 By translation $(X - 4, y + 3)$

4 In the opposite figure :

X, Y, Z are the midpoints of \overline{AB} , \overline{BC} , \overline{AC}

respectively where $XY = 4$ cm., $YZ = 2$ cm., $XZ = 3$ cm.

Find by proof : The perimeter of the triangle ABC



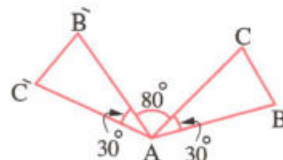
Accumulative test 12 till lesson 12 – unit 3

1 Choose the correct answer from those given :

1 In the opposite figure :

$\triangle \hat{A}\hat{B}\hat{C}$ is the image of $\triangle ABC$ by rotation about A with an angle of measure

- (a) -110° (b) 80°
(c) 110° (d) 140°



2 ABC is a triangle in which $m(\angle A) = m(\angle B) + m(\angle C)$ and $m(\angle B) = 35^\circ$, then $m(\angle C) = \dots\dots\dots$

- (a) 90° (b) 35° (c) 55° (d) 125°

3 The image of the point $(-3, 4)$ by translation of 3 units in the positive direction of X-axis is

- (a) $(0, 4)$ (b) $(-3, 4)$ (c) $(3, 4)$ (d) $(-3, 1)$

4 The image of the point $(-1, 3)$ is $(3, 1)$ by rotation about the origin point with an angle of measure

- (a) 90° (b) -90° (c) 180° (d) 360°

2 Complete each of the following :

1 ABCD is a parallelogram in which $m(\angle A) + m(\angle C) = 140^\circ$, then $m(\angle D) = \dots\dots\dots^\circ$

2 The image of the point $(-2, 3)$ by rotation about the origin point with an angle of measure 90° is

3 The image of the point $(-1, 4)$ by rotation about the origin point with an angle of measure 180° is

4 The identity rotation is a rotation with an angle of measure =

3 On a square lattice draw $\triangle ABC$ where A $(1, 1)$, B $(3, 4)$, C $(5, 2)$, then draw :

1 The image of $\triangle ABC$ by reflection in the origin point.

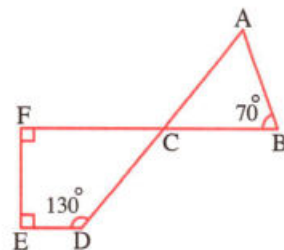
2 The image of $\triangle ABC$ by rotation about the origin point with an angle of measure 90°

4 [a] In the opposite figure :

$$m(\angle F) = m(\angle E) = 90^\circ$$

$$, m(\angle D) = 130^\circ , m(\angle B) = 70^\circ$$

Find by proof : $m(\angle A)$



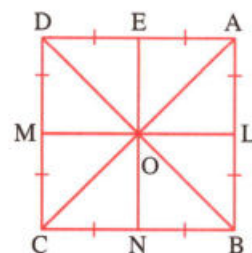
[b] In the opposite figure :

ABCD is a square of side

length 6 cm. , O is the point

of intersection of its diagonals , L , N , M and E

are the midpoints of \overline{AB} , \overline{BC} , \overline{CD} and \overline{DA} respectively.



Find : **1** The image of $\triangle AOL$ by translation of 3 cm. in the direction of \overrightarrow{AB}

2 The image of $\triangle AOL$ by reflection in \overleftrightarrow{EN}

3 The image of $\triangle AOL$ by rotation about O with an angle of measure -90°

Monthly Tests

on Geometry and measurement

October contents

* Unit Three : Geometry and measurement :

- Deductive proof.
- **The polygon :**
 - (convex - concave - regular)
 - The sum of measures of the interior angles of a polygon.
 - The sum of measures of the exterior angles of a polygon.
 - Parallelogram and its properties.
 - Parallelogram and its special cases.
- **Theorem (1) :** The sum of measure of the interior angles of a triangle is 180°

November contents

* Unit Three :

- The exterior angles of a triangle.
- **Theorem (2) :** The ray drawn from the midpoint of a side of a triangle parallel to.
- **Corollary :** The line segment joining the midpoints of two sides.
- **Theorem (3) :** The length of the line segment joining the midpoint of two sides.
- Pythagoras' theorem
- **Geometric transformations**
 - Reflection.
 - Translation.



Test

1

Total mark

10

Answer the following questions :

(3 Marks)

1 Choose the correct answer from the given ones :

- 1 The number of diagonals of the pentagon is
 (a) 5 (b) 9 (c) 15 (d) 2
- 2 If ABCD is a parallelogram , $m(\angle B) + m(\angle C) = \dots\dots\dots$
 (a) 70° (b) 180° (c) 90° (d) 360°
- 3 The parallelogram in which the two diagonals are equal in length is
 (a) a trapezium. (b) a rhombus. (c) a rectangle. (d) a square.

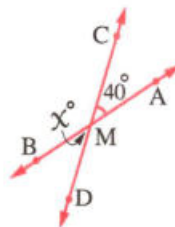
2 Complete :

(3 Marks)

- 1 The sum of measures of the interior angles of the quadrilateral equals $^\circ$
- 2 The measure of the exterior angle of the equilateral triangle at any one of its vertices equals

3 In the opposite figure :

If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$
 , then $X = \dots\dots\dots^\circ$



3 In the opposite figure :

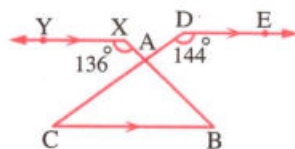
(2 Marks)

$\overline{BC} \parallel \overline{DE} \parallel \overline{XY}$

, $m(\angle D) = 144^\circ$

, $m(\angle X) = 136^\circ$

Find with proof : $m(\angle BAC)$



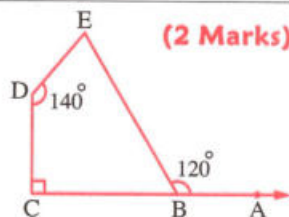
4 In the opposite figure :

(2 Marks)

$A \in \overline{CB}$, $m(\angle D) = 140^\circ$

, $m(\angle ABE) = 120^\circ$, $\overline{DC} \perp \overline{CB}$

Find : $m(\angle E)$



Answer the following questions :

(3 Marks)

1 Choose the correct answer from the given ones :

1 In $\triangle XYZ$: If $m(\angle X) = m(\angle Y) + m(\angle Z)$, then $\angle X$ is

- (a) acute. (b) right. (c) obtuse. (d) straight.

2 The rhombus in which its two diagonals are equal in length is called

- (a) a parallelogram. (b) a square.
(c) a rectangle. (d) a trapezium.

3 If two straight lines intersect , then each two vertically opposite angles are

- (a) equal in measure. (b) complementary.
(c) supplementary. (d) adjacent.

2 Complete :

(3 Marks)

1 The sum of measures of the exterior angle of a pentagon equals°

2 If ABCD is a parallelogram , $m(\angle C) = 70^\circ$, then $m(\angle B) =$ °

3 The number of sides of a regular polygon in which the measure of one of its interior angles 108° is sides.

3 In the opposite figure :

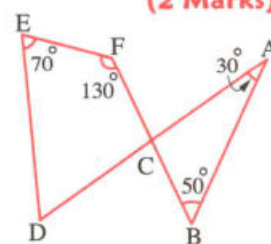
(2 Marks)

$$\overline{AD} \cap \overline{BF} = \{C\}$$

$$, m(\angle A) = 30^\circ , m(\angle B) = 50^\circ$$

$$, m(\angle F) = 130^\circ , m(\angle E) = 70^\circ$$

Find with proof : $m(\angle D)$



4 In the opposite figure :

(2 Marks)

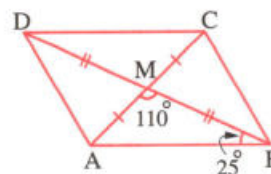
ABCD is a quadrilateral its two diagonals intersect at M

$$, m(\angle AMB) = 110^\circ , m(\angle MBA) = 25^\circ$$

$$, MA = MC , MB = MD$$

1 Prove that : ABCD is a parallelogram

2 Find : $m(\angle ACD)$



Test

1

Total mark

10

Answer the following questions :

(3 Marks)

1 Choose the correct answer from the given ones :

1 The image of the point (3 , 5) by reflection in the origin point is

- (a) (-3 , 5) (b) (5 , -3) (c) (-3 , -5) (d) (3 , -5)

2 In $\triangle XYZ$: M is the midpoint \overline{XY} , L is the midpoint of \overline{XZ} , $ML = 7$ cm. , then $YZ =$ cm.

- (a) 3.5 (b) 7 (c) 14 (d) 21

3 A rectangle of length 20 cm. , and the diagonal length is 25 cm. , then its width is cm.

- (a) 5 (b) 45 (c) 15 (d) 30

2 Complete :

(3 Marks)

1 The ray drawn from the midpoint of a side in a triangle parallel to another side

2 The image of the point (5 , -3) by translation of magnitude of 3 units in the negative direction of the X-axis is

3 The image of the point (2 , -1) by reflection in the y-axis is

3 On a square lattice , draw the image of $\triangle ABC$ where :

(2 Marks)

A (1 , 2) , B (4 , 3) , C (-1 , -2) by reflection in the X-axis

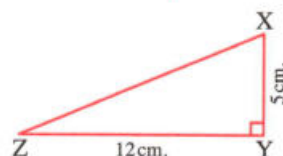
4 In the opposite figure :

(2 Marks)

XYZ is a right-angled triangle at Y

, $XY = 5$ cm. , $YZ = 12$ cm.

Find : The length of \overline{XZ}



Answer the following questions :

(3 Marks)

1 Choose the correct answer from the given ones :

- 1 The number of axes of symmetry of the isosceles triangle is
 (a) 1 (b) 2 (c) 3 (d) 4
- 2 The image of the point (3 , 0) by reflection in the is itself.
 (a) the X-axis (b) the y-axis (c) the origin point (d) the axis of symmetry
- 3 The image of the point (− 3 , 7) by the translation (− 2 , 1) is
 (a) (− 1 , 8) (b) (− 5 , 6) (c) (− 1 , 6) (d) (− 5 , 8)

2 Complete :

(3 Marks)

- 1 The area of the square drawn on the hypotenuse of the right-angled triangle equals
- 2 The image of the point (− 2 , − 9) by reflection in the y-axis is
- 3 The length of the line segment drawn between the midpoints of two sides in a triangle equals

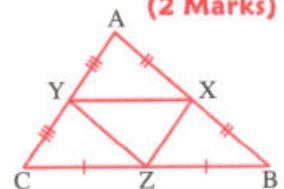
3 In the opposite figure :

(2 Marks)

ABC is a triangle in which :

X , Y , Z are the midpoints of \overline{AB} , \overline{AC} , \overline{BC} respectively

Prove that : The perimeter of $\triangle XYZ = \frac{1}{2}$ the perimeter of $\triangle ABC$



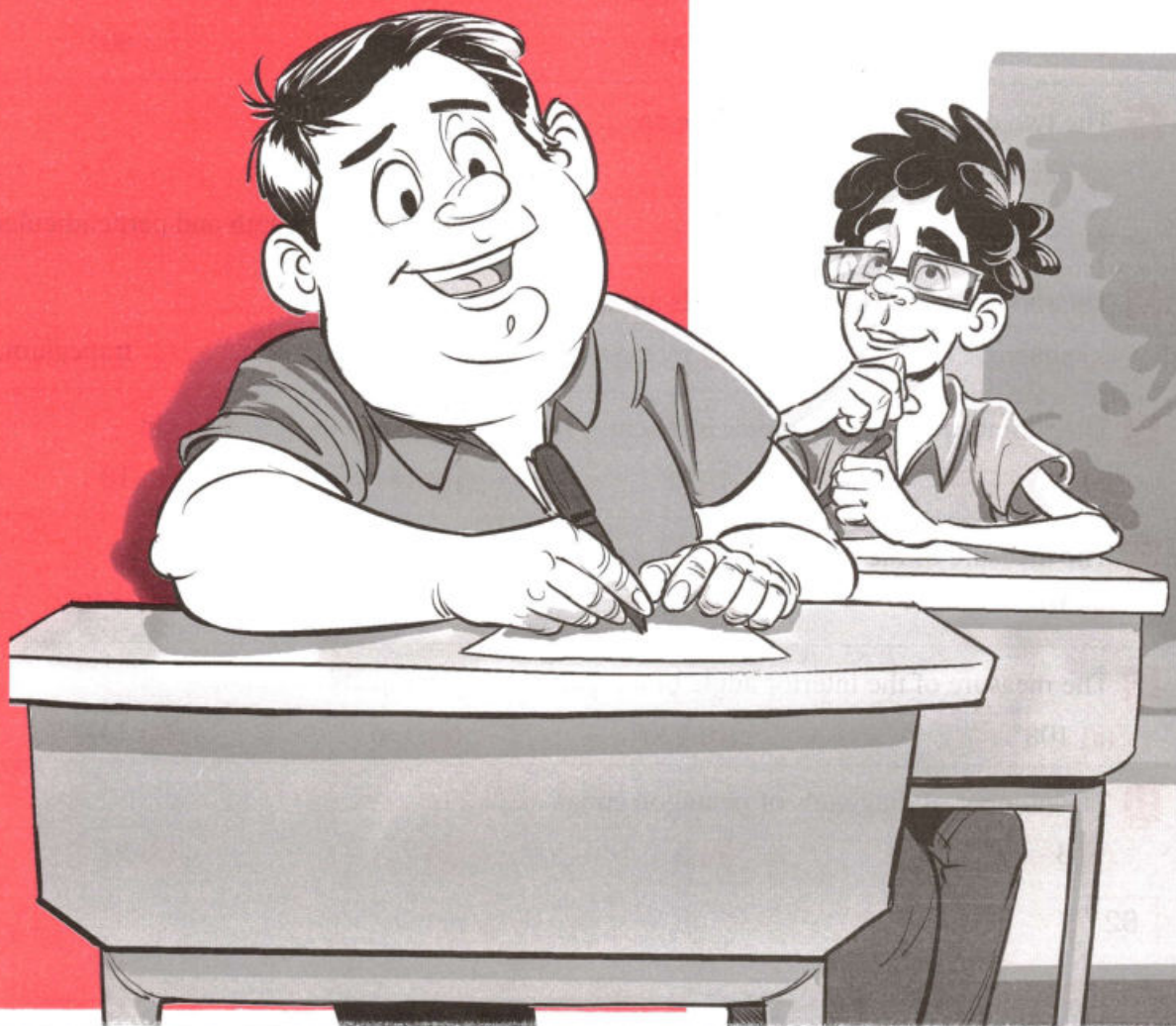
4 In the cartesian coordinate plane , draw $\triangle ABC$ where :

(2 Marks)

A (− 5 , 3) , B (− 2 , 1) and C (− 2 , 5) , then draw its image by the translation
 $(X , y) \longrightarrow (X + 3 , y - 3)$

Important Questions

on Geometry and Measurement



Important questions on Unit Three



Geometry and Measurement

First

Multiple choice questions

- 1 The two vertically opposite angles are
(a) complementary. (b) supplementary.
(c) adjacent. (d) equal in measure.

- 2 The sum of measures of the accumulative angles at a point equals
(a) 90° (b) 180° (c) 360° (d) 540°

- 3 The sum of measures of the interior angles of any quadrilateral equals
(a) 180° (b) 170° (c) 90° (d) 360°

- 4 In any polygon of n sides (the sum of measures of its interior angles + the sum of measures of its exterior angles) equals
(a) $(n - 2) \times 180^\circ$ (b) $n \times 180^\circ$ (c) $n \times 360^\circ$ (d) $(n - 2) \times 360^\circ$

- 5 If ABCD is a square, then $m(\angle CAD) =$
(a) 45° (b) 90° (c) 30° (d) 60°

- 6 The two diagonals in the rectangle are
(a) parallel. (b) perpendicular.
(c) equal in length. (d) equal in length and perpendicular.

- 7 The rhombus whose two diagonals are equal in length is called
(a) square. (b) rectangle. (c) parallelogram. (d) trapezium.

- 8 The rhombus whose perimeter is 60 cm., its side length equals cm.
(a) 20 (b) 18 (c) 15 (d) 10

- 9 The measure of the interior angle of a regular pentagon equals
(a) 108° (b) 180° (c) 135° (d) 540°

- 10 The measure of the interior angle of a regular hexagon equals
(a) 108° (b) 120° (c) 136° (d) 144°

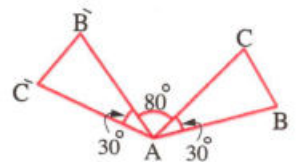
- 11 The number of diagonals of pentagon equals
(a) 3 (b) 5 (c) 7 (d) 9

- 12** The measure of the exterior angle of a regular polygon is 45° , then the number of its sides is
- (a) 3 sides. (b) 6 sides. (c) 8 sides. (d) 9 sides.
-
- 13** The measure of the interior angle of a regular polygon of 10 sides equals
- (a) 72° (b) 108° (c) 144° (d) 150°
-
- 14** The perimeter of a square of side length 5 cm. is cm.
- (a) 10 (b) 20 (c) 15 (d) 25
-
- 15** The parallelogram whose angle is right is called
- (a) square. (b) rhombus. (c) rectangle. (d) trapezium.
-
- 16** If two adjacent sides are equal in a parallelogram, then the figure is
- (a) square. (b) rhombus. (c) rectangle. (d) trapezium.
-
- 17** The two diagonals are equal in length and perpendicular in
- (a) rhombus. (b) rectangle. (c) square. (d) parallelogram.
-
- 18** The two diagonals are equal in length and not perpendicular in
- (a) square. (b) rectangle. (c) rhombus. (d) parallelogram.
-
- 19** The two diagonals are perpendicular and not equal in length in the
- (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
-
- 20** The diagonal of the square divides the vertex angle into two angles, the measure of each of them is
- (a) 45° (b) 30° (c) 90° (d) 60°
-
- 21** ABCD is a parallelogram in which $m(\angle A) = 70^\circ$, then $m(\angle B) =$
- (a) 90° (b) 110° (c) 70° (d) 80°
-
- 22** The sum of measures of the interior angles of a triangle equals the measure of angle.
- (a) right. (b) straight. (c) acute. (d) reflex.
-
- 23** Any triangle has at least two angles.
- (a) acute. (b) right. (c) obtuse. (d) straight.

Geometry and Measurement

- 24** ABC is a triangle in which $m(\angle A) = m(\angle B) + m(\angle C)$, then $m(\angle A) = \dots\dots\dots$
(a) 180° (b) 108° (c) 90° (d) 360°
-
- 25** In $\triangle ABC$, if $m(\angle B) = 3 m(\angle A) = 90^\circ$, then $m(\angle C) = \dots\dots\dots$
(a) 45° (b) 60° (c) 90° (d) 180°
-
- 26** $\triangle ABC$ in which X, Y are midpoints of \overline{AB} , \overline{AC} , $BC = 14$ cm., then $XY = \dots\dots\dots$
(a) 7 cm. (b) 6 cm. (c) 4 cm. (d) 14 cm.
-
- 27** $\triangle ABC$ is right-angled at B, if $AB = 20$, $AC = 25$, then $BC = \dots\dots\dots$ cm.
(a) 15 (b) 20 (c) 225 (d) 400
-
- 28** If ABC is a right-angled triangle at B, then the measure of its exterior angle at B equals $\dots\dots\dots$
(a) 90° (b) 360° (c) 180° (d) 45°
-
- 29** The image of the point $(2, -1)$ by reflection in X-axis is $\dots\dots\dots$
(a) $(2, 1)$ (b) $(1, 2)$ (c) $(-2, -1)$ (d) $(-1, 2)$
-
- 30** The image of the point $(3, -2)$ by reflection in y-axis is $\dots\dots\dots$
(a) $(2, 3)$ (b) $(-3, 2)$ (c) $(-3, -2)$ (d) $(3, 2)$
-
- 31** The image of the point $(3, 5)$ by reflection in the origin point is $\dots\dots\dots$
(a) $(-3, 5)$ (b) $(5, -3)$ (c) $(-3, -5)$ (d) $(3, -5)$
-
- 32** The image of the point $(-3, 0)$ is itself by reflection in $\dots\dots\dots$
(a) X-axis. (b) y-axis. (c) origin point. (d) axis of symmetry.
-
- 33** If \hat{A} is the image of point A by reflection in M, $MA = 6$ cm., then $AA\hat{A} = \dots\dots\dots$ cm.
(a) 6 (b) 3 (c) 12 (d) 9
-
- 34** If the image of the point $(a - 3, 7)$ by reflection in y-axis is itself, then $a = \dots\dots\dots$
(a) 10 (b) 3 (c) -3 (d) 7
-
- 35** The number of axes of symmetry of the parallelogram which has a right angle equals $\dots\dots\dots$
(a) zero (b) 1 (c) 2 (d) 3

- 36 The number of axes of symmetry of the square is
 (a) 1 (b) 2 (c) 3 (d) 4
-
- 37 The number of axes of symmetry of the equilateral triangle is
 (a) zero (b) 1 (c) 2 (d) 3
-
- 38 The number of axes of symmetry of the isosceles triangle is
 (a) 1 (b) 2 (c) 3 (d) 4
-
- 39 The number of axes of symmetry of the circle equals
 (a) 1 (b) 2 (c) 3 (d) infinite number
-
- 40 The image of the point A (3, -1) by translation (1, 2) is
 (a) \hat{A} (4, 1) (b) \hat{A} (2, -3) (c) \hat{A} (3, -2) (d) \hat{A} (2, 3)
-
- 41 The image of the point (3, 7) by translation ($X + 2$, $y - 1$) is
 (a) (5, 6) (b) (-3, 7) (c) (-3, 1) (d) (-1, -3)
-
- 42 The image of the point (5, -3) by rotation R (O, 90°) is
 (a) (3, 5) (b) (3, -5) (c) (5, 3) (d) (-3, -5)
-
- 43 The image of the triangle by rotation about the origin point with an angle of measure 180° is
 (a) a triangle. (b) a line segment. (c) a point. (d) straight line.
-
- 44 In the opposite figure :
 $\Delta \hat{A}\hat{B}\hat{C}$ is the image of ΔABC
 by rotation around A with an angle
 of measure
 (a) -110° (b) 80° (c) 110° (d) 140°
-
- 45 The geometric transformation which does not reserve the orientation of the vertices of a figure is
 (a) reflection in a straight line. (b) reflection in the origin point.
 (c) translation. (d) rotation.
-
- 46 The image of the point (2, 5) by rotation R (O, 270°) is
 (a) (2, 5) (b) (5, 2) (c) (-5, 2) (d) (5, -2)



Second Complete questions

- 1 The sum of measures of the exterior angles of a regular polygon of n sides =°
- 2 The number of sides of a regular polygon in which the measure of one of its interior angles 135° is sides.
- 3 ABCD is a parallelogram in which $m(\angle B) = 2m(\angle A)$, then $m(\angle A) = \dots\dots\dots^\circ$
- 4 The rectangle is a parallelogram with angle.
- 5 If ABCD is a rhombus, then \perp
- 6 The square is with a right angle.
- 7 The rectangle whose two diagonals are perpendicular is called
- 8 The parallelogram whose two diagonals are perpendicular and not equal in length is called
- 9 The quadrilateral in which only two sides are parallel is called a
- 10 The parallelogram whose perimeter is 30 cm. and the length of one of its sides is 7 cm., then the length of its adjacent side equals cm.
- 11 The rectangle whose perimeter is 20 cm. and its width is 4 cm., then its length equals cm.
- 12 ABCD is a rhombus in which $m(\angle A) + m(\angle C) = 120^\circ$, then $m(\angle B) = \dots\dots\dots^\circ$
- 13 The measure of the exterior angle of the equilateral triangle equals
- 14 The triangle ABC in which $m(\angle A) + m(\angle C) = 110^\circ$, then $m(\angle B) = \dots\dots\dots^\circ$
- 15 The line segment joining the midpoints of two sides of a triangle is the third side.
- 16 The length of the line segment drawn between the midpoints of two sides in a triangle equals the length of the third side.
- 17 If ABC is a triangle in which $m(\angle B) = 90^\circ$, then $(AC)^2 = \dots\dots\dots$
- 18 In the triangle ABC if $m(\angle B) = 90^\circ$, $(AB)^2 = \dots\dots\dots$

- 19 If the measure of an angle in a triangle is greater than the sum of measures of the other two angles, then the triangle is
-
- 20 If the triangle LMN is right-angled at M and $LM = 6$ cm, $MN = 8$ cm, then $(LN)^2 = \dots\dots\dots \text{cm}^2$.
-
- 21 ΔXYZ is right-angled at Y, if $XZ = 25$ cm, $YZ = 24$ cm, then $XY = \dots\dots\dots$
-
- 22 In ΔABC , if $m(\angle C) = m(\angle A) + m(\angle B)$, then $m(\angle C) = \dots\dots\dots^\circ$
-
- 23 The image of the point (3, 4) by reflection in y-axis is
-
- 24 The image of the point (2, 1) by reflection in X-axis is
-
- 25 The image of the point (-3, 5) by reflection in the origin point is
-
- 26 The image of the point (2, 0) by reflection in the origin point is
-
- 27 If the image of the point (X, y) by reflection in the origin point is (a, b), then $y + b = \dots\dots\dots$
-
- 28 If the reflection in a straight line transforms the figure to itself, then this straight line is called
-
- 29 The number of axes of symmetry of the isosceles trapezium equals
-
- 30 The image of the point (5, -3) by translation of magnitude 3 units in the negative direction of X-axis is
-
- 31 The image of the point (-1, 3) by translation (4, -2) is
-
- 32 The image of a line segment by a translation is another line segment is and
-
- 33 If the image of the point (-1, 3) by a translation is the point (1, 4), then the image of the point (3, -2) by the same translation is
-
- 34 The image of the point (5, 3) by translation $(X, y) \longrightarrow (X + 3, y - 1)$ is
-
- 35 The image of the point (1, 4) by rotation about the origin point with an angle of measure 90° is
-
- 36 The image of the point (3, 5) by rotation $R(0, -90^\circ)$ is
-

- 37** The image of the point $(5, -3)$ by rotation about the origin point with an angle of measure 180° is
- 38** The image of the point $(0, 3)$ by rotation $R(O, 90^\circ)$ is
- 39** The image of the point $(2, -1)$ by rotation about the origin point with an angle of measure -180° is
- 40** The image of the point $(-3, 5)$ by rotation about the origin point with an angle of measure 360° is

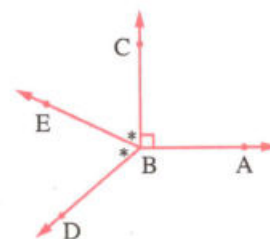
Third Essay questions

- 1** In the opposite figure :

If \overrightarrow{BE} bisects $\angle DBC$

$$, m(\angle EBC) = 65^\circ$$

Find with proof : $m(\angle ABD)$

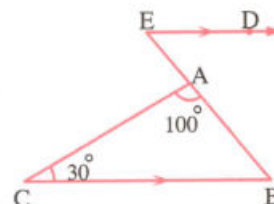


- 2** In the opposite figure :

$\overrightarrow{ED} \parallel \overrightarrow{BC}$, $m(\angle BAC) = 100^\circ$

$$, m(\angle C) = 30^\circ$$

Find : $m(\angle E)$



- 3** In the opposite figure :

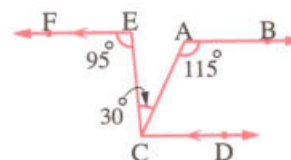
$\overrightarrow{EF} \parallel \overrightarrow{CD}$

$$, m(\angle CEF) = 95^\circ$$

$$, m(\angle ACE) = 30^\circ$$

$$, m(\angle BAC) = 115^\circ$$

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{EF}$



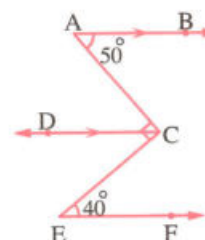
- 4** In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{CD}$

$$, m(\angle A) = 50^\circ , m(\angle ACE) = 90^\circ$$

$$, m(\angle E) = 40^\circ$$

Prove that : $\overrightarrow{CD} \parallel \overrightarrow{EF}$



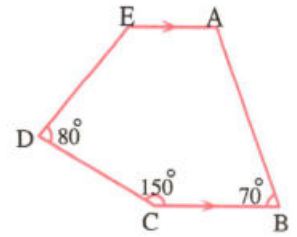
5 In the opposite figure :

$\overline{AE} \parallel \overline{BC}$, $m(\angle B) = 70^\circ$

, $m(\angle C) = 150^\circ$

, $m(\angle D) = 80^\circ$

Find with proof : $m(\angle E)$

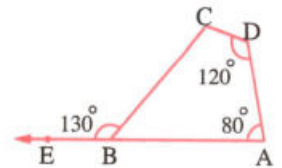


6 In the opposite figure :

$E \in \overrightarrow{AB}$, $m(\angle A) = 80^\circ$

, $m(\angle D) = 120^\circ$, $m(\angle CBE) = 130^\circ$

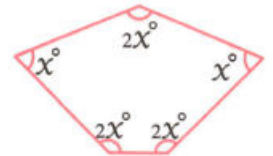
Find : $m(\angle C)$



7 In the opposite figure :

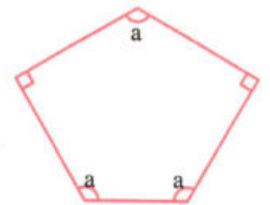
Find with giving reason

The value of x



8 In the opposite figure :

Find : The value of a

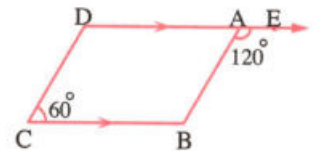


9 In the opposite figure :

$E \in \overrightarrow{DA}$, $m(\angle EAB) = 120^\circ$

, $m(\angle C) = 60^\circ$, $\overrightarrow{DA} \parallel \overrightarrow{CB}$

Prove that : ABCD is a parallelogram



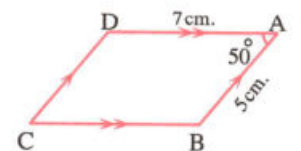
10 In the opposite figure :

If ABCD is a parallelogram in which :

$m(\angle A) = 50^\circ$, $AB = 5 \text{ cm}$, $AD = 7 \text{ cm}$.

Find : **1** $m(\angle B)$, $m(\angle C)$

2 The perimeter of the parallelogram.

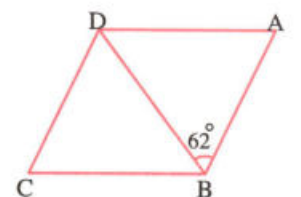


11 In the opposite figure :

ABCD is a rhombus in which :

, $m(\angle ABD) = 62^\circ$

Find with proof : $m(\angle A)$



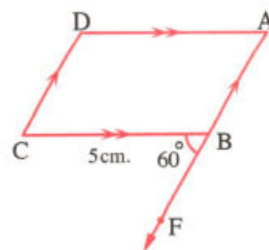
12 In the opposite figure :

ABCD is a parallelogram

$$, m(\angle CBF) = 60^\circ$$

$$, BC = 5 \text{ cm.}, F \in \overrightarrow{AB}$$

Find with proof : $m(\angle D)$, the length of \overline{AD}

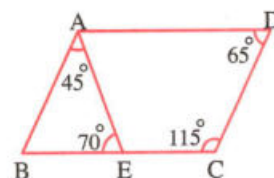


13 In the opposite figure :

$$m(\angle BAE) = 45^\circ, m(\angle AEB) = 70^\circ$$

$$, m(\angle D) = 65^\circ, m(\angle C) = 115^\circ$$

Prove that : ABCD is a parallelogram.



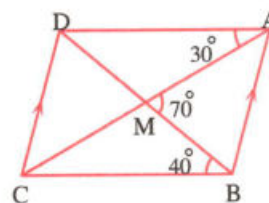
14 In the opposite figure :

$$\overline{AB} \parallel \overline{DC}, \overline{AC} \cap \overline{BD} = \{M\}$$

$$, m(\angle DAC) = 30^\circ, m(\angle DBC) = 40^\circ$$

$$, m(\angle AMB) = 70^\circ$$

Prove that : ABCD is a parallelogram.

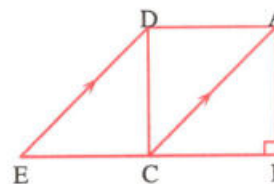


15 In the opposite figure :

ABCD is a square, $E \in \overrightarrow{BC}$

where : $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.

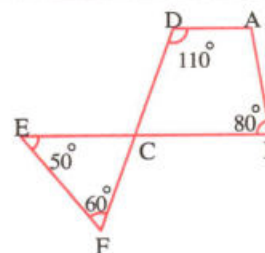


16 In the opposite figure :

$$m(\angle E) = 50^\circ, m(\angle F) = 60^\circ$$

$$, m(\angle B) = 80^\circ, m(\angle D) = 110^\circ$$

Find : $m(\angle A)$



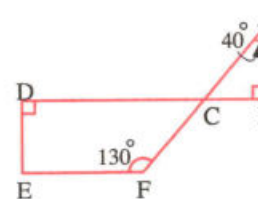
17 In the opposite figure :

$\overline{AB}, \overline{DE}$ are perpendicular

on $\overline{BD}, \overline{BD} \cap \overline{AF} = \{C\}$

$$, m(\angle A) = 40^\circ, m(\angle F) = 130^\circ$$

Find with proof : $m(\angle E)$

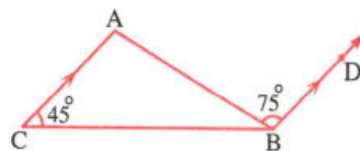


18 In the opposite figure :

$$\overrightarrow{BD} \parallel \overrightarrow{CA}, m(\angle C) = 45^\circ$$

$$, m(\angle ABD) = 75^\circ$$

Find : $m(\angle ABC)$

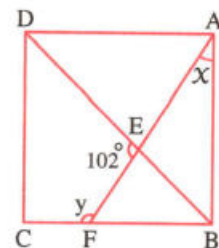


19 In the opposite figure :

ABCD is a square

Find in degrees :

The value of each of x and y

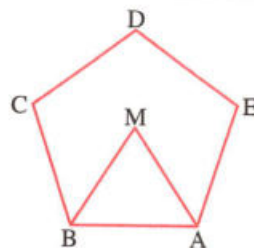


20 In the opposite figure :

ABCDE is a regular pentagon

, ABM is a regular triangle

Find : $m(\angle EAM)$ by proof.

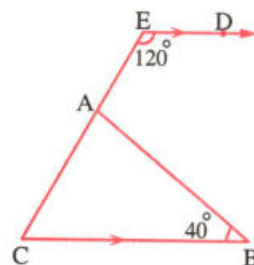


21 In the opposite figure :

$$\overrightarrow{ED} \parallel \overrightarrow{BC}, m(\angle E) = 120^\circ$$

$$, m(\angle B) = 40^\circ$$

Find : $m(\angle BAC)$

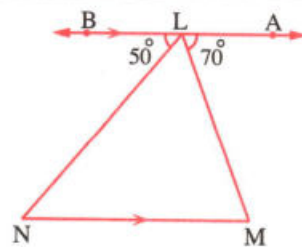


22 In the opposite figure :

$$\overrightarrow{AB} \parallel \overrightarrow{MN}, m(\angle ALM) = 70^\circ$$

$$, m(\angle BLN) = 50^\circ$$

Find : The measure of each of the interior angles of $\triangle LMN$



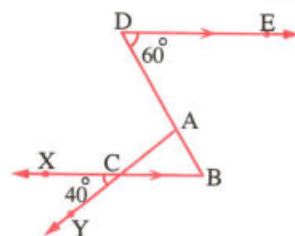
23 In the opposite figure :

$$\overrightarrow{DE} \parallel \overrightarrow{BC}$$

$$, m(\angle D) = 60^\circ$$

$$, m(\angle XCY) = 40^\circ$$

Calculate : The measures of the angles of $\triangle ABC$

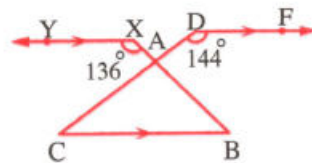


24 In the opposite figure :

$$\overline{BC} \parallel \overline{DF} \parallel \overline{XY}$$

$$, m(\angle D) = 144^\circ, m(\angle X) = 136^\circ$$

Find with proof : $m(\angle BAC)$



25 In the opposite figure :

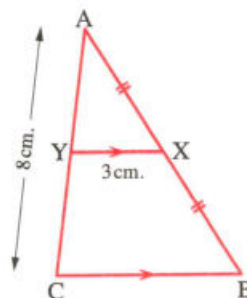
$$\overline{XY} \parallel \overline{BC}$$

, X is the midpoint of \overline{AB}

$$, AC = 8 \text{ cm.}$$

$$, XY = 3 \text{ cm.}$$

Find : The length of each of \overline{BC} , \overline{AY}



26 In the opposite figure :

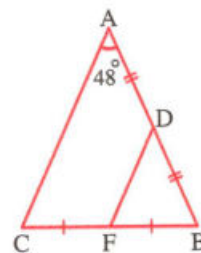
ABC is a triangle in which

D , F are the midpoints of \overline{AB}

, \overline{BC} respectively

1 Prove that : $\overline{DF} \parallel \overline{AC}$

2 If $m(\angle A) = 48^\circ$, find $m(\angle BDF)$

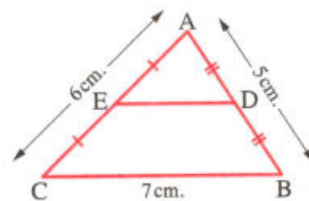


27 In the opposite figure :

ABC is a triangle in which : D is the midpoint of \overline{AB}

, E is the midpoint of \overline{AC} , if $AB = 5 \text{ cm.}$, $BC = 7 \text{ cm.}$, $AC = 6 \text{ cm.}$

Find : The perimeter of the triangle ADE

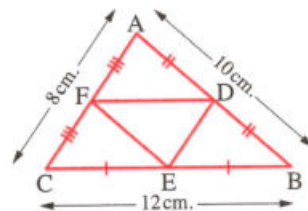


28 In the opposite figure :

$AB = 10 \text{ cm.}$, $BC = 12 \text{ cm.}$, $AC = 8 \text{ cm.}$

, D , E , F are the midpoints of \overline{AB} , \overline{BC} , \overline{AC} respectively.

Find : The perimeter of $\triangle DEF$



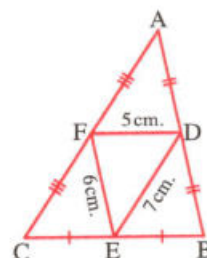
29 In the opposite figure :

D , E , F are the midpoints of

\overline{AB} , \overline{BC} , \overline{AC} respectively

, $DE = 7 \text{ cm.}$, $EF = 6 \text{ cm.}$, $DF = 5 \text{ cm.}$

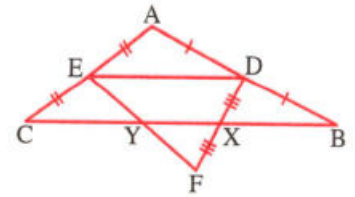
Find with proof : The perimeter of $\triangle ABC$



30 In the opposite figure :

D is the midpoint of \overline{AB} , E is the midpoint of \overline{AC} ,
 $\overline{DF} \cap \overline{BC} = \{X\}$ where $DX = XF$, $BC = 12$ cm.

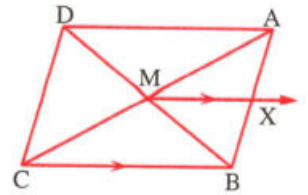
Find : The length of \overline{XY}



31 In the opposite figure :

ABCD is a parallelogram,
 , its diagonals intersect at M
 , $AB = 6$ cm. , $\overline{MX} \parallel \overline{CB}$

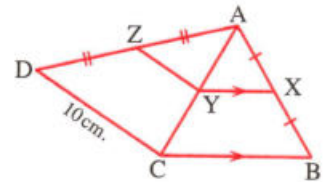
Find : The length of \overline{AX}



32 In the opposite figure :

X is the midpoint of \overline{AB} , $\overline{XY} \parallel \overline{BC}$
 , Z is the midpoint of \overline{AD} , $CD = 10$ cm.

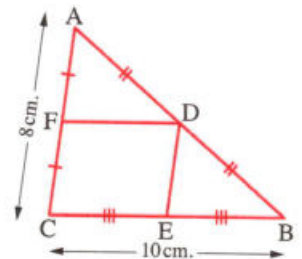
Find with proof : The length of \overline{YZ}



33 In the opposite figure :

D is the midpoint of \overline{AB}
 , E is the midpoint of \overline{BC}
 , F is the midpoint of \overline{AC}
 , $BC = 10$ cm. , $AC = 8$ cm.

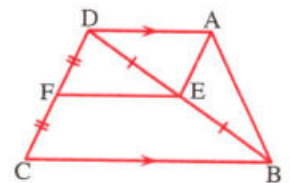
Find : The perimeter of the figure DECF



34 In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $BC = 2 AD$
 , E is the midpoint of \overline{DB}
 , F is the midpoint of \overline{CD}

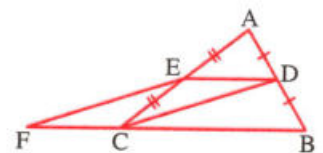
Prove that : AEFD is a parallelogram.



35 In the opposite figure :

D is the midpoint of \overline{AB}
 , E is the midpoint of \overline{AC}
 , $CF = \frac{1}{2} BC$, $F \in \overline{BC}$

Prove that : DCFE is a parallelogram.



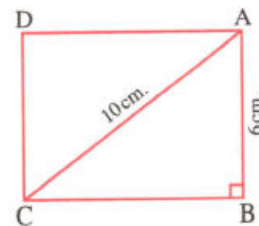
36 In the opposite figure :

ABCD is a rectangle in which

$AB = 6 \text{ cm.}$, $AC = 10 \text{ cm.}$

Find : **1** The length of \overline{BC}

2 The area of the rectangle ABCD

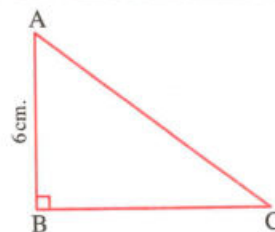


37 In the opposite figure :

ABC is a right-angled triangle at B

, its area is 24 cm^2 , $AB = 6 \text{ cm.}$

Find its perimeter.

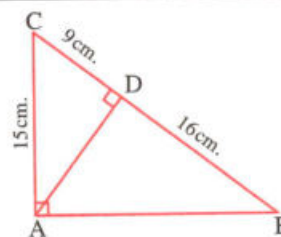


38 In the opposite figure :

$AC = 15 \text{ cm.}$, $DC = 9 \text{ cm.}$, $DB = 16 \text{ cm.}$, $m(\angle CAB) = 90^\circ$

, $\overline{AD} \perp \overline{CB}$

Find with proof : The length of each of \overline{AB} and \overline{AD}



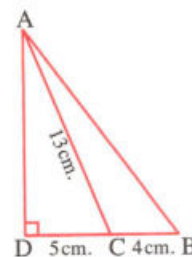
39 In the opposite figure :

$m(\angle D) = 90^\circ$, $AC = 13 \text{ cm.}$

, $CD = 5 \text{ cm.}$

, $BC = 4 \text{ cm.}$

Find : The length of each of \overline{AD} and \overline{AB}



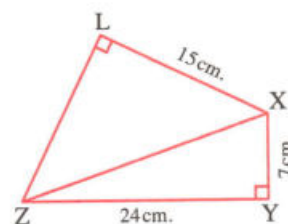
40 In the opposite figure :

$m(\angle Y) = m(\angle L) = 90^\circ$

, $XY = 7 \text{ cm.}$, $YZ = 24 \text{ cm.}$

, $XL = 15 \text{ cm.}$

Find : XZ , LZ

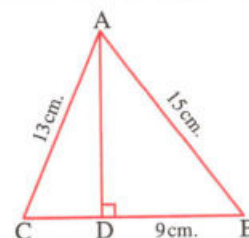


41 In the opposite figure :

$\overline{AD} \perp \overline{BC}$, $BD = 9 \text{ cm.}$

, $AB = 15 \text{ cm.}$, $AC = 13 \text{ cm.}$

Find : The length of each of \overline{AD} , \overline{DC}
and the area of the triangle ABC



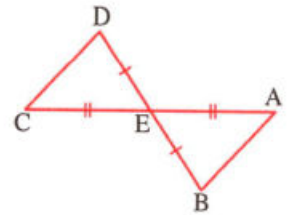
42 In the opposite figure :

$$\overline{AC} \cap \overline{BD} = \{E\}, AE = EC$$

$$, BE = ED, AB = (x + 5) \text{ cm.}$$

$$, CD = (2x - 3) \text{ cm.}$$

Using the geometric transformations find the length of \overline{CD}



43 On the square lattice, draw $\triangle ABC$ where $A(1, 2)$, $B(5, 2)$, $C(3, 5)$, then find its image :

1 By reflection in x -axis

2 By reflection in y -axis

44 Draw the isosceles triangle ABC in which :

$AB = BC = 4 \text{ cm.}$, $m(\angle ABC) = 90^\circ$, then find its image by reflection in point B

45 On the square lattice, plot the points $A(1, 4)$, $B(1, 1)$, $C(5, 1)$, then find the image of the triangle ABC by reflection in the origin point.

46 On the square lattice, draw \overline{AB} where $A(4, 3)$, $B(-1, 1)$, then draw its image by translation : $(x, y) \longrightarrow (x + 2, y - 1)$

47 On the square lattice, draw the triangle ABC where $A(1, 1)$, $B(4, 1)$, $C(4, 4)$, then find its image by translation $(2, 3)$

48 On the square lattice, draw the triangle ABC where $A(2, 4)$, $B(4, 0)$, $C(0, -1)$, then find its image by translation of a distance equals AB and in the direction of \overline{AB}

49 On the square lattice, draw $\triangle ABC$ where : $A(1, 1)$, $B(1, 4)$, $C(5, 1)$, then draw the image of $\triangle ABC$ by rotation $R(O, -90^\circ)$

50 Draw the triangle ABC , where : $A(5, 2)$, $B(2, 4)$, $C(1, 1)$, then find its image by rotation about the origin point with an angle of measure 90°

51 Draw $\triangle OBC$ in which : $O(0, 0)$, $B(4, 0)$, $C(4, 4)$, then find its image by rotation about the origin point with an angle of measure 180°

52 In the opposite figure :

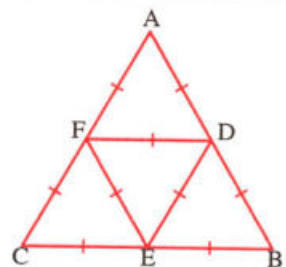
The triangles ADF , DBE and FEC are equilateral triangles

Find the image of $\triangle ADF$:

1 By translation \overrightarrow{AD} in direction of \overline{AD}

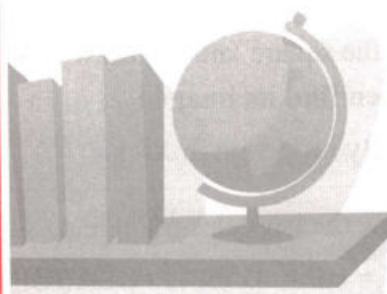
2 By reflection in \overleftrightarrow{DF}

3 By rotation $R(D, 60^\circ)$



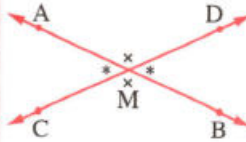
Final Revision

of Geometry and Measurement



Some relations between the angles

If two straight lines intersect, then the measures of each two vertically opposite angles are equal.



If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, then

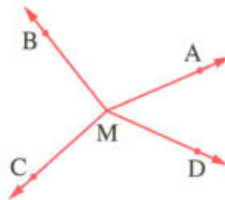
$$m(\angle AMC) = m(\angle BMD)$$

"Vertically opposite angles"

$$m(\angle AMD) = m(\angle CMB)$$

"Vertically opposite angles"

The sum of the measures of the accumulative angles at a point is equal to 360°



$$m(\angle AMD) + m(\angle DMC)$$

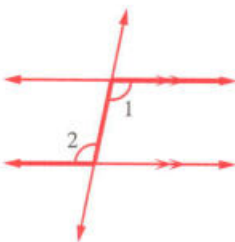
$$+ m(\angle CMB) + m(\angle BMA)$$

$$= 360^\circ$$

Parallelism

If a straight line intersects two parallel straight lines, then

Each two alternate angles are equal in measure.

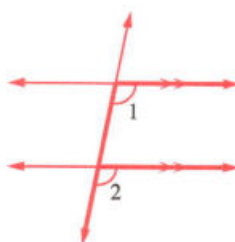


For example :

$$m(\angle 1) = m(\angle 2)$$

(alternate angles)

Each two corresponding angles are equal in measure.

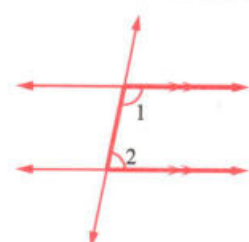


For example :

$$m(\angle 1) = m(\angle 2)$$

(corresponding angles)

Each two interior angles in the same side of the transversal are supplementary.



For example :

$$m(\angle 1) + m(\angle 2) = 180^\circ$$

Remember

How to prove the parallelism of two straight lines

The two straight lines are parallel if a third straight line intersects them (as a transversal) and **one** of the following cases is satisfied :

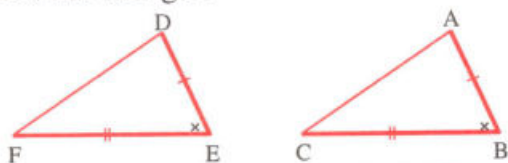
- 1 Two alternate angles have the same measure.
- or 2 Two corresponding angles have the same measure.
- or 3 Two interior angles in the same side of the transversal are supplementary.

Congruence of triangles

First case :

Two sides and the included angle (S.A.S.)

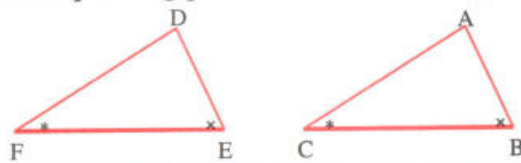
Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle.



Second case :

Two angles and one side (A.S.A.)

Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle.

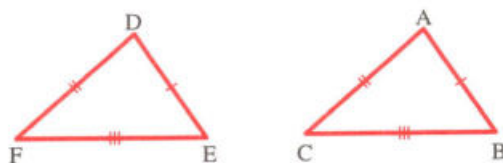


The cases of congruence
of two triangles

Third case :

Three sides (S.S.S.)

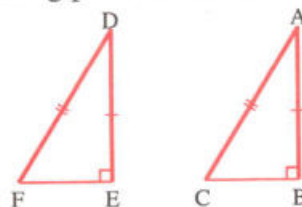
Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.



Fourth case :

Hypotenuse and one side in the right-angled triangle (R.H.S.)

Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.



The polygon

First rule

The sum of measures of the interior angles of a polygon of n sides $= (n - 2) \times 180^\circ$

Second rule

The measure of each interior angle of the regular polygon of n sides $= \frac{(n - 2) \times 180^\circ}{n}$

Third rule

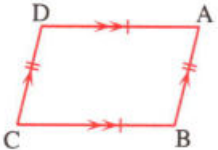
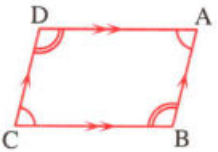
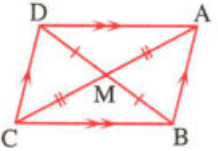
The sum of measures of the exterior angles of any convex polygon $= 360^\circ$

Fourth rule

The number of sides of the regular polygon in which the measure of one of its interior is $X^\circ = \frac{360^\circ}{180^\circ - X}$

Rules of the convex polygon

The parallelogram and its properties

1 Each two opposite sides are equal in length.		<ul style="list-style-type: none"> • $AB = DC$ • $AD = BC$
2 Each two opposite angles are equal in measure.		<ul style="list-style-type: none"> • $m(\angle A) = m(\angle C)$ • $m(\angle B) = m(\angle D)$
3 The sum of measures of each two consecutive angles is 180°		<ul style="list-style-type: none"> • $m(\angle A) + m(\angle B) = 180^\circ$ • $m(\angle B) + m(\angle C) = 180^\circ$ • $m(\angle C) + m(\angle D) = 180^\circ$ • $m(\angle D) + m(\angle A) = 180^\circ$
4 The two diagonals bisect each other.		<ul style="list-style-type: none"> • $AM = CM$ • $BM = DM$

When does a quadrilateral represent a parallelogram ?

A quadrilateral represents a parallelogram if one of the following conditions satisfies

Each two opposite sides are parallel.



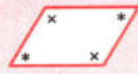
Each two opposite sides are equal in length.



Two opposite sides are parallel and equal in length.



Each two opposite angles are equal in measure.



The two diagonals bisect each other.



The parallelogram is

a rectangle

if

One of its angles is a right angle

or

Its two diagonals are equal in length.

a rhombus

if

Two adjacent sides are equal in length.

or

Its two diagonals are perpendicular.

a square

if

One of its angles is right and two adjacent sides are equal in length.

or

One of its angles is right and its two diagonals are perpendicular.

or

Its two diagonals are perpendicular and equal in length.

or

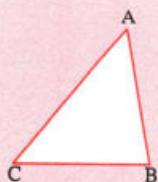
Two adjacent sides are equal in length and its two diagonals are equal in length.

Notice that

- * A square is a rectangle with two adjacent sides equal in length or with two perpendicular diagonals.
- * A square is a rhombus with a right angle or with two diagonals equal in length.
- * To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram.

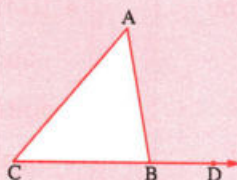
The triangle

- The sum of the measures of the interior angles of a triangle is 180°



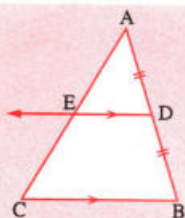
$$m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$$

- The measure of the exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.



If $D \in \overrightarrow{CB}$, then $\angle ABD$ is an exterior angle of $\triangle ABC$
 $m(\angle ABD) = m(\angle A) + m(\angle C)$

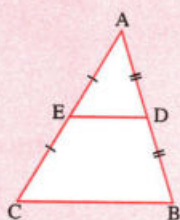
- The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.



If D is the midpoint of \overline{AB} ,
 $\overrightarrow{DE} \parallel \overrightarrow{BC}$, then E is the midpoint of \overline{AC}

- The line segment joining the midpoints of two sides of a triangle is parallel to the third side.

- The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.

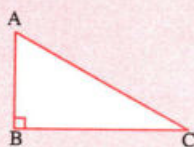


If D is the midpoint of \overline{AB} ,
 E is the midpoint of \overline{AC} ,
 then $\overline{DE} \parallel \overline{BC}$

If D the midpoint of \overline{AB} ,
 E is the midpoint of \overline{AC} ,
 then $DE = \frac{1}{2}BC$

Pythagoras' theorem :

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.



In $\triangle ABC$:

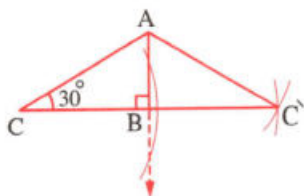
If $m(\angle B) = 90^\circ$

$$, \text{ then } (AC)^2 = (AB)^2 + (BC)^2$$

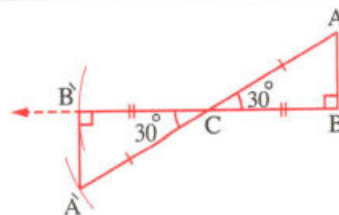
$$, (AB)^2 = (AC)^2 - (BC)^2$$

$$, (BC)^2 = (AC)^2 - (AB)^2$$

Geometric transformations



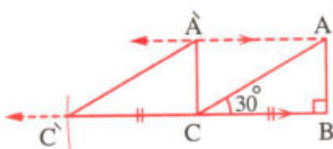
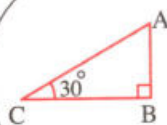
- Image of each of A and B are them.
- To find the image of C : We place the compasses at C , then we draw an arc cuts \overleftrightarrow{AB} at two points , then we place at each of them and draw two arcs intersect at C' , then $\triangle ABC'$ is the image of $\triangle ABC$ by reflection in \overleftrightarrow{AB}



- Image of C is itself.
- To find the image of A : We draw \overleftrightarrow{AC} , and by using the compasses take the point A' on \overleftrightarrow{AC} where $AC = A'C$
- Similarly , we find image of B , then $\triangle A'B'C$ is the image of $\triangle ABC$ by reflection in the point C

Reflection in \overleftrightarrow{AB}

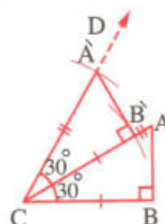
Reflection in the point C



- Image of B is C
- To find the image of C : We draw \overleftrightarrow{BC} and by using the compasses , we take point C' on \overleftrightarrow{BC} where $CC' = BC$
- To find the image of A : We draw from A a ray parallels to \overleftrightarrow{BC} , then by using the compasses we determine on it A' where $AA' = BC$, then $\triangle A'B'C$ is the image of $\triangle ABC$ by translation BC in the direction \overleftrightarrow{BC}

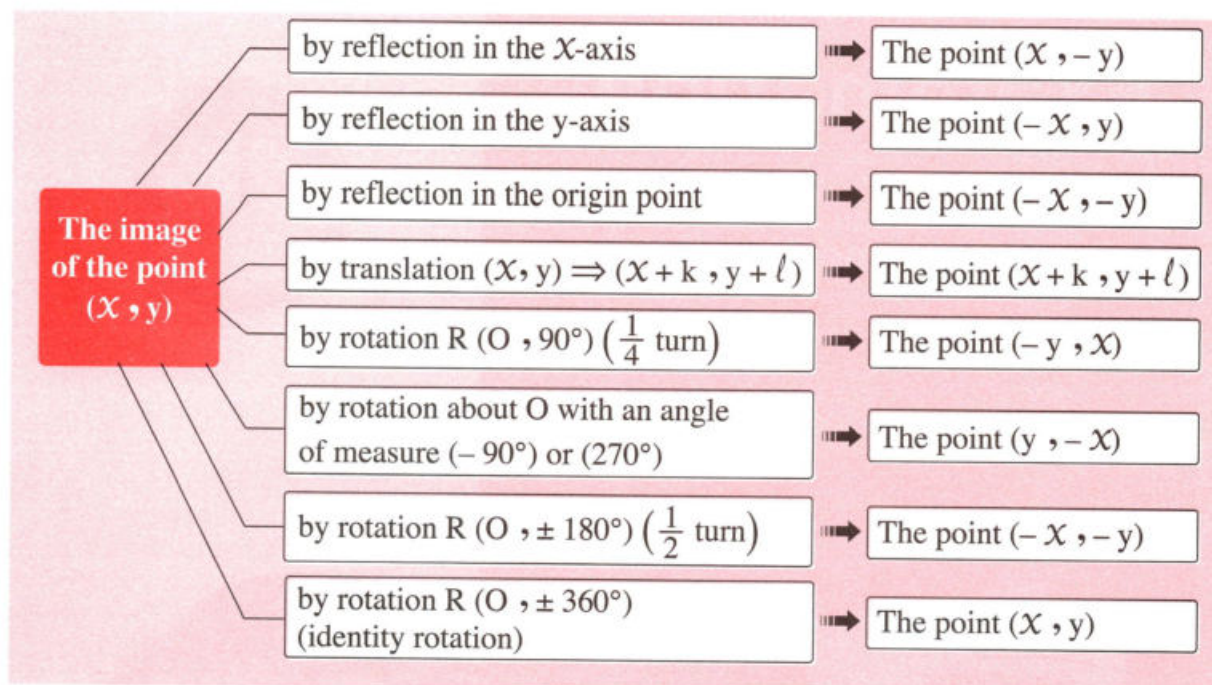
Translation in the direction \overleftrightarrow{BC}

Rotation around C with an angle 30°



- Image of C is itself.
- To find the image of A : Put the protractor with its straight edge on \overleftrightarrow{CA} and in the anticlockwise direction , draw \overleftrightarrow{CD} where $m(\angle ACD) = 30^\circ$, then take the point A' on \overleftrightarrow{CD} where $CA' = CA$
- Similarly , we find B' which is the image of B , $\triangle A'B'C$ is the image of $\triangle ABC$ by rotation around C with angle of measure 30°

Summary for geometrical transformations (reflection, translation, rotation) in the Cartesian plane :



Final Examinations

on Geometry and Measurement

- School book examinations
- Schools examinations



Model

1

Answer the following questions :

1 Choose the correct answer from those given :

1 Circumference of a circle of radius 7 cm. = cm. ($\pi = \frac{22}{7}$)

- (a) 11 (b) 22 (c) 44 (d) 88

2 The image of the point $(-1, 3)$ by translation $(4, -2)$ is

- (a) $(3, 1)$ (b) $(3, -1)$ (c) $(5, 1)$ (d) $(5, -5)$

3 The measure of the exterior angle of the equilateral triangle is

- (a) 30° (b) 45° (c) 60° (d) 120°

4 In a parallelogram if the adjacent sides are equal in the length , then the shape is

- (a) square. (b) rhombus. (c) rectangle. (d) trapezium.

5 The number of the diagonals of a pentagon is

- (a) 3 (b) 5 (c) 7 (d) 9

6 The number of axes of symmetry of an isosceles triangle =

- (a) zero (b) 1 (c) 2 (d) 3

2 Complete the following :

1 The image of the point $(2, 1)$ by reflection in X-axis is

2 In the opposite figure :

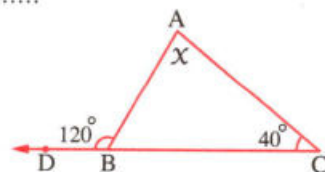
$x = \dots\dots\dots^\circ$

3 XYZ is a triangle in which $m(\angle Y) = 90^\circ$, $XY = 3$ cm.

, $XZ = 5$ cm. , then $YZ = \dots\dots\dots$ cm.

4 ABCD is a parallelogram in which $m(\angle A) = 100^\circ$, then $m(\angle B) + m(\angle D) = \dots\dots\dots^\circ$

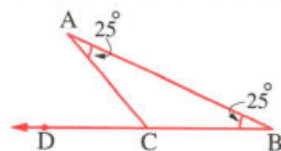
5 The sum of the measures of the interior angles of a triangle =



3 [a] In the opposite figure :

$m(\angle A) = m(\angle B) = 25^\circ$

Find : $m(\angle ACD)$

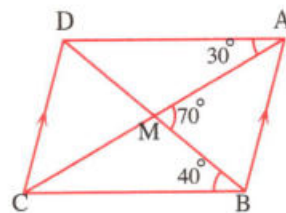


- [b]** Draw a triangle ABC in which $AB = 5$ cm. , $AC = 3$ cm. and $m(\angle A) = 40^\circ$
 , then draw \hat{C} is the image of C under rotation $R(A, 40^\circ)$, \hat{B} is the image
 of B under rotation $R(A, -40^\circ)$

4 [a] In the opposite figure :

$\overline{AB} \parallel \overline{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$,
 $m(\angle DAC) = 30^\circ$, $m(\angle DBC) = 40^\circ$
 and $m(\angle AMB) = 70^\circ$

Prove that : ABCD is a parallelogram.

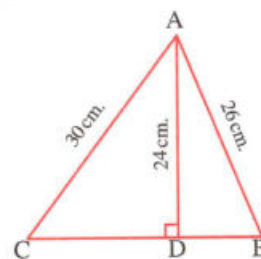


- [b]** Use the translation : $(x, y) \longrightarrow (x + 2, y + 3)$
 to find the point whose image is $(2, 3)$

5 [a] In the opposite figure :

$\overline{AD} \perp \overline{BC}$, if $AD = 24$ cm. , $AB = 26$ cm. , $AC = 30$ cm.

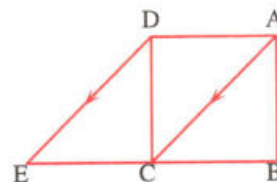
- 1 Find :** The length of \overline{BC}
2 Find : The area of $\triangle ABC$



[b] In the opposite figure :

ABCD is a square , $E \in \overline{BC}$, $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.



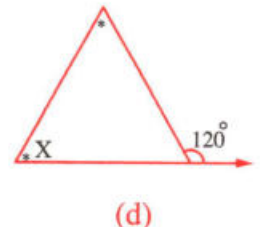
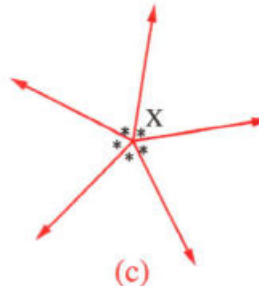
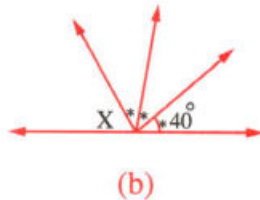
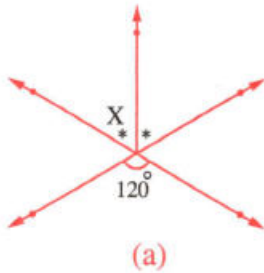
Model 2

Answer the following questions :

1 Choose the correct answer from those given :

- 1** ABC is a right-angled triangle at B , $AB = 6$ cm. , $BC = 8$ cm. , then $AC = \dots\dots\dots$ cm.
 (a) 10 (b) 28 (c) 100 (d) 160
- 2** The measure of each angle of regular hexagon equals
 (a) 60° (b) 108° (c) 120° (d) 135°
- 3** The two diagonals are equal in length and not perpendicular in
 (a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

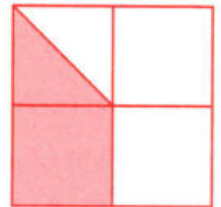
- 4 In all the following shapes $m(\angle X) = 60^\circ$ except the shape



- 5 In the opposite figure :

The area of the shaded part from the area of all shape equals

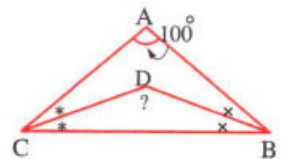
- (a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$



- 6 In the opposite figure :

$m(\angle BDC) = \dots^\circ$

- (a) 60 (b) 80 (c) 100 (d) 140

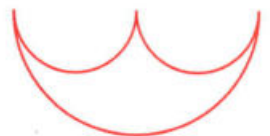


- 2 Complete the following :

- 1 In the opposite figure :

Semicircle of diameter 14 cm. and two semicircles the diameter of each is 7 cm.

, then the perimeter of the figure equals cm. $(\pi = \frac{22}{7})$



- 2 The image of the point (2, 3) by translation \overrightarrow{MN} , in direction \overrightarrow{MN} , where M (2, -1), N (5, 1) is

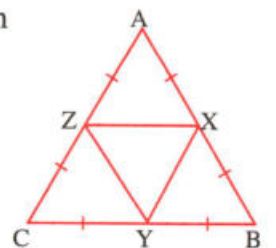
- 3 The volume of a cube of side length 1.2 m. = cm^3

- 4 The ray drawn parallel to one side of a triangle and passing through the midpoint of another side

- 5 In the opposite figure :

The image of the triangle XBY

by translation \overrightarrow{XZ} in direction \overrightarrow{XZ} is



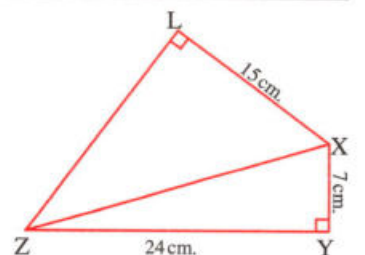
- 3 [a] In the opposite figure :

XYZL is a quadrilateral in which

$m(\angle Y) = m(\angle L) = 90^\circ$, $XY = 7 \text{ cm.}$,

$YZ = 24 \text{ cm.}$, $XL = 15 \text{ cm.}$

Find : The length of each of \overline{XZ} and \overline{LZ}



- [b] Using the square lattice, draw \overline{AB} where $A(4, 3)$, $B(-1, 1)$
then find the image of \overline{AB} by translation $(X, y) \longrightarrow (X + 2, y - 1)$

- 4 [a] Draw the image of triangle ABC where $A(1, 1)$, $B(3, 4)$, $C(5, 2)$
by reflection in X-axis.

- [b] In the opposite figure :

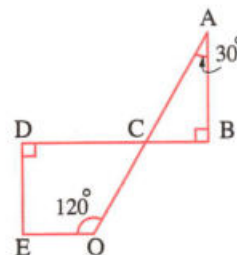
\overline{AB} and \overline{ED} are perpendicular to \overline{BD}

$$, \overline{BD} \cap \overline{AO} = \{C\} ,$$

$$m(\angle A) = 30^\circ$$

$$, m(\angle EOC) = 120^\circ ,$$

Find : $m(\angle E)$

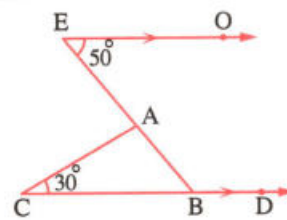


- 5 [a] In the opposite figure :

$$\overrightarrow{EO} \parallel \overrightarrow{CD} , m(\angle E) = 50^\circ$$

$$, m(\angle C) = 30^\circ ,$$

Find the measures of angles of $\triangle ABC$, $m(\angle ABD)$



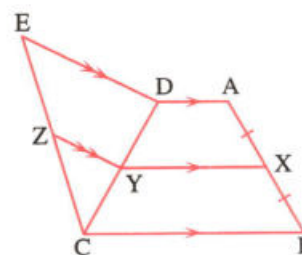
- [b] In the opposite figure :

X is the midpoint of \overline{AB}

$$, Y \in \overline{CD} , Z \in \overline{CE}$$

$$, \overline{AD} \parallel \overline{XY} \parallel \overline{BC} , \overline{YZ} \parallel \overline{DE}$$

Is $CZ = ZE$? giving reason



Model examination for the merge students

Answer the following questions :

1 Choose the correct answer :

- 1 The sum of the measures of the interior angles of a triangle =°
 (a) 90 (b) 360 (c) 180 (d) 540
- 2 The image of the point $(3, -2)$ by reflection in the y-axis is the point
 (a) $(3, 2)$ (b) $(-3, -2)$ (c) $(-3, 2)$ (d) $(-2, 3)$
- 3 The diagonals are equal and perpendicular in
 (a) rhombus. (b) square. (c) recangle. (d) parallelogram.

4 In the opposite figure :

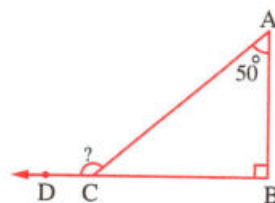
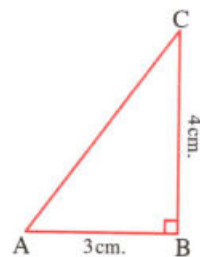
$AC = \dots\dots\dots$ cm.

- (a) 5 (b) 7
 (c) 25 (d) 625

5 In the opposite figure :

$m(\angle ACD) = \dots\dots\dots^\circ$

- (a) 40 (b) 140
 (c) 90 (d) 50

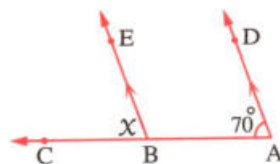


2 Complete each of the following :

- 1 The length of the line segment that joins two midpoints of two sides of a triangle equals the length of the third side.
- 2 The rectangle is a parallelogram in which one of it's angles is
- 3 The length of the side of a rhombus whose perimeter is 24 cm. equals cm.
- 4 The image of the point A $(-3, 2)$ by reflection in the origin point is the point \hat{A} (..... ,)

5 In the opposite figure :

$$x = \dots\dots\dots^\circ$$



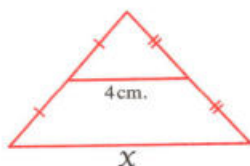
3 Put (✓) or (x) :

- 1** The image of the point (4 , 3) by reflection in the X-axis is the point (3 , - 4) ()
- 2** If ABC is a right-angled triangle at B , then $(AB)^2 = (BC)^2 + (AC)^2$ ()
- 3** The pentagon has 5 diagonals. ()
- 4** ABCD is a parallelogram , in which $m(\angle A) = 70^\circ$, then $m(\angle C) = 110^\circ$ ()
- 5** Any triangle contains at least two acute angles. ()

4 Join from the column (A) to the suitable in the column (B) :

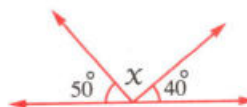
Column (A)	Column (B)
1 The sum of the measures of the interior angles of a quadrilateral =	• 120°
2 The measure of each angle of a regular hexagon =	• 360°
3 The image of the point (3 , 2) by translation (1 , - 2) is the point	• (- 1 , - 3)
4 The image of the point (1 , 3) by rotation about the origin point , of angle of measure 180° is the point (..... ,)	• 45
5 The diagonal of the square divides the angle of the vertex into two angles , the measure of each = $^\circ$	• (4 , 0)

5 Find the value of x :



$$x = \dots\dots\dots \text{ cm.}$$

Fig. (1)



$$x = \dots\dots\dots^\circ$$

Fig. (2)



1

Cairo Governorate



Heliopolis Educational Zone

Answer the following questions :

1 Choose the correct answer :

- 1 In $\triangle ABC$, if $m(\angle A) = m(\angle B) = 40^\circ$, then $m(\angle C) = \dots\dots\dots$
 (a) 70° (b) 30° (c) 50° (d) 100°
- 2 The image of the point $(0, -3)$ by reflection in the y -axis is $\dots\dots\dots$
 (a) $(0, 3)$ (b) $(-3, 0)$ (c) $(0, -3)$ (d) $(3, 0)$
- 3 The measure of the exterior angle of the equilateral triangle is $\dots\dots\dots$
 (a) 30° (b) 45° (c) 60° (d) 120°
- 4 The measure of the interior angle of the regular pentagon is $\dots\dots\dots$
 (a) 135° (b) 540° (c) 108° (d) 110°
- 5 The parallelogram whose diagonals are equal in length and perpendicular is a $\dots\dots\dots$
 (a) rectangle. (b) square. (c) rhombus. (d) trapezium.
- 6 The image of the point $(4, 4)$ by translation $(-4, 1)$ is $\dots\dots\dots$
 (a) $(0, 6)$ (b) $(4, 4)$ (c) $(0, 5)$ (d) $(0, -5)$

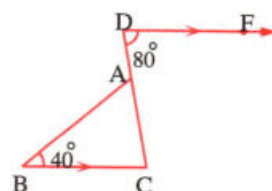
2 Complete the following :

- 1 The line segment joining the midpoints of two sides of a triangle $\dots\dots\dots$ to the third side.
- 2 ABCD is a parallelogram where $m(\angle A) = 60^\circ$, then $m(\angle B) = \dots\dots\dots^\circ$
- 3 The image of the point $(2, 1)$ by reflection in the X -axis is $\dots\dots\dots$
- 4 The measure of the angle of the identity rotation is $\dots\dots\dots^\circ$
- 5 The sum of the measures of the complementary angles is $\dots\dots\dots^\circ$
- 6 If the perimeter of a square is 40 cm. , then its side length is $\dots\dots\dots$

3 [a] In the opposite figure :

$$\overrightarrow{DF} \parallel \overrightarrow{BC}$$

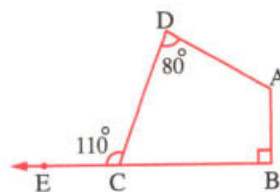
Find : $m(\angle BAC)$



[b] In the opposite figure :

$$E \in \overrightarrow{BC}$$

Find by proof : $m(\angle A)$

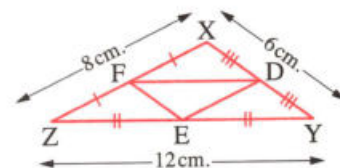


4 [a] In the opposite figure :

$$XY = 6 \text{ cm.}, XZ = 8 \text{ cm.}$$

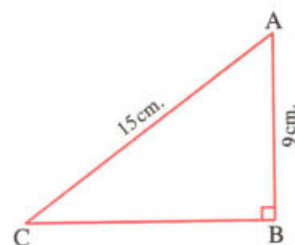
, $ZY = 12 \text{ cm.}$, D , F and E are the midpoints of \overline{XY} , \overline{XZ} , \overline{ZY} respectively.

Calculate : the perimeter of $\triangle DFE$



[b] In the opposite figure :

Find : The length of \overline{BC}



5 [a] Write two properties of the parallelogram.

[b] On a coordinate plane , draw \overline{AB} where A (4 , 1) , B (2 , 4) and draw its image by rotation (O , 180°)

2

Cairo Governorate



**Shoubra Educational Zone
Good Shepherd School**

Answer the following questions :

1 Choose the correct answer :

[1] The image of the point (- 2 , 3) by rotation around the origin point with an angle of measure 180° is

- (a) (2 , 3) (b) (2 , - 3) (c) (- 2 , - 3) (d) (- 3 , - 2)

[2] The image of the point (5 , 1) by reflection in the origin point is

- (a) (1 , 5) (b) (- 1 , - 5) (c) (- 5 , - 1) (d) (- 1 , 5)

[3] The parallelogram with equal sides is a

- (a) rectangle. (b) rhombus. (c) trapezium. (d) square.

[4] The measure of each interior angle of a regular hexagon equals

- (a) 120° (b) 135° (c) 108° (d) 90°

- 5 The two diagonals are equal in length in the
 (a) parallelogram. (b) trapezium. (c) rectangle. (d) rhombus.
- 6 The perimeter of the rhombus whose side length is 6 cm. equals cm.
 (a) 18 (b) 12 (c) 36 (d) 24

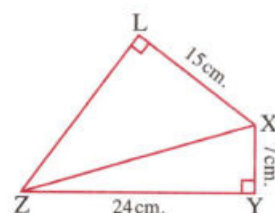
2 Complete :

- 1 The image of the point $(3, -5)$ by rotation around the origin point with an angle of measure 90° is
- 2 The length of the line segment joining the midpoints of two sides of a triangle equals the length of the third side.
- 3 XYZL is a parallelogram , if $m(\angle X) = m(\angle Y)$, then $m(\angle Y) = \dots\dots\dots^\circ$
- 4 The sum of measures of the exterior angles of a triangle equals $^\circ$
- 5 The sum of measures of the interior angles of a pentagon equals $^\circ$
- 6 The measure of the exterior angle of an equilateral triangle equals $^\circ$

3 [a] In the opposite figure :

$m(\angle XYZ) = m(\angle XLZ) = 90^\circ$
 , $XY = 7$ cm. , $XL = 15$ cm.
 and $YZ = 24$ cm.

Find : the lengths of \overline{XZ} and \overline{LZ}



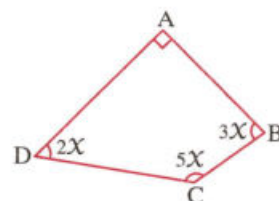
- [b] Draw $\triangle ABC$ where $A(1, 1)$, $B(4, 1)$ and $C(4, 5)$, then find its image by reflection in the y-axis and represent this image on the square lattice.

4 [a] In the opposite figure :

ABCD is a quadrilateral in which :

$m(\angle A) = 90^\circ$, $m(\angle B) = 3X$
 , $m(\angle C) = 5X$ and $m(\angle D) = 2X$

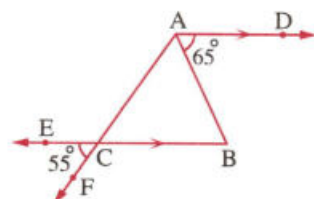
Find : the value of X



[b] In the opposite figure :

$\overrightarrow{AD} \parallel \overrightarrow{BC}$, $m(\angle DAB) = 65^\circ$
 , $m(\angle ECF) = 55^\circ$, $\overrightarrow{AF} \cap \overrightarrow{BC} = \{C\}$

Find : $m(\angle BAC)$

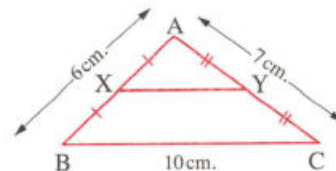


- 5 [a]** On a Cartesian plane, draw $\triangle ABC$ where $A(-2, 3)$, $B(2, 3)$ and $C(2, 6)$, then represent its image by translation $(X, y) \longrightarrow (X + 2, y - 1)$

[b] In the opposite figure :

X is the midpoint of \overline{AB} , Y is the midpoint of \overline{AC}
 $AB = 6$ cm. , $AC = 7$ cm. and $BC = 10$ cm.

Find : the perimeter of the figure $XBCY$



3

Giza Governorate



**Al-Agoza Directorate
El-Manar Islamic Language School**

Answer the following questions :

1 Choose the correct answer :

- 1** ABCD is a parallelogram, in which $m(\angle A) = 80^\circ$, then $m(\angle B) = \dots\dots\dots$
 (a) 80° (b) 90° (c) 100° (d) 120°
- 2** The diagonals are equal in length in the
 (a) parallelogram. (b) rhombus. (c) rectangle. (d) trapezium.
- 3** The image of the point $A(2, -1)$ by reflection in X -axis is
 (a) $(2, 1)$ (b) $(1, 2)$ (c) $(-2, -1)$ (d) $(-1, 2)$
- 4** $\triangle ABC$ is right-angled at B , if $AB = 6$ cm. , $BC = 8$ cm. , then $AC = \dots\dots\dots$ cm.
 (a) 20 (b) 25 (c) 15 (d) 10
- 5** The length of the line segment joining the two midpoints of two sides of a triangle is the length of the third side.
 (a) equal to (b) twice (c) half (d) quarter
- 6** The measure of each interior angle of the regular hexagon is
 (a) 720° (b) 360° (c) 120° (d) 60°

2 Complete :

- 1** If the measure of one angle of a triangle equals the sum of measures of the other two angles, then the measure of this angle is $^\circ$
- 2** The diagonals are equal in length and perpendicular in
- 3** The image of the point $A(1, 2)$ by translation $(3, -1)$ is
- 4** The sum of measures of the interior angles of a quadrilateral is $^\circ$
- 5** If the two alternate angles are equal in measure, then their straight lines are
- 6** The sum of measures of the accumulative angles at a point is $^\circ$

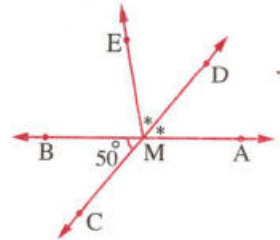
3 [a] In the opposite figure :

$$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$$

, \overleftrightarrow{MD} bisects $\angle AME$

$$, m(\angle CMB) = 50^\circ$$

Find with proof : $m(\angle EMB)$



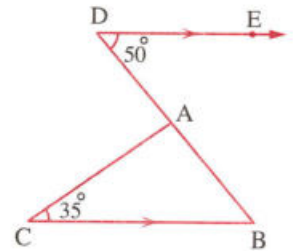
[b] In the opposite figure :

$$m(\angle D) = 50^\circ$$

$$, m(\angle C) = 35^\circ$$

$$, \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$$

Find : $m(\angle B)$, $m(\angle BAC)$



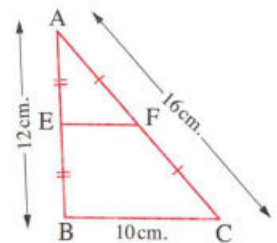
4 [a] In the opposite figure :

E , F are the midpoints of \overline{AB} , \overline{AC} respectively

, $BC = 10$ cm. , $AB = 12$ cm. , $AC = 16$ cm.

Find with proof :

the perimeter of the triangle AEF



[b] On the lattice , draw the triangle ABC such that : A (1 , 1) , B (4 , 1) , C (4 , 4)
 , then draw its image by translation (2 , 3)

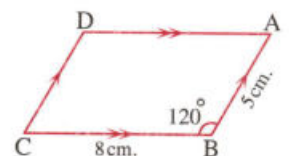
5 [a] In the opposite figure :

ABCD is a parallelogram , $AB = 5$ cm.

, $BC = 8$ cm. , $m(\angle B) = 120^\circ$

Find : **1** $m(\angle C)$

2 The perimeter of the parallelogram ABCD

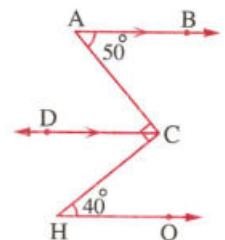


[b] In the opposite figure :

$$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} , m(\angle A) = 50^\circ$$

$$, \overleftrightarrow{AC} \perp \overleftrightarrow{CH} , m(\angle H) = 40^\circ$$

Prove that : $\overleftrightarrow{AB} \parallel \overleftrightarrow{HO}$





Answer the following questions :

1 Choose the correct answer :

- 1 The type of the angle whose measure is 180° is
 (a) obtuse. (b) right. (c) acute. (d) straight.
- 2 The sum of measures of the interior angles of the quadrilateral is
 (a) 360° (b) 180° (c) 720° (d) 150°
- 3 The image of the point (1 , 3) by translation (4 , - 1) is
 (a) (3 , 1) (b) (- 3 , - 1) (c) (5 , 2) (d) (6 , 0)
- 4 Any triangle has at least acute angles.
 (a) 4 (b) 3 (c) 1 (d) 2
- 5 The diagonals are equal in length and perpendicular in the
 (a) parallelogram. (b) rectangle. (c) rhombus. (d) square.
- 6 The image of the point (1 , 3) by reflection in the origin point is the point
 (a) (4 , 2) (b) (1 , - 3) (c) (- 1 , 3) (d) (- 1 , - 3)

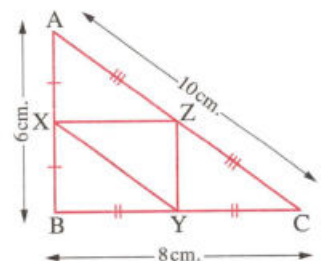
2 Complete the following :

- 1 The line segment joining the midpoints of two sides of a triangle is to the third side.
- 2 The sum of measures of the interior angles in any triangle equals $^\circ$
- 3 The image of (- 1 , 3) by reflection in y-axis is the point
- 4 The sum of the measures of the accumulative angles at a point equals $^\circ$
- 5 In $\triangle ABC$, if $m(\angle B) = 110^\circ$, then $m(\angle A) + m(\angle C) = \dots\dots\dots^\circ$
- 6 The ray drawn from the midpoint of a side of a triangle parallel to another side

3 [a] In the opposite figure :

X , Y , Z are the midpoints of \overline{AB} , \overline{BC} , \overline{AC} respectively
 , $AB = 6$ cm. , $BC = 8$ cm.
 , $AC = 10$ cm.

Find : the perimeter of $\triangle XYZ$



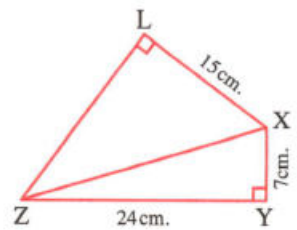
[b] In the opposite figure :

XYZL is a quadrilateral in which :

$$m(\angle Y) = m(\angle L) = 90^\circ$$

$$, XY = 7 \text{ cm.}, YZ = 24 \text{ cm.}, XL = 15 \text{ cm.}$$

Find : the lengths of \overline{XZ} , \overline{LZ}

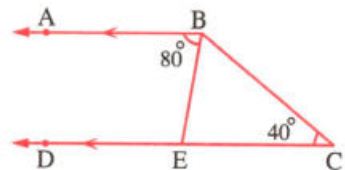


4 [a] In the opposite figure :

$$\overrightarrow{BA} \parallel \overrightarrow{CD}, m(\angle ABE) = 80^\circ$$

$$, m(\angle C) = 40^\circ$$

Find : $m(\angle EBC)$

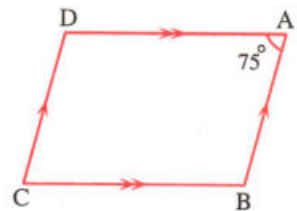


[b] In the opposite figure :

ABCD is a parallelogram

$$\text{where } m(\angle A) = 75^\circ$$

Find : $m(\angle B)$, $m(\angle C)$

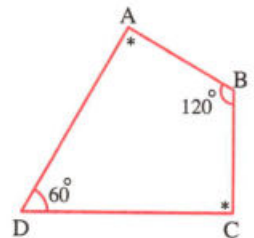


5 [a] In the opposite figure :

ABCD is a quadrilateral , $m(\angle B) = 120^\circ$

$$, m(\angle D) = 60^\circ , m(\angle A) = m(\angle C)$$

Find : $m(\angle C)$



[b] If $A(-1, -4)$, $B(2, -5)$, **find the images of** A , B **by reflection in** X -**axis.**

5

Alexandria Governorate



Agami Educational Zone
Mathematics Supervisor

Answer the following questions :

1 Choose the correct answer :

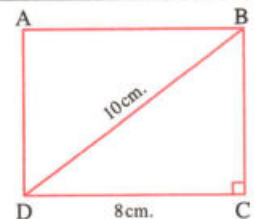
- [1]** The diagonals are equal in length and not perpendicular in the
 (a) rectangle. (b) rhombus. (c) square. (d) parallelogram.
- [2]** The image of the point $(-1, 3)$ by translation $(4, -2)$ is
 (a) $(3, 1)$ (b) $(3, -1)$ (c) $(5, 1)$ (d) $(5, -5)$
- [3]** The number of axes of symmetry of the equilateral triangle is
 (a) 1 (b) 2 (c) 3 (d) 4

- 4 Any triangle has at most obtuse angles.
 (a) 1 (b) 2 (c) 3 (d) 4
- 5 The sum of the measures of the exterior angles of a triangle equals
 (a) 180° (b) 360° (c) 630° (d) 90°
- 6 The measure of each interior angle of a regular hexagon equals
 (a) 120° (b) 90° (c) 108° (d) 60°

2 Complete the following :

1 In the opposite figure :

ABCD is a rectangle
 , its area = cm^2

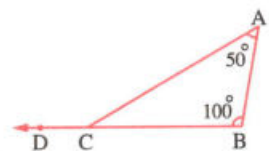


2 The sum of measures of the interior angles of the pentagon equals $^\circ$

3 A square is with two adjacent sides equal in length.

4 In the opposite figure :

$D \in \overrightarrow{BC}$, then $m(\angle ACD) = \dots\dots\dots^\circ$



5 The image of the point (1 , 4) by rotation $R(O, 180^\circ)$ is

6 The length of the line segment joining the midpoints of two sides of a triangle is the length of the third side.

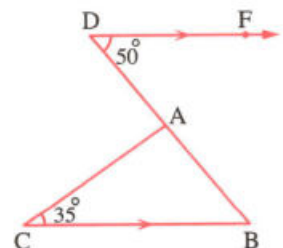
3 [a] In the opposite figure :

$\overrightarrow{DF} \parallel \overrightarrow{CB}$

, $m(\angle D) = 50^\circ$

, $m(\angle C) = 35^\circ$, $A \in \overrightarrow{BD}$

Find with proof : $m(\angle B)$ and $m(\angle DAC)$



[b] In the opposite figure :

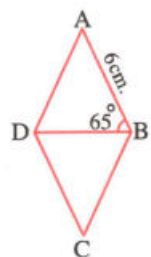
ABCD is a rhombus , $m(\angle ABD) = 65^\circ$

, $AB = 6 \text{ cm}$.

Find with proof :

1 $m(\angle C)$

2 The perimeter of the figure ABCD

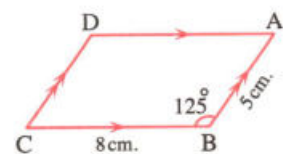


4 [a] In the opposite figure :

ABCD is a parallelogram in which :

$AB = 5 \text{ cm}$, $BC = 8 \text{ cm}$, $m(\angle B) = 125^\circ$

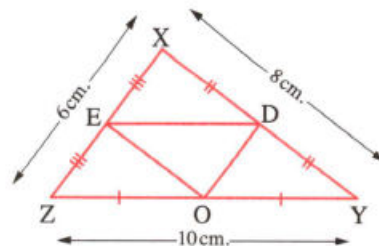
Find with proof : $m(\angle C)$, the perimeter of $\square ABCD$



[b] In the opposite figure :

XYZ is a triangle , E , D and O are the midpoints of \overline{XZ} , \overline{XY} and \overline{ZY} respectively
 $\text{, } XY = 8 \text{ cm. , } XZ = 6 \text{ cm. , } ZY = 10 \text{ cm.}$

Find : the perimeter of $\triangle DOE$



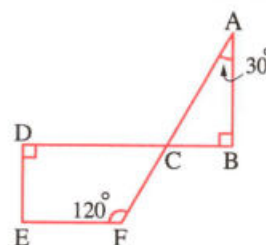
5 [a] In the opposite figure :

$\overline{AB} \perp \overline{BC}$, $\overline{BD} \cap \overline{AF} = \{C\}$

$\text{, } m(\angle A) = 30^\circ \text{ , } m(\angle D) = 90^\circ$

$\text{, } m(\angle F) = 120^\circ$

Find with proof : $m(\angle E)$



**[b] On the square lattice , draw $\triangle ABC$ where $A(1, 1)$, $B(4, 1)$, $C(5, 4)$
 , then draw its image by reflection in X-axis**

6

El-Kalyoubia Governorate



**Maths Supervision
 Official Language Schools**

Answer the following questions :

1 Choose the correct answer from those given :

[1] The right angle complements angle.

- (a) an obtuse (b) a right (c) an acute (d) a zero

[2] The parallelogram whose diagonals are perpendicular and equal in length is called a

- (a) rectangle. (b) rhombus. (c) square. (d) trapezium

**[3] If A and B are two supplementary angles where $m(\angle A) = 2m(\angle B)$
 , then $m(\angle B) = \dots\dots\dots$**

- (a) 30° (b) 60° (c) 90° (d) 120°

[4] The sum of measures of the interior angles of a polygon of n sides + the sum of measures of its exterior angles =

- (a) $(n - 2) \times 180^\circ$ (b) $n \times 180^\circ$ (c) $n \times 360^\circ$ (d) $(n - 2) \times 360^\circ$

[5] The identity rotation is a rotation with an angle of measure

- (a) 90° (b) -90° (c) 180° (d) 360°

- 6 The length of the line segment joining the midpoints of two sides of a triangle is equal to the length of the third side.

(a) quarter (b) third (c) half (d) twice

2 Complete the following :

- 1 DEF is a right-angled triangle at D , then $(DF)^2 + (DE)^2 = \dots\dots\dots$

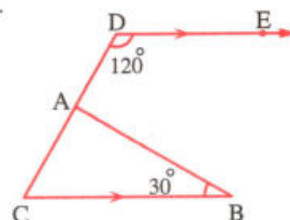
- 2 The image of the point (1 , 5) by translation (1 , - 2) is

- 3 In the opposite figure :

$\overrightarrow{DE} \parallel \overrightarrow{BC}$, $m(\angle B) = 30^\circ$

, $m(\angle D) = 120^\circ$

, then $m(\angle BAC) = \dots\dots\dots^\circ$



- 4 A rectangle , its width is 3 cm. and the length of its diagonal is 5 cm.

, then its area = cm^2

- 5 In $\triangle ABC$, if $m(\angle A) + m(\angle B) = 3 m(\angle C)$, then $m(\angle C) = \dots\dots\dots^\circ$

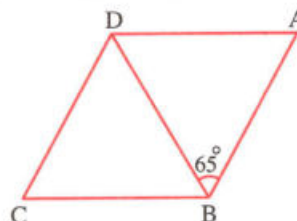
- 6 The measure of any of the exterior angles of an equilateral triangle equals $^\circ$

3 [a] In the opposite figure :

ABCD is a rhombus

, $m(\angle ABD) = 65^\circ$

Find with proof : $m(\angle A)$



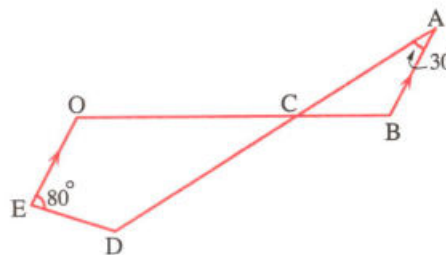
[b] In the opposite figure :

$\overline{AB} \parallel \overline{OE}$, $\overline{AD} \cap \overline{BO} = \{C\}$

, $m(\angle A) = 30^\circ$

, $m(\angle E) = 80^\circ$

Find with proof : $m(\angle D)$



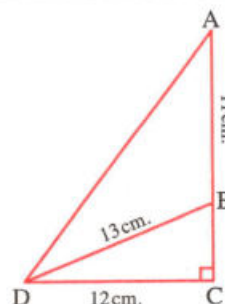
4 [a] In the opposite figure :

ACD is a right-angled triangle at C

, $AB = 11 \text{ cm.}$, $BD = 13 \text{ cm.}$

, $CD = 12 \text{ cm.}$

Find : the length of \overline{BC} and the perimeter of $\triangle ABD$



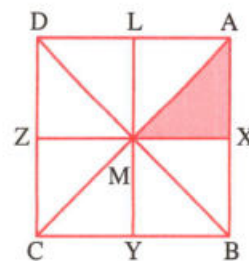
[b] In the opposite figure :

ABCD is a square , X , Y , Z and L are the midpoints of its sides.

Find the image of $\triangle AXM$ in each of the following cases :

[1] By reflection in \overleftrightarrow{LY}

[2] By rotation about M with an angle of measure 90°



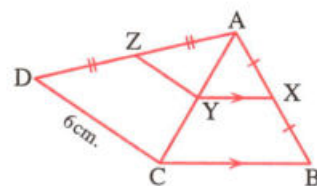
- 5 [a] Using the lattice , draw $\triangle ABC$ where A (2 , 1) , B (2 , 4) and C (4 , 2) , then draw its image by reflection in the X-axis

[b] In the opposite figure :

$\overline{XY} \parallel \overline{BC}$, X is the midpoint of \overline{AB}

, Z is the midpoint of \overline{AD} , $CD = 6$ cm.

Find with proof : the length of \overline{YZ}



7

El-Monofia Governorate



Tala Educational Administration
Mathematics Supervision

Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer from those given :

[1] The measure of each interior angle of the regular hexagon equals

(a) 60° (b) 108° (c) 135° (d) 120°

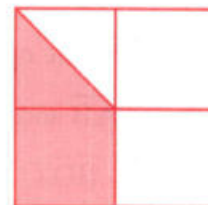
[2] The two diagonals are equal in length and not perpendicular in the

(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

[3] In the opposite figure :

The area of the shaded part from the area of all shape equals

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$
(c) $\frac{3}{8}$ (d) $\frac{3}{4}$



[4] ABC is a right-angled triangle at B , $AB = 6$ cm. , $BC = 8$ cm. , then $AC =$

(a) 10 cm. (b) 28 cm. (c) 100 cm. (d) 160 cm.

[5] The image of the point (3 , 3) by rotation R (O , 180°) is

(a) (-3 , 3) (b) (3 , 3) (c) (3 , -3) (d) (-3 , -3)

[6] The circumference of the circle of radius length 7 cm. equals ($\pi = \frac{22}{7}$)

(a) 11 cm. (b) 44 cm. (c) 22 cm. (d) 88 cm.

2 Complete the following :

1 The image of the point (2 , 1) by reflection in X-axis is

2 In the opposite figure :

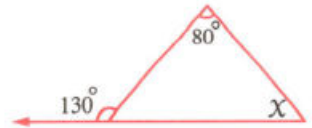
$$x = \dots\dots\dots^\circ$$

3 ABCD is a parallelogram in which $m(\angle A) = 100^\circ$
 , then $m(\angle B) + m(\angle D) = \dots\dots\dots^\circ$

4 The length of the line segment joining the two midpoints of two sides of a triangle is equal to the length of the third side.

5 A rectangle has the dimensions 3 cm. and 4 cm. , then its diagonal length is cm.

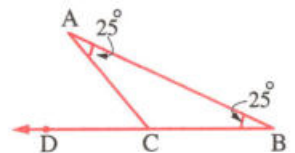
6 The identity rotation has an angle of measure $^\circ$



3 [a] In the opposite figure :

$$m(\angle A) = m(\angle B) = 25^\circ, D \in \overline{BC}$$

Find : $m(\angle ACD)$



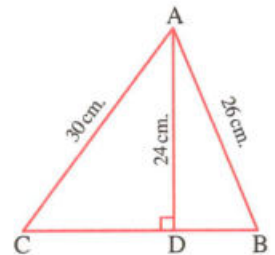
[b] In the opposite figure :

$$\overline{AD} \perp \overline{BC}, AD = 24 \text{ cm.}$$

$$, AB = 26 \text{ cm.}$$

$$, AC = 30 \text{ cm.}$$

Find : the length of \overline{BC}



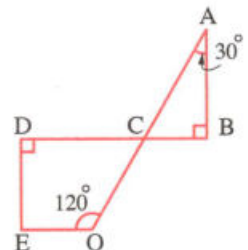
4 [a] In the opposite figure :

$$\overline{AB} \text{ and } \overline{ED} \text{ are perpendicular to } \overline{BD}$$

$$, \overline{BD} \cap \overline{AO} = \{C\}, m(\angle A) = 30^\circ$$

$$, m(\angle O) = 120^\circ$$

Find : $m(\angle E)$

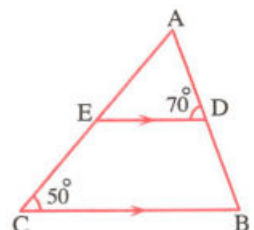


[b] In the opposite figure :

$$\overline{ED} \parallel \overline{CB}, m(\angle C) = 50^\circ$$

$$, m(\angle ADE) = 70^\circ$$

Find : $m(\angle A)$ in degrees with the proof.



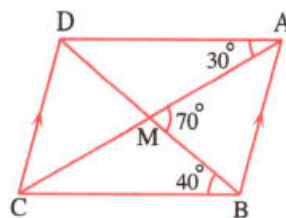
5 [a] In the opposite figure :

$$\overline{AB} \parallel \overline{DC}, \overline{AC} \cap \overline{BD} = \{M\}$$

$$, m(\angle DAC) = 30^\circ, m(\angle DBC) = 40^\circ$$

$$\text{and } m(\angle AMB) = 70^\circ$$

Prove that : ABCD is a parallelogram.



[b] On the square lattice , draw $\triangle ABC$ where $A(1, 1)$, $B(1, 4)$, $C(5, 1)$, then draw the image of $\triangle ABC$ by rotation $R(O, -90^\circ)$

8

El-Dakahlia Governorate



Maths Supervision

Answer the following questions : (Calculator is permitted)

1 Choose the correct answer from those given :

1 The image of $(-1, 3)$ by translation $(4, -2)$ is

- (a) $(3, 1)$ (b) $(3, -1)$ (c) $(5, 5)$ (d) $(5, -5)$

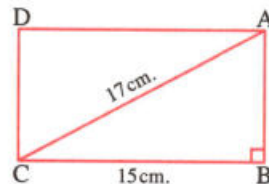
2 The measure of an interior angle of a regular octagon is

- (a) 120° (b) 135° (c) 108° (d) 720°

3 In the opposite figure :

ABCD is a rectangle , its area is cm^2

- (a) 255 (b) 120
(c) 100 (d) 8



4 In the parallelogram ABCD , $m(\angle A) = 2m(\angle B)$, then $m(\angle B) = \dots\dots\dots$

- (a) 60° (b) 120° (c) 180° (d) 30°

5 The measure of the exterior angle of the equilateral triangle equals

- (a) 60° (b) 90° (c) 120° (d) 360°

6 The image of the point $(3, 4)$ by rotation around the origin point with an angle of measure 90° is

- (a) $(-3, 4)$ (b) $(-4, 3)$ (c) $(-3, -4)$ (d) $(4, -3)$

2 Complete each of the following :

1 The image of the point $(-2, -7)$ by reflection in y-axis is

2 The line segment joining the midpoints of two sides of a triangle is to the third side.

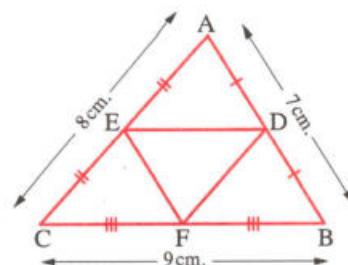
3 The parallelogram with right angle is called

- 4 The number of diagonals of the quadrilateral is
- 5 In any triangle, there are at least two angles.
- 6 The sum of measures of the interior angles of a heptagon equals°

3 [a] In the opposite figure :

ABC is a triangle, D, E, F are the midpoints of \overline{AB} , \overline{AC} , \overline{BC} respectively, $BC = 9$ cm., $AB = 7$ cm., $AC = 8$ cm.

Find by proof : The perimeter of $\triangle DEF$

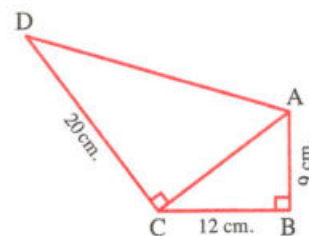


- [b]** On a coordinate plane, draw the rectangle ABCD where A (2, 1), B (6, 1), C (6, 3), D (2, 3), then draw the image of the rectangle ABCD by reflection in X-axis.

4 [a] In the opposite figure :

$m(\angle B) = m(\angle ACD) = 90^\circ$
 $AB = 9$ cm., $BC = 12$ cm.,
 $DC = 20$ cm.

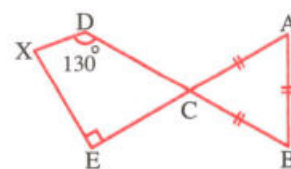
Find by proof : The lengths of \overline{AC} , \overline{AD}



[b] In the opposite figure :

$\triangle ABC$ is an equilateral triangle
 $\overline{BD} \cap \overline{AE} = \{C\}$, $m(\angle D) = 130^\circ$
 $m(\angle E) = 90^\circ$

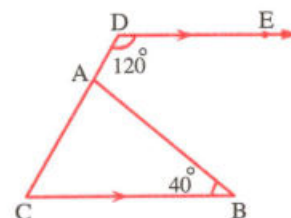
Find by proof : $m(\angle X)$



5 [a] In the opposite figure :

$\overline{DE} \parallel \overline{BC}$
 $m(\angle D) = 120^\circ$
 $m(\angle B) = 40^\circ$, $A \in \overline{CD}$

Find by proof : $m(\angle BAD)$



- [b]** On a square lattice, draw $\triangle ABC$ where A (3, 1), B (5, 2), C (2, 4), then draw its image by rotation $R(O, 180^\circ)$



Answer the following questions :

1 Choose the correct answer :

- 1 The sum of the measures of the interior angles of the triangle equals
 (a) 90° (b) 180° (c) 270° (d) 360°
- 2 The sum of the measures of the interior angles of any polygon which has n sides equals
 (a) $2n$ (b) $(n - 2) \times 180^\circ$ (c) $2n \times 180^\circ$ (d) $2n - 180^\circ$
- 3 The two diagonals are equal in length in the
 (a) trapezium. (b) rhombus. (c) parallelogram. (d) rectangle.
- 4 The image of the point $(2, -1)$ by translation $(3, 1)$ is
 (a) $(5, 0)$ (b) $(0, 5)$ (c) $(0, -1)$ (d) $(-1, 1)$
- 5 The number of diagonals of any pentagon equals
 (a) 3 (b) 4 (c) 5 (d) 6
- 6 The image of the point $(-3, -2)$ by reflection in X -axis is
 (a) $(3, 2)$ (b) $(3, -2)$ (c) $(-3, 2)$ (d) $(-2, -3)$
- 7 The length of the line segment which joins the midpoints of any two sides of a triangle equals the length of the third side.
 (a) quarter (b) third (c) fifth (d) half
- 8 The sum of the measures of the interior angles of a quadrilateral equals
 (a) 90° (b) 360° (c) 270° (d) 180°
- 9 The image of the point $(2, 2)$ by rotation about the origin point with an angle of measure 180° is
 (a) $(2, 2)$ (b) $(-2, -2)$ (c) $(-2, 2)$ (d) $(2, -2)$
- 10 The measure of each interior angle of a regular pentagon equals
 (a) 90° (b) 108° (c) 120° (d) 144°
- 11 ABCD is a parallelogram in which $m(\angle B) = 110^\circ$, then $m(\angle D) =$
 (a) 50° (b) 70° (c) 110° (d) 180°
- 12 If two lines intersect at a point, then each two angles are equal in measure.
 (a) corresponding (b) alternate
 (c) vertically opposite (d) adjacent

13 XYZ is a triangle in which $m(\angle X) = 80^\circ$, $m(\angle Y) = 40^\circ$, then $m(\angle Z) = \dots\dots\dots$

- (a) 60° (b) 90° (c) 20° (d) 50°

14 ABC is a triangle in which $m(\angle B) = 90^\circ$, $AB = 3$ cm., $BC = 4$ cm., then $AC = \dots\dots\dots$ cm.

- (a) 7 (b) 9 (c) 12 (d) 5

15 In any triangle, we have two $\dots\dots\dots$ angles at least.

- (a) right (b) acute (c) obtuse (d) straight

16 In the parallelogram, each two opposite angles are $\dots\dots\dots$

- (a) complementary. (b) reflex. (c) supplementary. (d) equal in measure.

17 The square is a $\dots\dots\dots$ with two diagonals equal in length.

- (a) rhombus. (b) rectangle. (c) trapezium. (d) parallelogram.

18 The quadrilateral with only two opposite parallel sides is called $\dots\dots\dots$

- (a) trapezium. (b) rhombus. (c) square. (d) parallelogram.

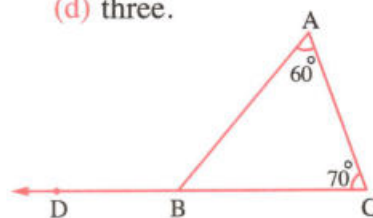
19 The number of symmetry axes of the isosceles triangle is $\dots\dots\dots$

- (a) zero. (b) one. (c) two. (d) three.

20 In the opposite figure :

$D \in \overrightarrow{CB}$, then $m(\angle ABD) = \dots\dots\dots$

- (a) 60° (b) 50°
(c) 130° (d) 70°



21 The image of the point $(-2, -3)$ by the geometrical transformation

$(x, y) \longrightarrow (x + 1, y)$ is $\dots\dots\dots$

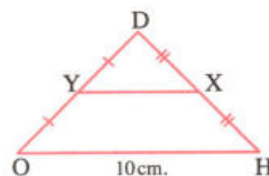
- (a) $(-1, -3)$ (b) $(-3, -1)$ (c) $(-3, 2)$ (d) $(-2, 3)$

2 In the opposite figure :

DHO is a triangle, X is the midpoint of \overline{DH}

, Y is the midpoint of \overline{DO} , $HO = 10$ cm.

Write with proof how to get XY

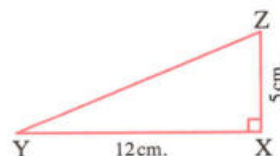


3 In the opposite figure :

$\triangle XYZ$ is right-angled to X

, $XY = 12$ cm., $XZ = 5$ cm.

Find : YZ, write the proof.



4 On a square lattice, draw \overline{AB} where $A(1, 2)$, $B(4, 3)$, then draw its image by reflection in y-axis.

10 Kafr El-Sheikh Governorate

Motobus Administration
General Math Supervision*Answer the following questions :***1 Choose the correct answer :**

- 1 The length of the line segment joining the two midpoints of two sides of a triangle is equal to the length of the third side.
 (a) third (b) twice (c) half (d) quarter
- 2 The image of A (1 , 5) by translation (1 , - 2) is
 (a) (1 , 5) (b) (2 , 3) (c) (3 , 2) (d) (2 , 7)
- 3 The measure of the complementary angle of an angle of measure 70° is
 (a) 20° (b) 50° (c) 110° (d) 130°
- 4 The measure of each interior angle of the regular hexagon equals
 (a) 60° (b) 108° (c) 120° (d) 135°
- 5 If the perimeter of the square = 20 cm. , then its side length = cm.
 (a) 4 (b) 5 (c) 80 (d) 10
- 6 If ABCD is a parallelogram in which $m(\angle B) = 50^\circ$, then $m(\angle C) =$
 (a) 40° (b) 50° (c) 90° (d) 130°

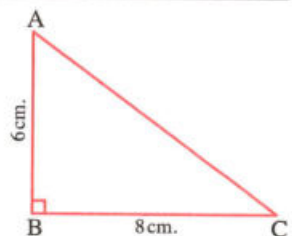
2 Complete :

- 1 The image of (3 , 5) by rotation R (O , 90°) is
- 2 The sum of the measures of the interior angles of a triangle equals $^\circ$
- 3 The pentagon has diagonals.
- 4 The two diagonals are perpendicular and not equal in length in
- 5 The image of the point (- 5 , 0) by reflection in the origin point is
- 6 The ray drawn from the midpoint of a side of a triangle parallel to another side the third side.

3 [a] In the opposite figure :

ABC is a triangle , $m(\angle B) = 90^\circ$
 , AB = 6 cm. , BC = 8 cm.

Find with proof : the length of \overline{AC}

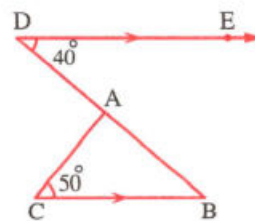


[b] In the opposite figure :

ABC is a triangle, $\overline{CB} \parallel \overline{DE}$

, $m(\angle D) = 40^\circ$, $m(\angle C) = 50^\circ$

Find with proof : **1** $m(\angle B)$ **2** $m(\angle CAB)$

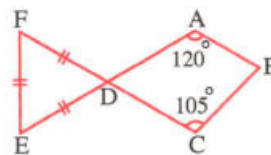


4 [a] In the opposite figure :

$\overline{AE} \cap \overline{CF} = \{D\}$, $\triangle DFE$ is an equilateral triangle

, $m(\angle A) = 120^\circ$, $m(\angle C) = 105^\circ$

Find : $m(\angle B)$

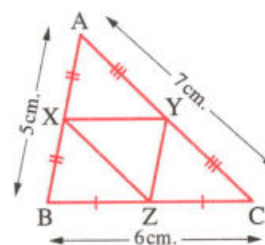


[b] In the opposite figure :

$AB = 5$ cm. , $BC = 6$ cm. , $AC = 7$ cm.

, X, Y, Z are the midpoints of \overline{AB} , \overline{AC} and \overline{BC} respectively

Find with proof : the perimeter of $\triangle XYZ$

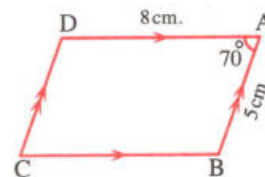


5 [a] In the opposite figure :

ABCD is a parallelogram, $AB = 5$ cm.

, $AD = 8$ cm. , $m(\angle A) = 70^\circ$

Find : **1** $m(\angle B)$ **2** The lengths of \overline{BC} , \overline{CD}



[b] On a square lattice , draw $\triangle ABC$ where A (1 , 3) , B (4 , 1) and C (0 , 1) , then draw the image of $\triangle ABC$ by reflection in the y-axis.

11

Souhag Governorate



**Akhmem Educational Management
Future Generation Language Schools**

Answer the following questions :

1 Choose the correct answer :

1 The parallelogram in which two adjacent sides are equal in length is called a

(a) rectangle. (b) square. (c) rhombus. (d) trapezium.

2 The sum of measures of the interior angles of the hexagon equals

(a) 720° (b) 270° (c) 120° (d) 360°

3 The perimeter of a square is 20 cm. , then its area is cm^2

(a) 5 (b) 25 (c) 100 (d) 16

- 4 If ABCD is a parallelogram in which $m(\angle A) + m(\angle C) = 140^\circ$, then $m(\angle B) = \dots\dots\dots$

(a) 40° (b) 110° (c) 70° (d) 60°

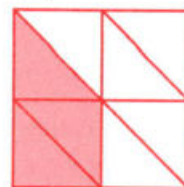
- 5 The image of the point $(-3, 5)$ by rotation about the origin point with an angle of measure 90° is $\dots\dots\dots$

(a) $(5, 3)$ (b) $(-5, 3)$ (c) $(3, 5)$ (d) $(-5, -3)$

- 6 In the opposite figure :

The area of the shaded part = $\dots\dots\dots$ the total area of the shape.

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{5}$



- 2 Complete the following :

- 1 The image of the point $(-5, 2)$ by reflection in X-axis is $\dots\dots\dots$
- 2 The length of the line segment joining the two midpoints of two sides of a triangle is equal to $\dots\dots\dots$ the length of the third side.
- 3 In the triangle ABC , if $m(\angle A) = 50^\circ$, $m(\angle B) = 100^\circ$, then $m(\angle C) = \dots\dots\dots^\circ$
- 4 If $\triangle XYZ$ is a right-angled triangle at X , $XY = 12$ cm. , $XZ = 9$ cm. , then $YZ = \dots\dots\dots$ cm.
- 5 The number of axes of symmetry of a rhombus equals $\dots\dots\dots$
- 6 The measure of the exterior angle of the equilateral triangle is $\dots\dots\dots^\circ$

- 3 [a] In the opposite figure :

ABCD is a parallelogram in which
 $AB = 5$ cm. , $BC = 8$ cm. , $m(\angle B) = 135^\circ$

Find : 1 $m(\angle D)$ 2 $m(\angle C)$

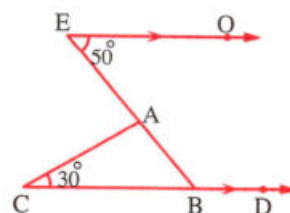
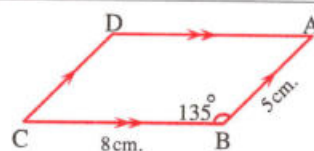
3 The perimeter of the parallelogram ABCD

- [b] In the opposite figure :

$\overrightarrow{EO} \parallel \overrightarrow{CD}$, $m(\angle E) = 50^\circ$, $m(\angle C) = 30^\circ$

Find : 1 The measures of angles of $\triangle ABC$

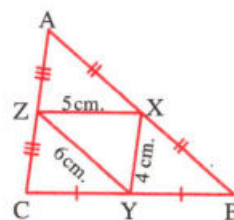
2 $m(\angle ABD)$



- 4 [a] In the opposite figure :

ABC is a triangle in which X , Y , Z are the midpoints of \overline{AB} , \overline{BC} , \overline{CA} respectively
 $XZ = 5$ cm. , $XY = 4$ cm. , $YZ = 6$ cm.

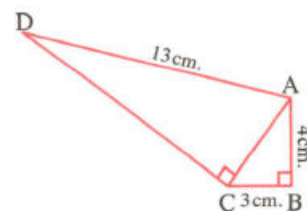
Find with proof : the perimeter of $\triangle ABC$



[b] In the opposite figure :

ABCD is a quadrilateral in which
 $m(\angle B) = m(\angle ACD) = 90^\circ$
 , AB = 4 cm. , BC = 3 cm. and AD = 13 cm.

Find : AC and DC



- 5 [a]** On a square lattice , draw the image of the triangle ABC where A (1 , 1) , B (2 , 3) , C (4 , 1) by reflection in the origin point.

- [b]** A line segment \overline{AB} where A (1 , 1) , B (2 , 3) Find its image by translation
 $(x, y) \longrightarrow (x + 2, y - 3)$

12

Aswan Governorate



Edfo Educational District
 Mathematics Supervision

Answer the following questions :

1 Choose the correct answer from the given ones :

- 1** The measure of the exterior angle of the equilateral triangle equals
 (a) 90° (b) 120° (c) 100° (d) 60°
- 2** The sum of measures of the interior angles of the triangle equals
 (a) 90° (b) 180° (c) 60° (d) 120°
- 3** The identity rotation is with an angle of measure
 (a) 90° (b) 180° (c) 270° (d) 360°
- 4** A square , its side length is 5 cm. , its area is cm^2
 (a) 5 (b) 25 (c) 40 (d) 13
- 5** The number of diagonals of the quadrilateral equals
 (a) 1 (b) 2 (c) 3 (d) 4
- 6** The measure of the straight angle equals
 (a) 90° (b) 180° (c) 360° (d) 270°

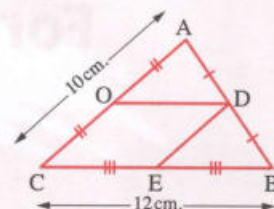
2 Complete the following :

- 1** The image of the point (4 , - 7) by reflection in the y-axis is
- 2** The type of the triangle whose measures of two angles are 50° and 70° is
- 3** In the right-angled triangle , the longest side is
- 4** In $\triangle XYZ$, if $m(\angle Y) > m(\angle X) + m(\angle Z)$, then the type of $\angle Y$ is
- 5** A square is with a right angle.
- 6** The measure of the interior angle of the regular hexagon equals $^\circ$

3 [a] In the opposite figure :

D, E, O are the midpoints of \overline{AB} , \overline{BC} , \overline{AC} respectively
 , $BC = 12$ cm. , $AC = 10$ cm.

Find : the perimeter of the shape DECO



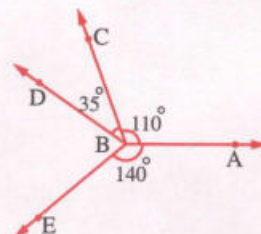
[b] In the opposite figure :

$$m(\angle ABC) = 110^\circ$$

$$, m(\angle CBD) = 35^\circ$$

$$, m(\angle ABE) = 140^\circ$$

Find : $m(\angle EBD)$



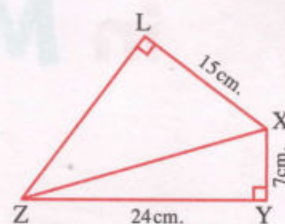
4 [a] In the opposite figure :

XYZL is a quadrilateral

$$, m(\angle Y) = m(\angle L) = 90^\circ$$

$$, XY = 7$$
 cm. , $YZ = 24$ cm. , $XL = 15$ cm.

Find : the lengths of \overline{XZ} , \overline{LZ}



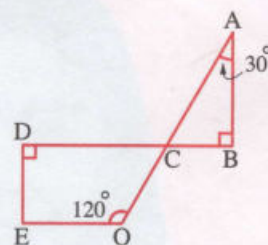
[b] Draw the image of $\triangle XYZ$, where $X(1, 1)$, $Y(3, 4)$, $Z(5, 2)$ by reflection in X-axis.

5 [a] In the opposite figure :

$$\overline{AB} \perp \overline{BD} , \overline{ED} \perp \overline{BD}$$

$$, \overline{BD} \cap \overline{AO} = \{C\} , m(\angle A) = 30^\circ , m(\angle O) = 120^\circ$$

Find : $m(\angle E)$



[b] If $\hat{A}(2, 3)$ is the image of the point A by the translation whose rule is $(x, y) \longrightarrow (x + 2, y + 3)$, **find :** A

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1

Cairo Governorate



El-Waily Educ. Admin
St. Joseph Maronite Language Schools

Answer the following questions :

1 Choose the correct answer :

- 1 If the image of the point $(5, -3)$ by rotation about the origin point is itself, then the measure of the rotation angle is
(a) 180° (b) 90° (c) 270° (d) 360°
- 2 The line segment joining the two midpoints of two sides of a triangle is the third side.
(a) intersecting (b) parallel to (c) perpendicular to (d) congruent to
- 3 The sum of measures of the accumulative angles at a point is
(a) 90 (b) 180 (c) 270 (d) 360
- 4 If ABC is a right-angled triangle at B and $AB = 4$ cm. , $BC = 3$ cm. , then $AC =$ cm.
(a) 16 (b) 25 (c) 9 (d) 5
- 5 The sum of measures of the interior angles of a hexagon is
(a) 360 (b) 540 (c) 720 (d) 120

2 Complete :

- 1 The sum of measures of the exterior angles of the pentagon is $^\circ$
- 2 If two lines intersect , then each two vertically opposite angles are
- 3 In $\triangle ABC$, if $m(\angle A) + m(\angle C) = m(\angle B)$, then $m(\angle B) =$ $^\circ$
- 4 The image of $(2, 3)$ by reflection in the X-axis is
- 5 The length of the line segment joining the two midpoints of two sides of a triangle equals the length of the third side.

3 [a] In the opposite figure :

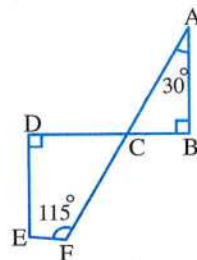
$$\overline{AF} \cap \overline{BD} = \{C\} , \overline{AB} \perp \overline{BC}$$

$$, \overline{CD} \perp \overline{ED}$$

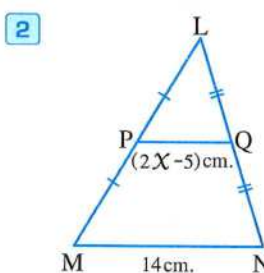
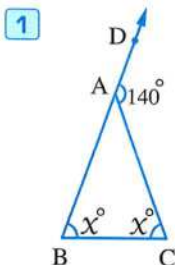
$$, m(\angle A) = 30^\circ$$

$$, m(\angle F) = 115^\circ$$

Find with proof : $m(\angle E)$



[b] In each of the following figures , find with proof the value of X :

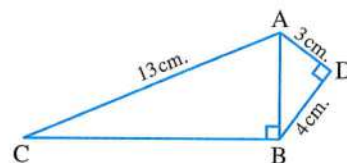


- 4 [a]** If $A(4, 3)$, $B(-1, 1)$, find the image of each of A and B by translation :
 $(X, y) \longrightarrow (X + 2, y - 2)$

[b] In the opposite figure :

$m(\angle ABC) = 90^\circ$, $m(\angle ADB) = 90^\circ$
 $AC = 13$ cm. , $AD = 3$ cm. , $BD = 4$ cm.

Find with proof : The length of each of \overline{AB} and \overline{BC}



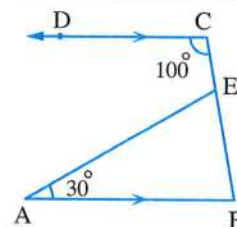
- 5 [a] In the opposite figure :**

$\overline{AB} \parallel \overline{CD}$

, $m(\angle A) = 30^\circ$

, $m(\angle C) = 100^\circ$ and $E \in \overline{BC}$

Find with proof : $m(\angle AEC)$



- [b]** On the square lattice , find the image of $\triangle ABC$ where $A(1, -3)$, $B(3, 3)$ and $C(5, 0)$ by reflection in the y-axis.

2

Cairo Governorate



Hadayek El-Kobba Zone
 El-Nokrashi Official Language School

Answer the following questions :

- 1 Choose the correct answer :**

- 1** The two diagonals are equal in length and perpendicular in the
 (a) trapezium. (b) square. (c) rectangle. (d) parallelogram.
- 2** The image of the point $(-1, 3)$ by translation $(4, -2)$ is
 (a) $(3, 1)$ (b) $(3, -1)$ (c) $(5, 1)$ (d) $(5, -5)$
- 3** The measure of the exterior angle of the equilateral triangle equals
 (a) 60° (b) 100° (c) 120° (d) 150°
- 4** The number of lines of symmetry of the square equals
 (a) 1 (b) 2 (c) 3 (d) 4

- 5 The measure of each angle of the regular pentagon equals
 (a) 108° (b) 100° (c) 90° (d) 60°

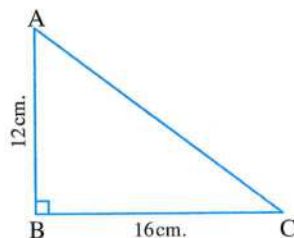
2 Complete each of the following :

- 1 The rhombus with a right angle is
 2 The length of the line segment joining the two midpoints of two sides of a triangle is equal to the length of the third side.
 3 If ABCD is a parallelogram in which $BC = 8$ cm. and $CD = 6$ cm., then its perimeter = cm.
 4 The ray drawn from the midpoint of a side of a triangle parallel to another side
 5 If two adjacent sides are equal in length in a parallelogram , then it becomes

3 [a] In the opposite figure :

ABC is a right-angled triangle at B
 , $AB = 12$ cm. , $BC = 16$ cm.

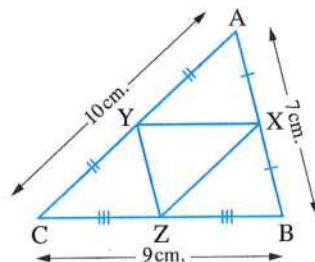
Find with steps : The length of \overline{AC}



[b] In the opposite figure :

X , Y , Z are the midpoints of \overline{AB} , \overline{AC}
 , \overline{BC} respectively , $AB = 7$ cm.
 , $BC = 9$ cm. , $AC = 10$ cm.

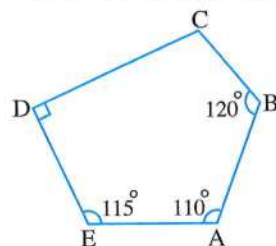
Find : The perimeter of $\triangle XYZ$



4 [a] In the opposite figure :

ABCDE is a pentagon in which :
 $m(\angle A) = 110^\circ$, $m(\angle B) = 120^\circ$
 , $m(\angle E) = 115^\circ$, $m(\angle D) = 90^\circ$

Find with proof : $m(\angle C)$



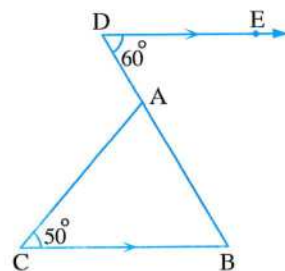
[b] In the opposite figure :

$\overline{DE} \parallel \overline{CB}$

, $m(\angle D) = 60^\circ$

, $m(\angle C) = 50^\circ$

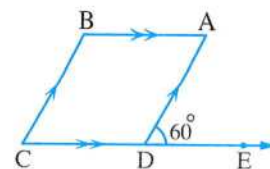
Find with proof : $m(\angle BAC)$



5 [a] In the opposite figure :

ABCD is a parallelogram , $E \in \overrightarrow{CD}$, $m(\angle ADE) = 60^\circ$

Find with proof : $m(\angle B)$



- [b]** Using the square lattice , draw $\triangle ABC$ where $A(1, 3)$, $B(5, 3)$ and $C(3, 6)$, then find the image of $\triangle ABC$ by reflection in the origin point.

3
Giza Governorate

Math Inspection

Answer the following questions :

1 Choose the correct answer :

- [1]** The image of $(3, -5)$ by rotation about the origin point with an angle of measure 270° is
 (a) $(5, 3)$ (b) $(-5, 3)$ (c) $(3, 5)$ (d) $(-5, -3)$
- [2]** The parallelogram whose diagonals are perpendicular and equal in length is called a
 (a) rhombus. (b) square. (c) rectangle. (d) trapezium.
- [3]** The sum of the measures of the interior angles of a hexagon equals
 (a) 270 (b) 180 (c) 720 (d) 360
- [4]** The side length of a rhombus whose perimeter is 8 cm. equals
 (a) 32 cm. (b) 2 cm. (c) 16 cm. (d) 12 cm.
- [5]** Any triangle has at least acute angles.
 (a) zero (b) 1 (c) 2 (d) 3

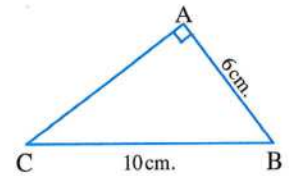
2 Complete :

- [1]** The rectangle is a parallelogram in which one of its angles is
- [2]** If ABCD is a parallelogram in which $m(\angle A) = 60^\circ$, then $m(\angle B) =$
- [3]** The image of the point $(3, 2)$ by reflection in the origin point is
- [4]** If ABCD is a parallelogram in which $BC = 8$ cm. and $CD = 6$ cm. , then its perimeter =
- [5]** The length of the line segment joining the two midpoints of two sides of a triangle is equal to the length of the third side.

3 [a] In the opposite figure :

ABC is a right-angled triangle at A
 , BC = 10 cm. , AB = 6 cm.

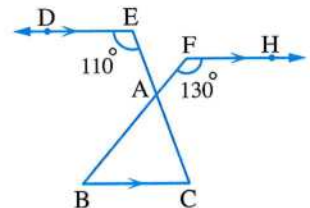
Find : The length of \overline{AC}



[b] In the opposite figure :

$\overrightarrow{ED} \parallel \overrightarrow{BC} \parallel \overrightarrow{FH}$, $m(\angle E) = 110^\circ$
 , $m(\angle F) = 130^\circ$

Find with proof : $m(\angle BAC)$



4 [a] The ratio among the measures of the angles of a quadrilateral is 2 : 2 : 3 : 5

Calculate the measure of the biggest angle.

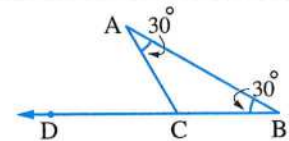
[b] Draw the image of the rectangle XYZL where

X (- 5 , 1) , Y (- 5 , 4) , Z (- 1 , 4) , L (- 1 , 1) by reflection in the X-axis.

5 [a] In the opposite figure :

$m(\angle A) = m(\angle B) = 30^\circ$, $C \in \overline{BD}$

Find with steps : $m(\angle ACD)$

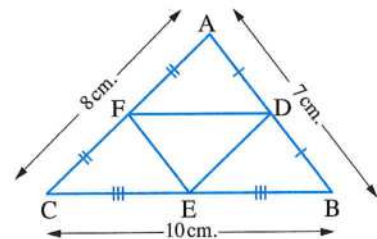


[b] In the opposite figure :

BC = 10 cm. , AB = 7 cm. , AC = 8 cm.

, E , D and F are the midpoints of
 \overline{BC} , \overline{AB} and \overline{AC} respectively

Find with proof : The perimeter of $\triangle FDE$



4

Giza Governorate



Abu El-Nomros Directorate

Answer the following questions :

1 Choose the correct answer :

[1] The image of the point (2 , - 5) by rotation about the origin point with an angle of measure 90° is

- (a) (2 , - 5) (b) (2 , 5) (c) (- 2 , - 5) (d) (5 , 2)

[2] The two diagonals are equal in length and not perpendicular in the

- (a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

[3] The measure of each angle of a regular hexagon equals

- (a) 60° (b) 108° (c) 120° (d) 135°

4 The triangle contains at least two angles.

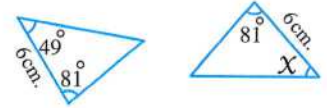
- (a) obtuse (b) right (c) acute (d) reflex

5 In the opposite figure :

If the two triangles are congruent

, then $X = \dots\dots\dots$

- (a) 49° (b) 50° (c) 81° (d) 70°

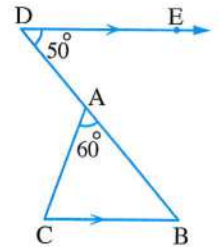


2 Complete each of the following :

- 1 The line segment joining the midpoints of two sides of a triangle is
 2 In $\triangle XYZ$, if $m(\angle Y) = 90^\circ$, then $(XZ)^2 = \dots\dots\dots + \dots\dots\dots$
 3 The image of the point $(5, -3)$ by translation 3 units in the negative direction of the X -axis is
 4 ABCD is a parallelogram in which $m(\angle A) = 60^\circ$, then $m(\angle B) = \dots\dots\dots^\circ$

5 In the opposite figure :

$m(\angle C) = \dots\dots\dots^\circ$



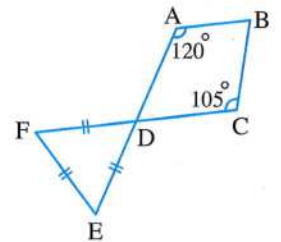
3 [a] In the opposite figure :

$$\overline{AE} \cap \overline{CF} = \{D\}$$

, DEF is an equilateral triangle

, $m(\angle A) = 120^\circ$, $m(\angle C) = 105^\circ$

Find : $m(\angle B)$

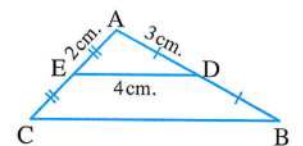


[b] In the opposite figure :

ABC is a triangle in which $AD = 3$ cm., $AE = 2$ cm.

, $ED = 4$ cm., D and E are the midpoints of \overline{AB} and \overline{AC}

Find : The perimeter of the figure DBCE



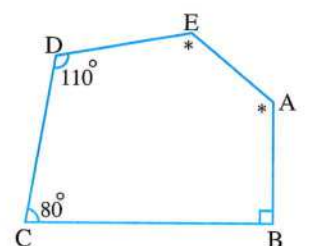
4 [a] In the opposite figure :

$\overline{AB} \perp \overline{BC}$, $m(\angle C) = 80^\circ$

, $m(\angle D) = 110^\circ$

, $m(\angle A) = m(\angle E)$

Find with proof : $m(\angle A)$



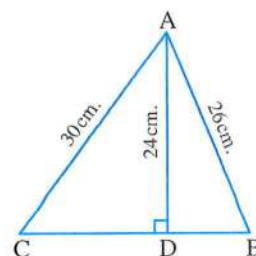
[b] In the opposite figure :

$\overline{AD} \perp \overline{BC}$, $AD = 24$ cm.

, $AB = 26$ cm. , $AC = 30$ cm. ,

Find : **[1]** The length of \overline{BC}

[2] The area of the triangle ABC

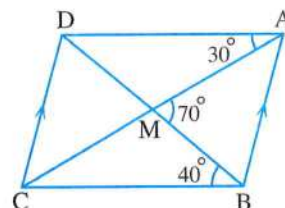


5 [a] In the opposite figure :

$\overline{AB} \parallel \overline{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$, $m(\angle DAC) = 30^\circ$

, $m(\angle DBC) = 40^\circ$, $m(\angle AMB) = 70^\circ$

Prove that : ABCD is a parallelogram.



[b] Using the translation $(x, y) \longrightarrow (x + 2, y + 3)$

, find the image of the point : (2 , 4)

[c] Draw the image of $\triangle ABC$ where $A(1, 1)$, $B(3, 4)$, $C(5, 2)$ by reflection in the X -axis.

5

Alexandria Governorate



East Educational Directorate
Math Supervision (B)

Answer the following questions :

1 Choose the correct answer :

[1] The image of the point (1 , 3) by translation (4 , - 2) is

(a) (3 , 1)

(b) (3 , - 1)

(c) (5 , 1)

(d) (5 , - 5)

[2] The square has line of symmetry.

(a) 1

(b) 2

(c) 3

(d) 4

[3] The sum of the measures of the interior angles of the triangle equals

(a) 90°

(b) 180°

(c) 360°

(d) 108°

[4] The image of the point (- 3 , 4) by rotation about the origin point with an angle of measure 90° is

(a) (4 , 3)

(b) (- 4 , 3)

(c) (3 , 4)

(d) (- 4 , - 3)

[5] The sum of measures of the interior angles of a hexagon is

(a) 180°

(b) 360°

(c) 540°

(d) 720°

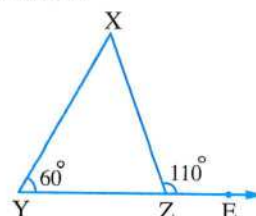
2 Complete :

- 1 The line segment joining the midpoints of two sides of a triangle
- 2 The image of the point $(5, -3)$ by reflection in the y-axis is
- 3 The perimeter of the rhombus whose side length is 8 cm. equals

4 In the opposite figure :

$$m(\angle YXZ) = \dots\dots\dots^\circ$$

- 5 The sum of the measures of the interior angles of the quadrilateral equals

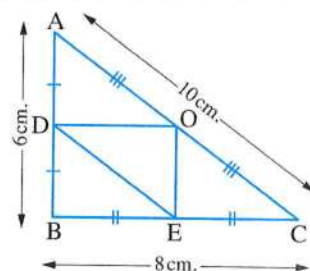


3 [a] In the opposite figure :

D, E, O are the midpoints of \overline{AB} , \overline{BC} , \overline{AC} respectively, $AB = 6$ cm.

, $BC = 8$ cm. , $AC = 10$ cm.

Find : The perimeter of $\triangle DEO$



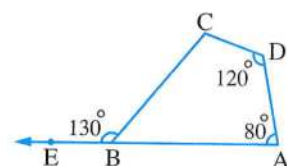
[b] In the opposite figure :

$$m(\angle A) = 80^\circ$$

$$, m(\angle D) = 120^\circ$$

$$, m(\angle CBE) = 130^\circ, B \in \overline{AE}$$

Find : $m(\angle C)$



- 4 [a] Using the lattice, draw $\triangle ABC$ where $A(2, 0)$, $B(0, 3)$, $C(-3, 2)$, then draw its image by reflection in the X-axis.

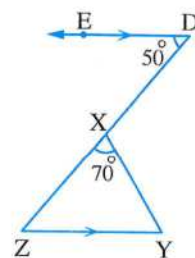
[b] In the opposite figure :

$$\overline{DE} \parallel \overline{YZ}$$

$$, m(\angle ZDE) = 50^\circ$$

$$, m(\angle YXZ) = 70^\circ, X \in \overline{DZ}$$

Find with proof : $m(\angle Z)$, $m(\angle Y)$, $m(\angle YXD)$



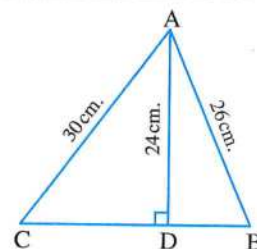
5 [a] In the opposite figure :

$$\overline{AD} \perp \overline{BC}, AD = 24 \text{ cm.}$$

$$, AB = 26 \text{ cm.}, AC = 30 \text{ cm.}$$

Find : 1 The length of \overline{BC}

2 The area of $\triangle ABC$



- [b] Using the square lattice, draw $\triangle ABC$ where $A(5, 4)$, $B(-2, 1)$, $C(1, 6)$, then find its image by the translation $(2, -2)$



Answer the following questions :

1 Choose the correct answer :

- 1 The rhombus whose two diagonals are equal in length is called a
 (a) square. (b) rectangle. (c) parallelogram. (d) trapezium.
- 2 The image of the point (1 , 5) by rotation about the origin point by an angle of measure 90° is the point
 (a) (5 , 1) (b) (-1 , 5) (c) (-5 , 1) (d) (1 , -5)
- 3 The sum of measures of the accumulative angles at a point equals $^\circ$
 (a) 360 (b) 180 (c) 120 (d) 90
- 4 The number of axes of symmetry of the isosceles triangle is
 (a) 1 (b) 2 (c) 3 (d) 4
- 5 In $\triangle ABC$, if $m(\angle A) = 50^\circ$ and $m(\angle B) = 100^\circ$, then $m(\angle C) =$ $^\circ$
 (a) 100 (b) 80 (c) 50 (d) 30

2 Complete each of the following :

- 1 The quadrilateral whose two diagonals bisect each other is called
- 2 The ray drawn from the midpoint of a side of a triangle parallel to another side is
- 3 The measure of the identity rotation angle is $^\circ$
- 4 The image of the point (3 , -2) by translation $(X - 1 , y + 6)$ is
- 5 The triangle has two angles at least.

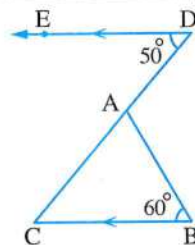
3 [a] In the opposite figure :

$$\overrightarrow{DE} \parallel \overrightarrow{CB}$$

$$, m(\angle D) = 50^\circ$$

$$\text{and } m(\angle B) = 60^\circ$$

Find : $m(\angle C)$ and $m(\angle BAC)$ (Show the steps)



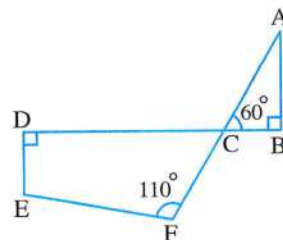
[b] In the opposite figure :

$$\overline{AF} \cap \overline{BD} = \{C\} , m(\angle B) = m(\angle D) = 90^\circ$$

$$, m(\angle ACB) = 60^\circ$$

$$\text{and } m(\angle F) = 110^\circ$$

Find with proof : $m(\angle A)$ and $m(\angle E)$

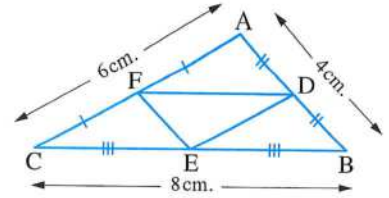


4 [a] In the opposite figure :

D is the midpoint of \overline{AB} , E is the midpoint of \overline{BC} , and F is the midpoint of \overline{AC}

, $AB = 4$ cm. , $BC = 8$ cm. and $AC = 6$ cm.

Find : The perimeter of $\triangle DEF$



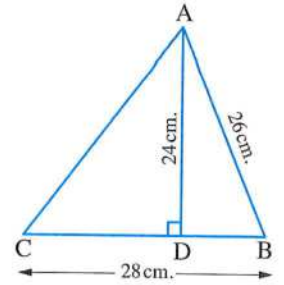
[b] In the opposite figure :

$\overline{AD} \perp \overline{BC}$, $AB = 26$ cm.

, $AD = 24$ cm. and $BC = 28$ cm.

Find :

The length of each of \overline{BD} and \overline{AC}



- 5 [a]** In the coordinates plane , draw $\triangle ABC$ where $A(2, 0)$, $B(3, 2)$, $C(1, 4)$, then find its image by reflection in the y-axis.

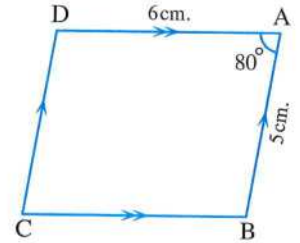
[b] In the opposite figure :

ABCD is a parallelogram

where $m(\angle A) = 80^\circ$

, $AB = 5$ cm. and $AD = 6$ cm.

Find : $m(\angle B)$, $m(\angle C)$ and the perimeter of the parallelogram.



7

El-Sharkia Governorate



East Zagazig Educational Administration
Ahmed Waheed Language School

Answer the following questions :

1 Choose the correct answer :

- 1** The sum of the measures of the interior angles of a triangle equals°
(a) 90 (b) 270 (c) 180 (d) 360
- 2** The quadrilateral in which only two sides are parallel is the
(a) parallelogram. (b) rhombus. (c) rectangle. (d) trapezium.
- 3** ABCD is a parallelogram. If $m(\angle C) = 130^\circ$, then $m(\angle A) = \dots\dots\dots^\circ$
(a) 130 (b) 50 (c) 65 (d) 40
- 4** ABC is a right-angled triangle at B , then $(AB)^2 = \dots\dots\dots$
(a) $(AC)^2 - (BC)^2$ (b) $(AC)^2 = (BC)^2$ (c) $(AC)^2 + (BC)^2$ (d) non of them.

- 5 The image of the point $(-1, 3)$ by translation $(-3, 0)$ is

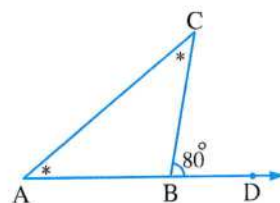
(a) $(-2, 3)$ (b) $(-4, 3)$ (c) $(-4, 0)$ (d) $(-1, 0)$

2 Complete each of the following :

- 1 The ray drawn from the midpoint of a side of a triangle parallel to another side the third side.

- 2 In the opposite figure :

$m(\angle C) = \dots\dots\dots^\circ$



- 3 The sum of measures of the accumulative angles at a point equals $^\circ$

- 4 is a rectangle whose two diagonals are perpendicular.

- 5 The image of the point $(5, -4)$ by rotation about the origin point with an angle of measure 90° is

- 3 [a] Draw the triangle ABC where $A = (1, 1)$, $B = (4, 1)$ and $C = (3, -1)$

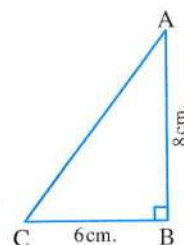
, then find its image by rotation about the origin point with an angle of measure 180°

- [b] In the opposite figure :

ABC is a right-angled triangle at B

, $BC = 6$ cm. , $AB = 8$ cm.

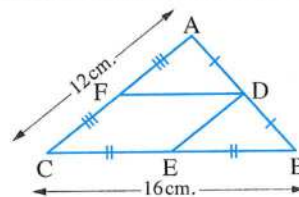
Find : The length of \overline{AC}



- 4 [a] In the opposite figure :

ABC is a triangle in which D, E and F are the midpoints of \overline{AB} , \overline{BC} and \overline{CA} respectively, $BC = 16$ cm. , $AC = 12$ cm.

Find : The perimeter of the quadrilateral DECF with proof.



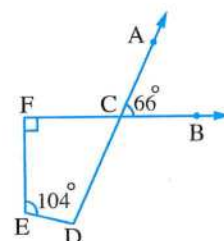
- [b] In the opposite figure :

$\overline{EF} \perp \overline{FB}$, $\overline{DA} \cap \overline{FB} = \{C\}$

, $m(\angle ACB) = 66^\circ$

, $m(\angle E) = 104^\circ$

Find : $m(\angle D)$

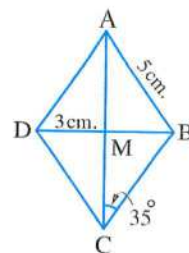


5 [a] In the opposite figure :

ABCD is a rhombus in which $m(\angle ACB) = 35^\circ$
 $\overline{AC} \cap \overline{BD} = \{M\}$, $AB = 5$ cm. , $DM = 3$ cm.

Find : **1** $m(\angle DCA)$

2 The perimeter of $\triangle ABD$



[b] On a square lattice , draw $\triangle ABC$ where $A(1, 1)$, $B(4, 1)$, $C(4, 4)$, then draw its image by reflection in the X -axis.

8 El-Monofia Governorate



Tala Educational Administration
 Mathematics Orientation

Answer the following questions :

1 Choose the correct answer from the given ones :

1 The measure of the interior angle of a regular pentagon is

- (a) 60° (b) 90° (c) 120° (d) 108°

2 The image of the point $(3, 3)$ by rotation $R(O, 90^\circ)$ is

- (a) $(-3, 3)$ (b) $(3, 3)$ (c) $(3, -3)$ (d) $(-3, -3)$

3 In the triangle XYZ , if $m(\angle Y) = 90^\circ$, then $(XY)^2 + (YZ)^2 =$

- (a) $(XY)^2$ (b) $(YZ)^2$ (c) $(XZ)^2$ (d) XZ

4 If the diagonals of a parallelogram are perpendicular , then the shape is

- (a) rectangle. (b) square. (c) rhombus. (d) trapezium.

5 The length of the line segment joining the midpoints of two sides of a triangle equals the length of the third side.

- (a) fourth (b) half (c) third (d) fifth

2 Complete the following statements :

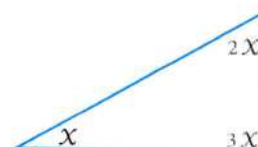
1 The identity rotation has an angle of measure $^\circ$

2 The measure of the exterior angle of a triangle is equal to of the measures of the two interior angles not adjacent to it.

3 The image of the point $(3, 5)$ by the translation $(2, -1)$ is

4 In the opposite figure :

The value of $X =$



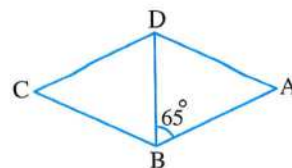
- 5 A rectangle has the dimensions 3 cm. and 4 cm.
 , then its diagonal length is cm.

3 [a] In the opposite figure :

ABCD is a rhombus

, $m(\angle ABD) = 65^\circ$

Find : $m(\angle A)$ with the proof.



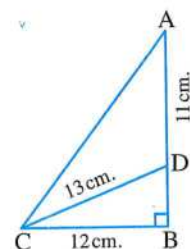
[b] In the opposite figure :

$m(\angle B) = 90^\circ$, $D \in \overline{AB}$

, $AD = 11$ cm.

, $BC = 12$ cm. , $DC = 13$ cm.

Find : The length of each of \overline{DB} and \overline{AC} with the proof.



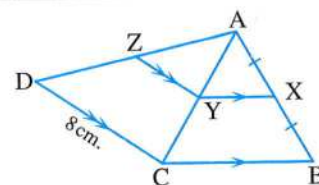
4 [a] In the opposite figure :

X is the midpoint of \overline{AB} , $\overline{XY} \parallel \overline{BC}$

, $\overline{YZ} \parallel \overline{CD}$, $CD = 8$ cm.

1 **Prove that :** Y is the midpoint of \overline{AC}

2 **Find :** The length of \overline{YZ}

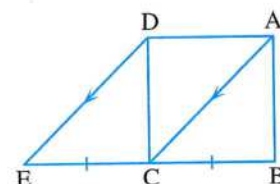


[b] In the opposite figure :

ABCD is a square , $E \in \overline{BC}$

, $BC = CE$, $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.



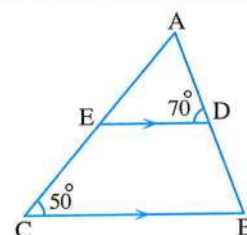
5 [a] In the opposite figure :

$\overline{ED} \parallel \overline{CB}$

, $m(\angle C) = 50^\circ$

, $m(\angle ADE) = 70^\circ$

Find : $m(\angle A)$ in degrees with the proof.



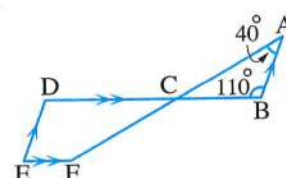
[b] In the opposite figure :

$\overline{BD} \cap \overline{AF} = \{C\}$, $m(\angle B) = 110^\circ$

, $m(\angle A) = 40^\circ$

, $\overline{BD} \parallel \overline{EF}$, $\overline{AB} \parallel \overline{DE}$

Find with the proof : $m(\angle CFE)$



9

El-Gharbia Governorate

Central Maths Supervision
Tanta Modern School

Answer the following questions :

1 Complete :

- 1 The ray drawn parallel to one side of a triangle and passing through the midpoint of another side
- 2 The measure of each exterior angle of a regular hexagon equals°
- 3 The square is with a right angle.
- 4 The number of axes of symmetry of a rectangle equals
- 5 The number of sides of a regular polygon if the measure of its exterior angle equals 120° is sides.

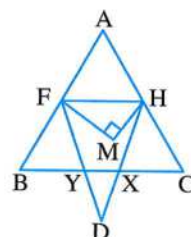
2 Choose the correct answer :

- 1 The image of the point $(3, -1)$ by translation one unit in the positive direction of y-axis followed by reflection in y-axis is
 - (a) $(1, 3)$
 - (b) $(-3, 0)$
 - (c) $(3, 1)$
 - (d) $(3, 0)$
- 2 The number of diagonals of the pentagon is
 - (a) 3
 - (b) 5
 - (c) 7
 - (d) 9
- 3 The parallelogram which has two adjacent perpendicular sides is a
 - (a) square.
 - (b) trapezium.
 - (c) rectangle.
 - (d) rhombus.
- 4 Any triangle has at least acute angles.
 - (a) 1
 - (b) 2
 - (c) 3
 - (d) 4
- 5 If two lines intersect , then each two vertically opposite angles are
 - (a) supplementary.
 - (b) complementary.
 - (c) equal in measure.
 - (d) parallel.

3 [a] In the opposite figure :

F , H , X , Y are the midpoints
of \overline{AB} , \overline{AC} , \overline{DH} , \overline{DF} respectively.
 $BC = 20$ cm.
 $m(\angle FMH) = 90^\circ$, $MH = 6$ cm.

Find with proof : FM , YX



- [b] On a square lattice , draw the image of the square ABCD where ,
A (1 , 1) , B (3 , 1) , C (3 , 3) , D (1 , 3) by reflection in the y-axis.

4 [a] In the opposite figure :

ABCD is a quadrilateral in which

$$m(\angle A) = 90^\circ$$

Find : The value of x

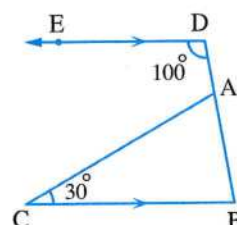
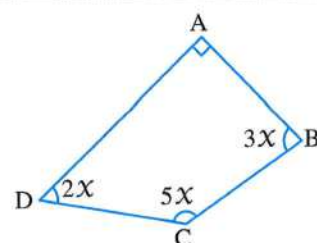
[b] In the opposite figure :

$$\overrightarrow{DE} \parallel \overrightarrow{BC}$$

$$, m(\angle D) = 100^\circ$$

$$, m(\angle C) = 30^\circ \text{ and } A \in \overline{DB}$$

Find : $m(\angle BAC)$



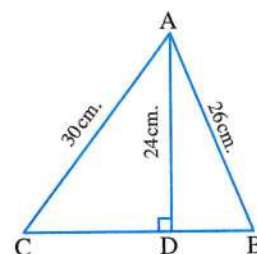
5 [a] In the opposite figure :

$$\overline{AD} \perp \overline{BC} , AD = 24 \text{ cm.}$$

$$, AB = 26 \text{ cm.} , AC = 30 \text{ cm.}$$

Find : 1 The length of \overline{BC}

2 The area of $\triangle ABC$

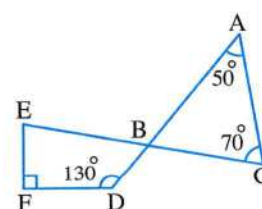


[b] In the opposite figure :

$$\overline{CE} \cap \overline{AD} = \{B\} , m(\angle A) = 50^\circ , m(\angle C) = 70^\circ$$

$$, m(\angle D) = 130^\circ \text{ and } m(\angle F) = 90^\circ$$

Find : $m(\angle E)$



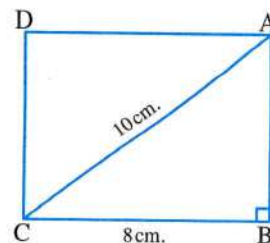
Answer the following questions :

1 Complete each of the following :

- 1 The square is with a right angle.
- 2 The image of the point $(-1, 3)$ by translation $(4, -2)$ is
- 3 The line segment joining the midpoints of two sides of a triangle is to the third side.
- 4 The measure of any of the exterior angles of an equilateral triangle equals°

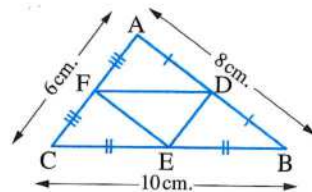
5 In the opposite figure :

ABCD is a rectangle
 , its area is cm^2



2 [a] In the opposite figure :

ABC is a triangle , D , E , F are the midpoints
 of \overline{AB} , \overline{BC} , \overline{CA} respectively
 , AB = 8 cm. , BC = 10 cm. , CA = 6 cm.



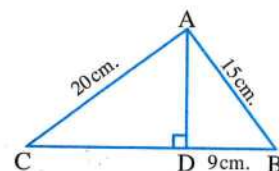
Find by proof : The perimeter of $\triangle DEF$

[b] Using the lattice , draw $\triangle ABC$ where A (1 , 0) , B (0 , 2) and C (− 3 , 1)
 , then draw its image by reflection in the X-axis.

3 [a] In the opposite figure :

$\overline{AD} \perp \overline{BC}$, AB = 15 cm.
 , BD = 9 cm. , AC = 20 cm.

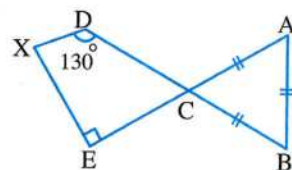
Find by proof : The length of each of \overline{AD} , \overline{DC}



[b] In the opposite figure :

$\triangle ABC$ is an equilateral triangle , $\overline{BD} \cap \overline{AE} = \{C\}$
 , $m(\angle D) = 130^\circ$, $m(\angle E) = 90^\circ$

Find by proof : $m(\angle X)$



4 Choose the correct answer from those given :

1 The image of the point (2 , − 5) by reflection in the y-axis is

- (a) (2 , − 5) (b) (2 , 5) (c) (− 2 , − 5) (d) (5 , 2)

2 The sum of measures of the interior angles of a pentagon equals $^\circ$

- (a) 180 (b) 360 (c) 540 (d) 720

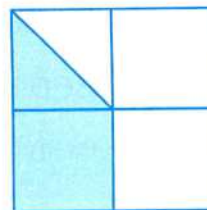
3 ABCD is a parallelogram , if $m(\angle D) = 110^\circ$, then $m(\angle C) =$

- (a) 40 (b) 70 (c) 110 (d) 180

4 In the opposite figure :

The area of the shaded part = the total area of the shape.

- (a) $\frac{1}{2}$ (b) $\frac{3}{4}$
(c) $\frac{3}{8}$ (d) $\frac{3}{5}$



5 The image of the point (3 , - 4) by rotation around the origin point with an angle of measure 90° is

- (a) (- 3 , 4) (b) (4 , 3) (c) (- 3 , - 4) (d) (3 , 4)

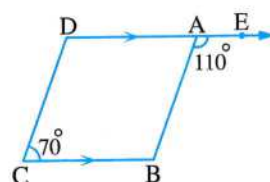
5 [a] In the opposite figure :

$$E \in \overrightarrow{DA}, \overrightarrow{DA} \parallel \overrightarrow{CB}$$

$$, m(\angle EAB) = 110^\circ$$

$$, m(\angle C) = 70^\circ$$

Prove that : ABCD is a parallelogram



[b] Using the lattice , draw the line segment \overline{AB} where A (5 , 0) , B (0 , 4) , then find its image by rotation about the origin point with an angle of measure $(- 90^\circ)$.

11

Port Said Governorate



Educational Directorate

Answer the following questions :

1 Choose the correct answer from those given :

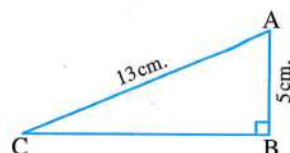
- 1** The sum of measures of the interior angles of a triangle equals
(a) 90° (b) 360° (c) 180° (d) 540°
- 2** The measure of each angle of a regular hexagon equals
(a) 720° (b) 120° (c) 135° (d) 90°
- 3** The image of the point (2 , - 3) by reflection in the X-axis is the point
(a) (- 2 , - 3) (b) (2 , 3) (c) (- 2 , 3) (d) (- 3 , 2)
- 4** The diagonals are equal in length and perpendicular in the
(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.
- 5** The measure of the exterior angle of the equilateral triangle equals
(a) 30° (b) 45° (c) 60° (d) 120°

2 Complete each of the following :

- 1 The number of diagonals of a pentagon is
- 2 The image of the point (2 , 5) by translation (1 , 2) is the point
- 3 The ray drawn parallel to one side of a triangle and passing through the midpoint of another side the third side.
- 4 The image of the point (1 , 7) by rotation about the origin point with an angle of measure 180° is the point

5 In the opposite figure :

$m(\angle B) = 90^\circ$, $AB = 5$ cm. , $AC = 13$ cm.
 , then $BC =$


3 [a] In the opposite figure :

$\overline{EC} \cap \overline{FA} = \{D\}$, $m(\angle A) = 85^\circ$, $m(\angle C) = 110^\circ$
 , $m(\angle F) = 95^\circ$, $m(\angle E) = 30^\circ$

Find : $m(\angle B)$

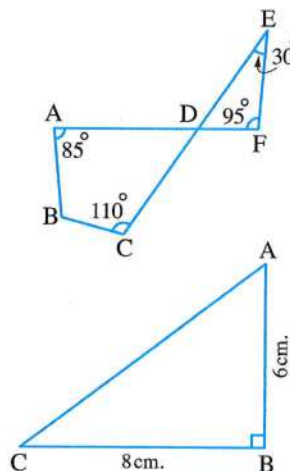
[b] In the opposite figure :

$m(\angle B) = 90^\circ$

, $AB = 6$ cm.

, $BC = 8$ cm.

Find : The length of \overline{AC}


4 [a] In the opposite figure :

$\overline{DE} \parallel \overline{BC}$

, $m(\angle D) = 55^\circ$

, $m(\angle BAC) = 85^\circ$

Find : $m(\angle C)$, $m(\angle B)$

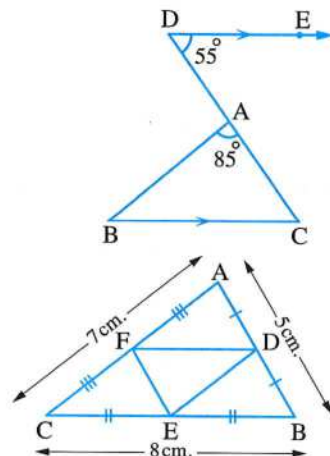
[b] In the opposite figure :

$AB = 5$ cm. , $BC = 8$ cm. , $AC = 7$ cm.

, D , E and F are the midpoints of \overline{AB}

, \overline{BC} and \overline{AC} respectively.

Find : The perimeter of $\triangle DEF$



- 5 [a]** On the square lattice , draw the triangle ABC where A (1 , 1) , B (5 , 1) , C (3 , 5)
 , then draw the image of $\triangle ABC$ by reflection in the X-axis.

- [b]** On the square lattice , draw \overline{AB} where A (4 , 3) , B (-1 , 1) , then find the image of \overline{AB} by translation (2 , -1)



Answer the following questions :

1 Complete the following :

- 1 The image of the point $(2, 3)$ by translation $(X - 3, y - 2)$ is
- 2 The measure of each angle of a regular hexagon equals°
- 3 ABC is a right-angled triangle at B , $AB = 6$ cm. , $BC = 8$ cm. , then $AC =$ cm.
- 4 The image of the point $(2, 1)$ by reflection in the X -axis is
- 5 The number of axes of symmetry of an isosceles triangle is

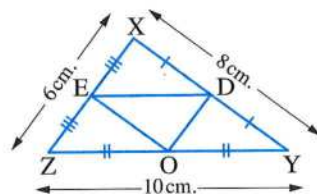
2 Choose the correct answer from those given :

- 1 The measure of the exterior angle of an equilateral triangle is°
(a) 30 (b) 45 (c) 60 (d) 120
- 2 The diagonals are equal in length and not perpendicular in the
(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.
- 3 The edge length of a cube whose volume is 27 cm^3 is cm.
(a) 9 (b) 3 (c) 27 (d) 6
- 4 The length of the line segment joining the midpoints of two sides of a triangle equals the third side.
(a) half (b) twice (c) fourth (d) third
- 5 ΔABC is right-angled at B , $m(\angle A) = 50^\circ$, then $m(\angle C) =$ °
(a) 90 (b) 50 (c) 40 (d) 60

3 [a] In the opposite figure :

XYZ is a triangle in which : $XY = 8$ cm. , $XZ = 6$ cm. , $YZ = 10$ cm. , D , O , E are the midpoints of \overline{XY} , \overline{YZ} , \overline{XZ} respectively.

Find with proof : The perimeter of ΔDOE



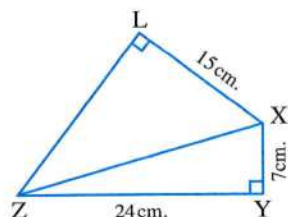
[b] In the opposite figure :

XYZL is a quadrilateral in which

$m(\angle Y) = m(\angle L) = 90^\circ$

, $XY = 7$ cm. , $YZ = 24$ cm. , $XL = 15$ cm.

Find : The length of each of \overline{XZ} and \overline{LZ}



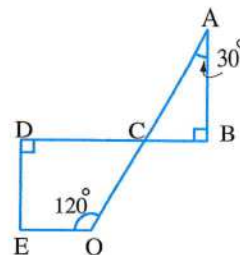
4 [a] In the opposite figure :

\overline{AB} and \overline{ED} are perpendicular to \overline{BD}

$$\overline{BD} \cap \overline{AO} = \{C\}$$

$$m(\angle A) = 30^\circ, m(\angle O) = 120^\circ$$

Find : $m(\angle E)$



- [b]** Draw the triangle ABC and its image $\hat{A} \hat{B} \hat{C}$ where $A(1, 1)$, $B(3, 4)$, $C(5, 2)$ by reflection in the X -axis.

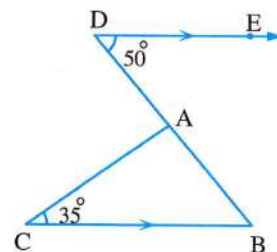
5 [a] In the opposite figure :

$$m(\angle D) = 50^\circ$$

$$m(\angle C) = 35^\circ$$

$$\overrightarrow{DE} \parallel \overrightarrow{BC}$$

Find : $m(\angle B)$ and $m(\angle BAC)$



- [b]** On the square lattice, draw the triangle whose vertices are $A(4, 4)$, $B(4, 2)$, $C(1, 2)$, then find its image by the translation $(X - 4, y - 2)$

13 El-Menia Governorate

Samalout Directorate
El-Ahd El-Geded Language Schools

Answer the following questions :

1 Choose the correct answer :

- [1]** The image of the point $(-1, 3)$ by translation $(4, -2)$ is
 (a) $(3, 1)$ (b) $(3, -1)$ (c) $(5, 1)$ (d) $(5, -5)$
- [2]** The measure of the exterior angle of the equilateral triangle equals °
 (a) 60 (b) 45 (c) 30 (d) 120
- [3]** In a parallelogram, if the adjacent sides are equal in length, then the shape is
 (a) trapezium. (b) rhombus. (c) square. (d) rectangle.
- [4]** The number of the diagonals of a pentagon is
 (a) 3 (b) 5 (c) 7 (d) 9
- [5]** Any triangle has at least acute angles.
 (a) zero (b) 1 (c) 2 (d) 3

2 Complete each of the following :

- [1]** The sum of measures of the interior angles of a triangle equals°
- [2]** The line segment joining the midpoints of two sides of a triangle is the third side.

- 3 If ΔXYZ is a right-angled triangle at X , $XY = 12$ cm. and $XZ = 9$ cm.
 , then $YZ = \dots\dots\dots$ cm.
- 4 The image of the point $(-1, 2)$ by rotation about the origin point with an angle of measure 90° is $\dots\dots\dots$
- 5 The sum of the measures of the accumulative angles at a point equals $\dots\dots\dots^\circ$

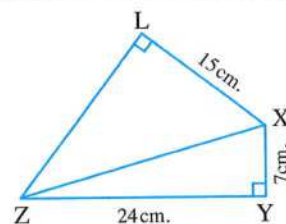
3 [a] In the opposite figure :

$XYZL$ is a quadrilateral in which :

$$m(\angle Y) = m(\angle L) = 90^\circ$$

, $XY = 7$ cm. , $YZ = 24$ cm. , $XL = 15$ cm.

Find : The length of each of \overline{XZ} and \overline{LZ}



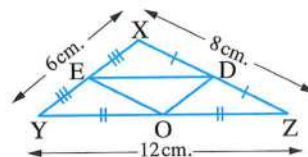
[b] In the opposite figure :

XYZ is a triangle in which $XY = 6$ cm.

, $XZ = 8$ cm. , $YZ = 12$ cm.

, E, D, O are the midpoints of \overline{XY} , \overline{XZ} , \overline{YZ} respectively.

Find : The perimeter of ΔEDO



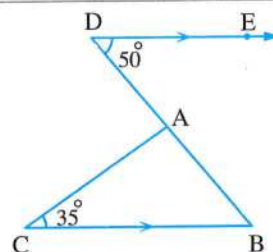
4 [a] In the opposite figure :

$$\overrightarrow{DE} \parallel \overrightarrow{CB}$$

$$, m(\angle D) = 50^\circ$$

$$, m(\angle C) = 35^\circ$$

Find : $m(\angle B)$ and $m(\angle BAC)$

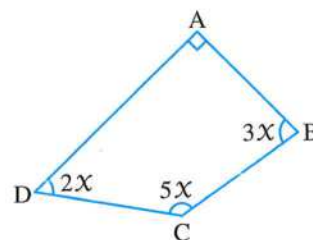


[b] In the opposite figure :

$$m(\angle A) = 90^\circ$$

Find :

The value of X

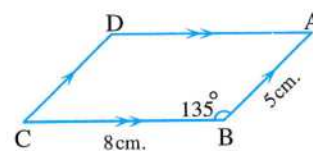


5 [a] In the opposite figure :

$ABCD$ is a parallelogram in which $AB = 5$ cm.

, $BC = 8$ cm. , $m(\angle B) = 135^\circ$

Find : 1 $m(\angle C)$ 2 The perimeter of the parallelogram $ABCD$



- [b] On the square grid , draw ΔABC where $A(1, 1)$, $B(4, 1)$, $C(4, 4)$
 , then find the image of ΔABC by reflection in the X -axis.

14

Assiut Governorate

Administration of Distinguished
& Governmental Language Schools

Answer the following questions :

1 Choose the correct answer from those given :

- 1 The image of the point $(3, -2)$ by reflection in the y -axis is
 (a) $(3, 2)$ (b) $(-3, -2)$ (c) $(-3, 2)$ (d) $(-2, 3)$
- 2 The diagonals are equal in length and perpendicular in the
 (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
- 3 If the image of $(5, -3)$ by rotation about the origin point is itself, then the measure of rotation angle is
 (a) 90° (b) 180° (c) 270° (d) 360°
- 4 The measure of each angle of the regular hexagon equals
 (a) 60° (b) 108° (c) 120° (d) 135°
- 5 The angle whose measure is 135° is
 (a) obtuse. (b) right. (c) acute. (d) straight.

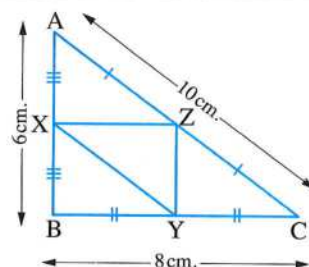
2 Complete each of the following :

- 1 The measure of the exterior angle of the equilateral triangle is $^\circ$
- 2 In $\triangle XYZ$, if $m(\angle Y) = 90^\circ$, then $(XZ)^2 = \dots + \dots$
- 3 The ray drawn from the midpoint of a side of a triangle parallel to another side
- 4 The length of the line segment that joins the two midpoints of two sides of a triangle equals the length of the third side.
- 5 The sum of the measures of the interior angles of a triangle equals $^\circ$

3 [a] In the opposite figure :

X, Y, Z are the midpoints of $\overline{AB}, \overline{BC}, \overline{AC}$ respectively
 $, AB = 6 \text{ cm.}$
 $, BC = 8 \text{ cm.}$
 $, AC = 10 \text{ cm.}$

Find : The perimeter of $\triangle XYZ$

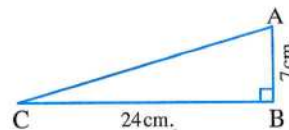


- [b]** Draw on a graph paper $\triangle ABC$ in which $A(1, 1), B(3, 4), C(5, 2)$
 , then draw its image by reflection in the X -axis

4 [a] In the opposite figure :

ABC is a triangle in which
 $m(\angle B) = 90^\circ$, $AB = 7$ cm. , $BC = 24$ cm.

Find : The length of \overline{AC}

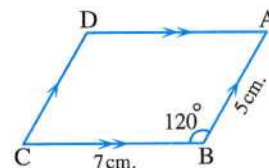


[b] In the opposite figure :

ABCD is a parallelogram , $AB = 5$ cm.
 , $BC = 7$ cm. , $m(\angle B) = 120^\circ$

Find : 1 $m(\angle C)$, $m(\angle D)$

2 The length of each of \overline{AD} and \overline{DC}



5 [a] Using the translation $(X, y) \longrightarrow (X + 3, y - 2)$, find the image of the point $(-3, 5)$

[b] In the opposite figure :

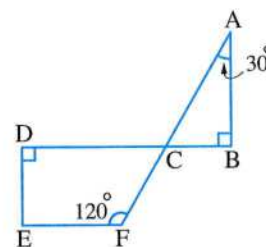
$\overline{AB} \perp \overline{BC}$, $\overline{BD} \cap \overline{AF} = \{C\}$

, $m(\angle A) = 30^\circ$, $m(\angle D) = 90^\circ$

, $m(\angle F) = 120^\circ$

Find with proof : 1 $m(\angle ACB)$

2 $m(\angle E)$



15

Luxor Governorate



Directorate of Education
 Maths Supervision

Answer the following questions : (Calculator is permitted)

1 Choose the correct answer :

1 The sum of the measures of the exterior angles of the triangle is

- (a) 90° (b) 180° (c) 360° (d) 270°

2 The diagonals are equal in length and not perpendicular in the

- (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

3 The measure of each angle of the regular pentagon equals

- (a) 60° (b) 108° (c) 120° (d) 135°

4 The image of the point $(3, 5)$ by rotation about the origin point is itself , then the measure of the angle of rotation is

- (a) 90° (b) 180° (c) 270° (d) 360°

5 The triangle has at least two angles.

- (a) acute (b) right (c) obtuse (d) reflex

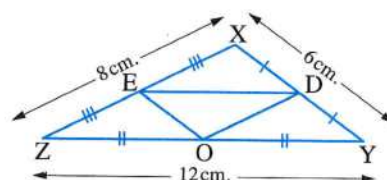
2 Complete :

- 1 The sum of the measures of the interior angles of a quadrilateral equals°
- 2 The image of the point (4, -1) by rotation about the origin point by an angle of measure 90° is
- 3 In $\triangle ABC$, if $m(\angle B) = 90^\circ$, then $(AC)^2 = (AB)^2 + (\dots\dots\dots)^2$
- 4 The line segment joining the midpoints of two sides of a triangle is to the third side.
- 5 The image of the point (3, 1) by translation : $(X, y) \longrightarrow (X, y - 1)$ is

3 [a] In the opposite figure :

XYZ is a triangle in which $XY = 6$ cm.
 $XZ = 8$ cm. , $YZ = 12$ cm. , D , O , E are
 the midpoints of \overline{XY} , \overline{YZ} , \overline{XZ} respectively

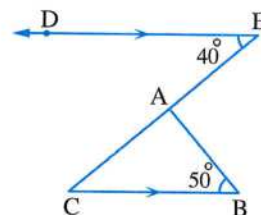
Find : The perimeter of $\triangle DOE$



[b] In the opposite figure :

$\overline{BC} \parallel \overline{ED}$, $m(\angle E) = 40^\circ$
 $m(\angle B) = 50^\circ$

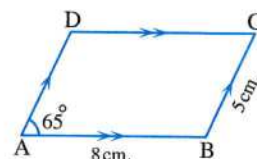
Find : $m(\angle BAC)$



4 [a] In the opposite figure :

ABCD is a parallelogram , $m(\angle A) = 65^\circ$
 $AB = 8$ cm. , $BC = 5$ cm.

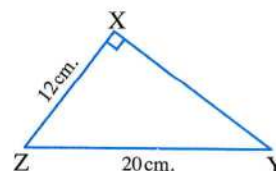
Find : 1 $m(\angle B)$ 2 The perimeter of ABCD



[b] In the opposite figure :

$m(\angle X) = 90^\circ$, $XZ = 12$ cm.
 $ZY = 20$ cm.

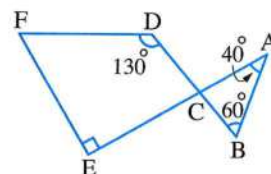
Find : The length of \overline{XY}



5 [a] In the opposite figure :

$\overline{AE} \cap \overline{BD} = \{C\}$
 $m(\angle A) = 40^\circ$, $m(\angle B) = 60^\circ$
 $m(\angle D) = 130^\circ$, $m(\angle E) = 90^\circ$

Find : $m(\angle F)$



- [b] On a square lattice , draw the image of the triangle ABC where : A (1, 1) , B (3, 4)
 , C (5, 2) by reflection in the y-axis



By a group of supervisors

GUIDE ANSWERS

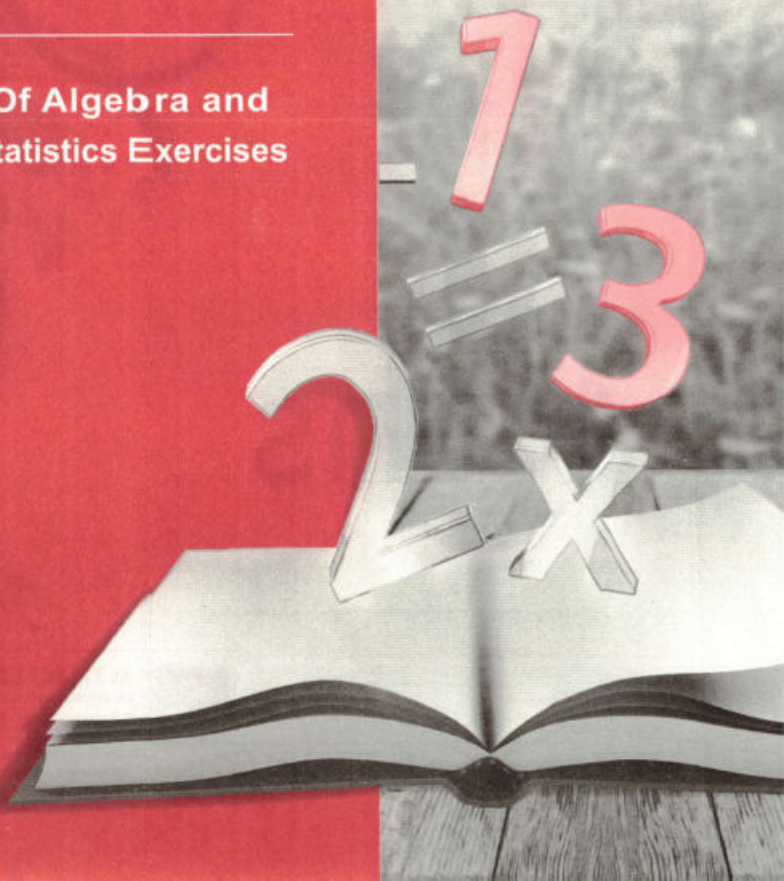
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Maths

Guide Answers

Of Algebra and
Statistics Exercises





Answers of unit one

Answers of Exercise 1

1

$$[1] \left(\frac{1}{3}\right)^4 = \frac{1^4}{3^4} = \frac{1}{81} \quad [2] \left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{9}{25}$$

$$[3] \left(-\frac{1}{7}\right)^3 = -\frac{1^3}{7^3} = -\frac{1}{343} \quad [4] \left(-\frac{3}{4}\right)^4 = \frac{3^4}{4^4} = \frac{81}{256}$$

$$[5] \left(\frac{5}{9}\right)^0 = 1$$

$$[6] \left(-2\frac{1}{2}\right)^3 = \left(-\frac{5}{2}\right)^3 = -\frac{125}{8}$$

$$[7] (0.04)^2 = \left(\frac{4}{100}\right)^2 = \left(\frac{1}{25}\right)^2 = \frac{1}{625}$$

$$[8] (1.5)^3 = \left(\frac{15}{10}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

$$[9] (-3.2)^2 = \left(-\frac{16}{5}\right)^2 = \frac{256}{25}$$

2

$$[1] 8 \times \frac{1}{8} = 1 \quad [2] \frac{9}{16} \times \frac{8}{27} = \frac{1}{6}$$

$$[3] -\frac{27}{125} \times \left(-\frac{25}{27}\right) = \frac{1}{5}$$

$$[4] \frac{9}{25} \div \left(-\frac{9}{125}\right) = \frac{9}{25} \times \left(-\frac{125}{9}\right) = -5$$

$$[5] \frac{16}{9} \times \frac{27}{8} = 6$$

$$[6] \frac{25}{36} \div \frac{15}{4} = \frac{25}{36} \times \frac{4}{15} = \frac{5}{27}$$

$$[7] \left(\frac{5}{2}\right)^2 \times \frac{4}{25} = \frac{25}{4} \times \frac{4}{25} = 1$$

$$[8] \frac{25}{9} \div \left(-\frac{5}{3}\right)^2 = \frac{25}{9} \div \frac{25}{9} = 1$$

3

$$[1] \frac{16}{25} \times \frac{5}{16} \times 1 = \frac{1}{5}$$

$$[2] \frac{3}{4} \times \left(-\frac{8}{27}\right) \times \frac{9}{4} = -\frac{1}{2}$$

$$[3] \frac{625}{81} \times \left(-\frac{27}{125}\right) \times (-1) = \frac{5}{3}$$

$$[4] -\frac{8}{27} \times \frac{1}{27} \div \frac{4}{81} = -\frac{8}{27} \times \frac{1}{27} \times \frac{81}{4} = -\frac{2}{9}$$

$$[5] \left(\frac{125}{8} + \frac{81}{16}\right) \times \frac{27}{125} = \frac{125}{8} \times \frac{16}{81} \times \frac{27}{125} = \frac{2}{3}$$

$$[6] -\frac{1}{8} \div (-3) = \frac{1}{8} \times \frac{1}{3} = \frac{1}{24}$$

4

$$[1] (c) \quad [2] (d) \quad [3] (a) \quad [4] (b) \quad [5] (c)$$

$$[6] (b) \quad [7] (c) \quad [8] (c) \quad [9] (b) \quad [10] (b)$$

5

$$[1] 3 \quad [2] 2 \quad [3] 3 \quad [4] 2$$

$$[5] 3 \quad [6] 2 \quad [7] -\frac{8}{125} \quad [8] \frac{1}{9}$$

$$[9] -\frac{3}{8} \quad [10] 3 \quad [11] \frac{81}{256}, \frac{243}{1024} \quad [12] \left(\frac{1}{4}\right)^2$$

6

$$x^2 + y^3 = \left(-\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^3 = \frac{4}{9} - \frac{1}{27} = \frac{12}{27} - \frac{1}{27} = \frac{11}{27}$$

$$[7] a^3 + b^3 = \left(\frac{2}{3}\right)^3 + \left(-\frac{4}{3}\right)^3 = \frac{8}{27} + \left(-\frac{64}{27}\right)$$

$$= \frac{8}{27} - \frac{64}{27} = -\frac{56}{27}$$

$$|a^3 + b^3| = \left|-\frac{56}{27}\right| = \frac{56}{27}$$

8

$$[8] 9xy^2 - z^3 = 9 \times \frac{1}{2} \times \left(-\frac{2}{3}\right)^2 - (-3)^3$$

$$= 9 \times \frac{1}{2} \times \frac{4}{9} + 27 = 2 + 27 = 29$$

9

$$[9] a^3b^2 + b^2c - 8abc$$

$$= \left(-\frac{1}{2}\right)^3 \times 2^2 + 2^2 \times \frac{3}{4} - 8 \times \left(-\frac{1}{2}\right) \times 2 \times \frac{3}{4}$$

$$= -\frac{1}{8} \times 4 + 4 \times \frac{3}{4} + 6 = -\frac{1}{2} + 3 + 6 = 8\frac{1}{2}$$

10

$$[1] x^2y^2z^2 = \left(-\frac{3}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 \times \left(-\frac{4}{3}\right)^2$$

$$= \frac{9}{4} \times \frac{1}{4} \times \frac{16}{9} = 1$$

$$[2] x^2 \div z^2 = \left(-\frac{3}{2}\right)^2 \div \left(-\frac{4}{3}\right)^2$$

$$= \frac{9}{4} \div \frac{16}{9} = \frac{9}{4} \times \frac{9}{16} = \frac{81}{64}$$

$$[3] x^2 - yz^2 = \left(-\frac{3}{2}\right)^2 - \frac{1}{2} \times \left(-\frac{4}{3}\right)^2 = \frac{9}{4} - \frac{1}{2} \times \frac{16}{9}$$

$$= \frac{9}{4} - \frac{16}{18} = \frac{81}{36} - \frac{32}{36} = \frac{49}{36}$$

$$[4] x^2y^2z^2 = 1 \text{ from (1)}$$

$$\therefore x + y = -\frac{3}{2} + \frac{1}{2} = -1$$

$$\text{i.e. } \frac{x^2y^2z^2}{x+y} = \frac{1}{-1} = -1$$

11 The volume of the cube = $(1\frac{1}{2})^3 = (\frac{3}{2})^3 = \frac{27}{8} \text{ cm}^3$

12 1 (a)

2 (b)

13 The order is: $(-\frac{2}{3})^3, (-\frac{1}{3})^3, (-\frac{1}{3})^2, (\frac{2}{3})^2$

Answers of Exercise 2

1

1 $(\frac{2}{3})^{3+2} = (\frac{2}{3})^5 = \frac{32}{243}$

2 $-(\frac{2}{3})^{3+2} = -(\frac{2}{3})^5 = -\frac{32}{243}$

3 $(\frac{1}{5})^{1+4} = (\frac{1}{5})^5 = \frac{1}{3125}$

4 $(\frac{1}{6})^{9-8} = \frac{1}{6}$ 5 $(\frac{2}{7})^{5-3} = (\frac{2}{7})^2 = \frac{4}{49}$

6 $-(\frac{3}{5})^{7-5} = -(\frac{3}{5})^2 = -\frac{9}{25}$

7 $(\frac{5}{2})^2 \div \frac{5}{2} = (\frac{5}{2})^{2-1} = \frac{5}{2}$

8 $(\frac{1}{2})^{2+1+3} = (\frac{1}{2})^6 = \frac{1}{64}$

9 $(\frac{4}{5})^{8-6+1} = (\frac{4}{5})^3 = \frac{64}{125}$

2

1 $3^{7+3-6} = 3^4 = 81$

2 $2^{6+1-3-4} = 2^0 = 1$

3 $5^{4+2-3} = 5^3 = 125$

4 $2^{5+4-3-2} = 2^4 = 16$

5 $(-3)^{5-3} \times (-2)^{7-5} = (-3)^2 \times (-2)^2 = 9 \times 4 = 36$

6 $x^{4+5-6} \times y^{3-2} = x^3 y$

3 1 $\frac{a^5 b^3}{c^5}$

2 $\frac{5^2 x^2}{3^2 y^2} = \frac{25 x^2}{9 y^2}$

3 $\frac{2^4 a^4 b^4}{3^4 c^4} = \frac{16 a^4 b^4}{81 c^4}$

4 $\frac{x^{2 \times 2}}{y^{3 \times 2}} = \frac{x^4}{y^6}$

5 $\frac{a^3 \times 3 \times b^{2 \times 3}}{c^{5 \times 3}} = \frac{a^9 b^6}{c^{15}}$

6 $-\frac{c^{2 \times 3}}{d^3} = -\frac{c^6}{d^3}$

7 $\frac{x^{3 \times 2}}{y^{2 \times 2}} = \frac{x^6}{y^4}$

8 $(\frac{4 x^3 y^2}{2 x^2 y})^7 = (2 x y)^7 = 2^7 x^7 y^7 = 128 x^7 y^7$

9 $\frac{2^3 a^3 \times 2^4 a^4}{2^6 a^6 \times a} = 2^{3+4-6} \times a^{3+4-6-1} = 2 \times a^0 = 2 \times 1 = 2$

4 1 $(\frac{1}{2})^{2 \times 2} = (\frac{1}{2})^4 = \frac{1}{16}$

2 $(\frac{3}{2})^{2 \times 5} = (\frac{3}{2})^{10} = \frac{59049}{1024}$

3 $[(\frac{5}{2})^3]^2 = (\frac{5}{2})^{3 \times 2} = (\frac{5}{2})^6 = \frac{15625}{64}$

4 $(\frac{3}{5} \times \frac{5}{3})^{10} = 1^{10} = 1$

5 $(\frac{2}{7})^6 \times (\frac{7}{2})^6 = (\frac{2}{7} \times \frac{7}{2})^6 = 1^6 = 1$

6 $(\frac{5}{2})^2 \times (\frac{2}{5})^2 = (\frac{5}{2} \times \frac{2}{5})^2 = 1^2 = 1$

5 1 (a)

2 (d)

3 (c)

4 (d)

5 (c)

6 (a)

7 (d)

8 (c)

9 (d)

10 (c)

11 (c)

12 (d)

13 (c)

6 $\frac{2^4 y^4 \times 3^2 y^2}{12 y^5} = \frac{16 y^4 \times 9 y^2}{12 y^5} = \frac{144 y^6}{12 y^5} = 12 y$

At: $y = -\frac{1}{6}$

The result = $12 \times (-\frac{1}{6}) = -2$

7

1 $\frac{(a^2 c^2)^2}{b} = \frac{a^4 c^4}{b} = (\frac{5}{3})^4 \times (\frac{2}{5})^4 \div (-\frac{3}{2})$
 $= (\frac{5}{3} \times \frac{2}{5})^4 \div (-\frac{3}{2})$
 $= (\frac{2}{3})^4 \times (-\frac{2}{3}) = -(\frac{2}{3})^5 = -\frac{32}{243}$

2 $(\frac{2ab}{5c})^3 = (\frac{2 \times \frac{5}{3} \times (-\frac{3}{2})}{5 \times \frac{2}{5}})^3 = (-\frac{5}{2})^3 = -\frac{125}{8}$

8 1 $x^3 y^2 = (-\frac{1}{2})^3 \times (\frac{3}{4})^2 = -\frac{1}{8} \times \frac{9}{16} = -\frac{9}{128}$

2 $y^3 x^2 = (\frac{3}{4})^3 \times (-\frac{1}{2})^2 = \frac{27}{64} \times \frac{1}{4} = \frac{27}{256}$

3 $\frac{x^3}{y^2 z^2} = \frac{(-\frac{1}{2})^3}{(\frac{3}{4})^2 \times (-\frac{3}{2})^2} = \frac{-\frac{1}{8}}{\frac{9}{16} \times \frac{9}{4}}$
 $= \frac{-\frac{1}{8}}{\frac{81}{64}} = -\frac{1}{8} \times \frac{64}{81} = -\frac{8}{81}$

9 1 24

2 $\frac{9}{16}$

3 $((-3)^2)^4$

4 zero

5 4

6 $2 x$

10

The area of the square = $(\frac{2x}{5})^2 = \frac{2^2 x^2}{5^2} = \frac{4 x^2}{25} \text{ cm}^2$



$$\begin{aligned} 11 \quad \text{The volume of the cube} &= \left(\frac{3a^3}{7}\right)^3 = \frac{3^3(a^3)^3}{7^3} \\ &= \frac{27a^9}{343} \text{ cm}^3. \end{aligned}$$

12

The area of the shaded part

= the area of the great square - the area of the small square

$$= \left(\frac{3X}{2}\right)^2 - \left(\frac{X}{2}\right)^2 = \frac{9X^2}{4} - \frac{X^2}{4} = \frac{8X^2}{4} = 2X^2 \text{ cm}^2.$$

$$13 \quad \text{The number is } 4^3 \div 4 = 4^2$$

$$\text{Then } \frac{3}{4} \text{ the number} = \frac{3}{4} \times 4^2 = 12$$

$$14 \quad X^{15}y^{14} = X \times X^{14} \times y^{14} = X(Xy)^{14}$$

$$= \frac{1}{5} \left(\frac{1}{5} \times 5\right)^{14} = \frac{1}{5} \times (1)^{14} = \frac{1}{5}$$

15

$$1 \quad \text{L.H.S.} = 5^X(5^2 - 5) = 5^X(25 - 5) = 20 \times 5^X = \text{R.H.S.}$$

$$2 \quad 3^{15} + 3^{14} = 3^{14}(3 + 1) = 4 \times 3^{14}$$

i.e. $3^{15} + 3^{14}$ is divisible by 4

Answers of Exercise 3

$$1 \quad 1) 4^{-1} = \frac{1}{4} \quad 2) 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$3) \left(\frac{1}{2}\right)^{-1} = 2 \quad 4) \left(-\frac{2}{3}\right)^{-2} = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$5) (0.2)^{-2} = \left(\frac{2}{10}\right)^{-2} = \left(\frac{10}{2}\right)^2 = 25$$

$$6) (1.2)^{-1} = \left(\frac{6}{5}\right)^{-1} = \frac{5}{6}$$

$$2 \quad 1) 3^7 \times 3^{-3} = 3^{7+(-3)} = 3^4 = 81$$

$$2) 2^{-2} \times 2^{-3} = 2^{-2+(-3)} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

$$3) \frac{3}{3^{-2}} = 3^{1-(-2)} = 3^3 = 27$$

$$4) \frac{6^{-2}}{6^{-3}} = 6^{-2-(-3)} = 6$$

$$3 \quad 1) (5^{-1})^{-3} = 5^3 = 125 \quad 2) (3^{-2})^2 = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$$

$$3) (0.25)^{-2} = \left(\frac{1}{4}\right)^{-2} = 4^2 = 16$$

$$4) (2^{-1} \times 2^{-2})^3 = (2^{-3})^3 = 2^{-9} = \frac{1}{2^9} = \frac{1}{512}$$

$$5) \left(\frac{3^{-1}}{3}\right)^2 = \frac{3^{-2}}{3^2} = 3^{-2-2} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$$

$$6) \left(\frac{8^4}{8^4}\right)^0 = 1$$

4

$$1) 8^{1-2-(-3)} = 8^2 = 64 \quad 2) 7^{-2+5-3} = 7^0 = 1$$

$$3) 2^{5-2-(-4)-3} = 2^4 = 16$$

$$4) \frac{2^3 \times 2^{-3}}{2^4} = 2^{3-3-4} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$5) \frac{3^{-6}}{3^{-2} \times 3^{-6}} = \frac{1}{3^{-2}} = 3^2 = 9$$

$$6) (9^{3+1-5})^{-3} = (9^{-1})^{-3} = 9^3 = 729$$

$$7) (2^{5-3} \times 3^{2-4})^{-1} = (2^2 \times 3^{-2})^{-1} = 2^{-2} \times 3^2 = \frac{9}{4}$$

$$8) 1 \times 2^4 = 1 \times 16 = 16$$

$$9) \frac{(10)^2 \times (10^{-2})^3}{(10)^{-3}} = \frac{(10)^2 \times (10)^{-6}}{(10)^{-3}} = (10)^{2-6+3} = (10)^{-1} = \frac{1}{10}$$

5

$$1) \frac{7}{X}$$

$$2) \frac{y^2}{X}$$

$$3) \frac{1}{a^2b^3}$$

$$4) X^{3-5} = X^{-2} = \frac{1}{X^2}$$

$$5) X^{3-2-1} = X^0 = 1$$

$$6) c^{-5-2} = c^{-7} = \frac{1}{c^7}$$

$$7) a^{-6} = \frac{1}{a^6}$$

$$8) b^3$$

$$9) (a^{-2} \cdot 5)^2 = (a^{-3})^2 = a^{-6} = \frac{1}{a^6}$$

$$10) X^{-6} \times X^6 = X^0 = 1$$

$$11) (y^{5-(-2)})^{-3} = (y^7)^{-3} = y^{-21} = \frac{1}{y^{21}}$$

$$12) X^{2-3-(-4)-1} = X^2$$

$$13) \frac{X^{-6} \times X^{-2}}{X^{-3} \times X^{-4}} = X^{-6-2-(-3)-(-4)} = X^{-1} = \frac{1}{X}$$

$$14) \frac{a^{-1}}{b^2} \times \frac{a^2}{2^{-2}b^{-4}} = \frac{2^2 \times a^{-1+2}}{b^{2-4}} = \frac{4a}{b^{-2}} = 4ab^2$$

$$15) X^2 + 2XX^{-1} + (X^{-1})^2 = X^2 + 2 + X^{-2} = X^2 + \frac{1}{X^2} + 2$$

6

$$1) \frac{1}{8}$$

$$2) 3$$

$$3) X^3$$

$$4) -2 \times X^2$$

$$5) \frac{y^4}{9}$$

$$6) 3a^2$$

$$7) X^2y^3$$

$$8) \frac{y}{X}$$

$$9) 1$$

$$10) -2$$

$$11) 0$$

$$12) 1 + a^5$$

$$13) 4$$

7

$$1) (b)$$

$$2) (c)$$

$$3) (c)$$

$$4) (c)$$

$$5) (b)$$

$$6) (b)$$

$$7) (d)$$

$$8) (d)$$

$$9) (d)$$

$$10) (d)$$

$$11) (b)$$

$$12) (a)$$

8 1) > 2) < 3) < 4) > 5) > 6) =

9 $b^{-3} = \frac{1}{b^3}$ substituting $b = 0$ then $\frac{1}{b^3} = \frac{1}{0}$ (undefined).

10 1) At $X = -2, y = 2$

$$\begin{aligned}\text{Then: } \left(-\frac{3}{5}\right)^{-2} \times \left(\frac{3}{5}\right)^2 &= \left(\frac{5}{3}\right)^2 \times \left(\frac{3}{5}\right)^2 \\ &= \left(\frac{5}{3} \times \frac{3}{5}\right)^2 = 1^2 = 1\end{aligned}$$

2) At $X = -1, y = 2$

$$\text{Then: } \left(-\frac{3}{5}\right)^{-1} \times \left(\frac{3}{5}\right)^2 = -\frac{5}{3} \times \frac{9}{25} = -\frac{3}{5}$$

11 $\left(\frac{y}{x^2}\right)^{-2} = \left(\frac{2}{3} \div \left(-\frac{1}{3}\right)^2\right)^{-2} = \left(\frac{2}{3} \div \frac{1}{9}\right)^{-2}$

$$= \left(\frac{2}{3} \times \frac{9}{1}\right)^{-2} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$$

12 $\frac{2^{10} \times 3^4}{(2 \times 2 \times 3)^5} = \frac{2^{10} \times 3^4}{(2^2 \times 3)^5} = \frac{2^{10} \times 3^4}{2^{10} \times 3^5}$

$$= 2^{10-10} \times 3^{4-5} = 3^{-1} = \frac{1}{3}$$

13
$$\frac{(2 \times 3)^{2n+1} \times (2^2)^{-n}}{2^n \times 3^{2n+1}} = \frac{2^{2n+1} \times 3^{2n+1} \times 2^{-2n}}{2^n \times 3^{2n+1}}$$

$$= 2^{2n+1-2n-n} \times 3^{2n+1-2n-1}$$

$$= 2^{1-n} \times 3^0 = 2^{1-n}$$

When $n = 3$, then $2^{1-n} = 2^{1-3} = 2^{-2} = \frac{1}{4}$

14 The height of the jump $= 2^3 \div 2^{-4}$

$$= 2^{3-(-4)} = 2^7$$

$$= 128 \text{ times of its length}$$

15 1) In 2 years the population will be $2(1.03)^2 = 2.1218$ millions.
 2) The population now $= 2(1.03)^0 = 2$ millions.
 3) The population last year was $2(1.03)^{-1} = \frac{2}{1.03} \approx 1.94$ millions.

16 1) $2^{n+1} = 2^n \times 2 = 3 \times 2 = 6$
 2) $4^n = (2 \times 2)^n = 2^n \times 2^n = 3 \times 3 = 9$
 3) $4^{-n} = (2 \times 2)^{-n} = 2^{-n} \times 2^{-n} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
 4) $2^{n-1} = 2^n \times 2^{-1} = 3 \times \frac{1}{2} = \frac{3}{2}$

17 $a^{51} b^{50} = a \times a^{50} b^{50} = a(a b)^{50}$

$$= 5 \left(5 \times \frac{1}{5}\right)^{50} = 5 \times 1 = 5$$

18 The order is: $(-5)^{15}, (-2)^{15}, (-2)^{-15}, 2^{-20}, (-2)^{20}, (-5)^{20}$

Answers of Exercise 4

1 The numbers which are in the standard form are (1), (4), (6) and (8)

2

1) 6×10^5 2) -2×10^4 3) 7×10^6

4) 1.9×10^7 5) 4.687×10^7 6) 5.8×10

3

1) 6×10^{-4} 2) 5.3×10^{-5} 3) 8.64×10^{-4}

4) 4.21×10^{-1} 5) 2.50003×10 6) -3.00501×10^2

4) $(5.1 \times 10^8) \text{ km}^2$

5) $(1.67 \times 10^{-24}) \text{ gm}$.

6 The velocity of light $= 300\,000 \times 1000$

$$= 300\,000\,000$$

$$= 3 \times 10^8 \text{ m/sec.}$$

7 1×10^{-15} seconds

8 18 zeroes

9 1) 6.8×10^6 2) 6.8×10^{-4}
 3) 7.2×10^8 4) 7.5×10^{-7}
 5) -3.24×10^5 6) -7.025×10^{-6}
 7) 4×10^{-11} 8) 5×10^{11}
 9) 3.6×10^{-7} 10) 2.0205×10^9

10 1) > 2) < 3) < 4) <
 5) > 6) > 7) > 8) >

11 The order is $8.35 \times 10^{-2}, 1 \times 10^{-2}, 3.6 \times 10^{-3}, 5.2 \times 10^{-5}, 6.08 \times 10^{-8}$

12 1) (d) 2) (b) 3) (c) 4) (b)
 5) (c) 6) (b) 7) (a) 8) (b)
 9) (c) 10) (c) 11) (d)



- 13**
- 1 9.6×10^{13} 2 $17.22 \times 10^3 = 1.722 \times 10^4$
 3 $0.502 \times 10^{-7} = 5.02 \times 10^{-8}$
 4 $(4.4 \times 10^3) \times (32 \times 10^5) = 140.8 \times 10^8 = 1.408 \times 10^{10}$
 5 2×10^2 6 $25.1 \times 10^{-7} = 2.51 \times 10^{-6}$
 7 1×10^3
 8 $(625 \times 10^4) \div (2.5 \times 10^{-3}) = 250 \times 10^7 = 2.5 \times 10^9$
- 14**
- 1 $10^4 (3.8 \times 10 + 4.6) = 10^4 (38 + 4.6)$
 $= 42.6 \times 10^4 = 4.26 \times 10^5$
 2 $10^3 (4.54 \times 10 + 3.76) = 10^3 \times 49.16 = 4.916 \times 10^4$
 3 $10^7 (5.3 \times 10 - 0.8) = 10^7 \times 52.2 = 5.22 \times 10^8$
 4 $10^{-3} (2.65 \times 10 - 6.34) = 10^{-3} \times 20.16 = 2.016 \times 10^{-2}$
- 15**
- 1 $5 \times 10^3 \times 3 \times 10^3 = 15 \times 10^6 = 1.5 \times 10^7$
 2 $4 \times 10^2 \times 7 \times 10^{-5} = 28 \times 10^{-3} = 2.8 \times 10^{-2}$
 3 $8 \times 10^3 \div 4 \times 10^{-3} = 2 \times 10^6$
 4 $3.3 \times 10^{-5} \div 5 \times 10^2 = 0.66 \times 10^{-7} = 6.6 \times 10^{-8}$
 5 $(2 \times 10^4)^3 = 8 \times 10^{12}$
 6 $(2 \times 10^{-3})^2 = 4 \times 10^{-6}$
 7 $(1 \times 10^{-1})^{-8} = 10^8$
- 16**
- 1 $800\,000 = 8 \times 10^5$ $\therefore n = 5$
 2 $0.00000006 = 6 \times 10^{-8}$ $\therefore n = -8$
 3 $0.00052 = 5.2 \times 10^{-4}$ $\therefore n = -4$
 4 $0.000357 = 3.57 \times 10^{-4}$ $\therefore n = -4$
 5 $(0.004)^2 = (4 \times 10^{-3})^2$
 $= 16 \times 10^{-6}$
 $= 1.6 \times 10^{-5}$ $\therefore n = -5$
 6 $76293 = 7.6293 \times 10^4$ $\therefore n = 7.6293$
- 17**
- The earth is the greater
 and the difference between the two diameter lengths
 $= (1.27 \times 10^4) - (6.79 \times 10^3)$
 $= 10^3 (1.27 \times 10 - 6.79) = 5.91 \times 10^3 \text{ km.}$
- 18**
- 1 The distance from the Sun to the Earth
 $= v \times t = 3 \times 10^8 \times 8 \times 60$
 $= 1440 \times 10^8 = 1.44 \times 10^{11} \text{ m.}$
 2 The time elapsed $= \frac{d}{v} = \frac{108 \times 10^6 \times 10^3}{3 \times 10^8}$
 $= 36 \times 10 = 360 \text{ seconds} = 6 \text{ minutes}$
- 19**
- $\frac{10^3 (9.02 + 4.98 \times 10)}{2.5 \times 10^{-5}} = \frac{58.82 \times 10^3}{2.5 \times 10^{-5}}$
 $= \frac{5.882 \times 10^4}{2.5 \times 10^{-5}} = 2.3528 \times 10^9$
- 20**
- 1 $10^{29} - 10^{28} = 10^{28} (10 - 1) = 9 \times 10^{28}$
 2 $2^{19} \times 5^{15} = 2^{15} \times 2^4 \times 5^{15} = (2 \times 5)^{15} \times 2^4$
 $= 10^{15} \times 16 = 1.6 \times 10^{16}$
- 21**
- $X = 5 + 30 + 400 + 6000 + 90\,000 + 400\,000 + 2\,000\,000$
 $\therefore X = 2496435 = 2.496435 \times 10^6$

Answers of Exercise 5

- 1**
- 1 5 2 10 3 19
 4 12 5 45 6 143
- 2**
- 1 49 2 18 3 9 4 36
 5 3 6 40 7 378 8 11
 9 108 10 97 11 51 12 -13
- 3**
- 1 zero 2 zero 3 12 4 13
 5 9 6 10 7 86 8 22
 9 -14 10 -7
- 4**
- 1 2 2 6 3 -20
 4 2 5 2 6 zero
 7 5 8 23 9 1
- 5**
- 2 $(\frac{5 \times 3 + 3}{4 \times 3 - 3}) = 4$
- 6**
- 1 $(2 + 5)^2 = 49$ 2 $(5 - 2)^3 = 27$
 3 $(\frac{5}{2})^3 = \frac{125}{8}$ 4 $\frac{6^2}{5 - 1} = 9$
 5 $\frac{5 - 2}{5^3} = \frac{3}{125}$ 6 $\frac{12}{4 \times 25} = \frac{3}{25}$

7 $16 \times 9 + (4 \times 6) + 3 \times 6 \times 9 = 168$

8 $X = 32, y = 8 \quad \therefore X - 4y = 32 - 4 \times 8 = \text{zero}$

9 $X = 15, y = 30 \quad \therefore \left(\frac{y}{X}\right)^{-3} = \left(\frac{30}{15}\right)^{-3} = \frac{1}{8}$

10 1 $T = 6 \text{ s}^2 = 6 \times 3^2 = 54 \text{ m}^2$

2 $T = 6 \text{ s}^2 = 6 \times (0.8)^2 = 3.84 \text{ cm}^2$

11 1 $A = \frac{1}{2} h (a + b) = \frac{1}{2} \times 2 \left(\frac{3}{4} + \frac{1}{4}\right) = 1 \text{ m}^2$

2 $A = \frac{1}{2} h (a + b) = \frac{1}{2} \times 4 \left(\frac{1}{2} + \frac{1}{2}\right) = 2 \text{ m}^2$

12 1 $3 + 96 + (12 \times 4) = 5$

2 $3 + (96 + 12) \times 4 = 35$

3 $(3 + 96) \div 12 \times 4 = 33$

Answers of Exercise 6

1 1 4 2 -5 3 ± 50

4 ± 200 5 $\frac{3}{7}$ 6 $-\frac{8}{5}$

7 $\sqrt{\frac{81}{100}} = \frac{9}{10}$ 8 $\pm \sqrt{\frac{144}{100}} = \pm \frac{12}{10} = \pm \frac{6}{5}$

9 $\sqrt{\frac{25}{4}} = \frac{5}{2}$ 10 $-\sqrt{\frac{36}{25}} = -\frac{6}{5}$ 11 -4

12 ± 8 13 $\frac{81}{100}$ 14 $|- \frac{3}{4}| = \frac{3}{4}$

15 $\pm \frac{24}{35}$ 16 $-\sqrt{\frac{25}{400}} = -\sqrt{\frac{1}{16}} = -\frac{1}{4}$

17 $|\frac{7a^2}{5b^3}|$ 18 $|\frac{4b^4}{11h}|$ 19 $|\frac{7a^2b}{3}|$

20 $|\frac{5xy}{6}|$

2 1 ± 8 2 ± 12

3 $\pm \sqrt{\frac{25}{4}} = \pm \frac{5}{2}$ 4 ± 0.5

3 1 $3 + 4 = 7$ 2 $\sqrt{100} = 10$

3 $-\sqrt{144} = -12$ 4 $\sqrt{9 + 16} = \sqrt{25} = 5$

5 $-\sqrt{100 - 64} = -\sqrt{36} = -6$

6 $\sqrt{\frac{9}{16} + \frac{16}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

7 $\sqrt{5^4 + 3^2 - 5} = \sqrt{5^2} = 5$

8 $\sqrt{\left(\frac{1}{2} \times \frac{1}{3}\right)^4} = \sqrt{\left(\frac{1}{6}\right)^4} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$

9 $\sqrt{\left(\frac{1}{4}\right)^5} = \sqrt{\left(\left(\frac{1}{2}\right)^2\right)^5} = \sqrt{\left(\frac{1}{2}\right)^{10}} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

4 1 1 2 $\frac{2}{3}$ 3 1 4 100

5 $\frac{5}{2}$ 6 $\frac{10}{7}$ 7 $\frac{1}{2}$ 8 $\frac{3}{4}$

9 $\left(\frac{5}{2}\right)^2$ or $\left(-\frac{5}{2}\right)^2$ 10 3 11 $a^2 b^4$

12 $\frac{3}{4}$ 13 3 14 -2

5 1 (c) 2 (b) 3 (b) 4 (d) 5 (a)

6 (c) 7 (b) 8 (a) 9 (d) 10 (b)

6 1 $\frac{7}{2} \times 1 \times \frac{2^2}{7^2} = \frac{7}{2} \times \frac{4}{49} = \frac{2}{7}$

2 $\frac{2}{5} \times \frac{3}{4} \div \left(-\frac{1^3}{2^3}\right) = \frac{2}{5} \times \frac{3}{4} \times \left(-\frac{8}{1}\right) = -\frac{12}{5}$

3 $\frac{1^2}{3^2} + \frac{8}{9} - 1 = \frac{1}{9} + \frac{8}{9} - \frac{9}{9} = \text{zero}$

4 $\frac{3}{4} \times \left(-\frac{2^3}{3^3}\right) \times \frac{9}{4} = -\frac{3}{4} \times \frac{8}{27} \times \frac{9}{4} = -\frac{1}{2}$

7 1 $4 + 5 = 9$ 2 $\sqrt{4 + 5} = \sqrt{9} = 3$

3 $\sqrt{(4 + 5)^2} = \sqrt{(9)^2} = 9$

8 $\sqrt{\frac{4}{9}} = \frac{2}{3}$

\therefore The L.C.M. of 3 and 4 is 12

$\therefore \frac{2}{3} = \frac{8}{12}, \frac{3}{4} = \frac{9}{12}, \frac{8}{12} = \frac{24}{36}, \frac{9}{12} = \frac{27}{36}$

\therefore The two numbers are $\frac{25}{36}$ and $\frac{26}{36}$

9 1 $\sqrt{25 - 10 + 1} = \sqrt{16} = 4$

Another solution : $\sqrt{(5-1)^2} = \sqrt{4^2} = 4$

2 $\sqrt{\frac{1}{16} - \frac{1}{2} + 1} = \sqrt{\frac{1}{16} - \frac{8}{16} + \frac{16}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

Another solution :

$\sqrt{\left(\frac{1}{4} - 1\right)^2} = \sqrt{\left(-\frac{3}{4}\right)^2} = |-\frac{3}{4}| = \frac{3}{4}$

3 $\sqrt{9} = 3$ 4 $\sqrt{16} = 4$

10

1 $\therefore (XY)^2 = 25 \quad \therefore XY = \sqrt{25} = 5 \text{ cm.}$

\therefore E is the midpoint of XY

\therefore XE = 2.5 cm.



- 2 $\because (AB)^2 = 144 \quad \therefore AB = \sqrt{144} = 12 \text{ cm.}$
 $\because (BC)^2 = 625 \quad \therefore BC = \sqrt{625} = 25 \text{ cm.}$
 $\therefore AC = 37 \text{ cm.}$
- 3 The side length of the square = $\sqrt{0.49} = 0.7 \text{ cm.}$
 \therefore Its perimeter = the side length $\times 4$
 $= 0.7 \times 4 = 2.8 \text{ cm.}$
- 4 The area of the square = the area of the triangle
 $= \frac{1}{2} \times 9 \times 8 = 36 \text{ cm}^2$
 \therefore The side length of the square = $\sqrt{36} = 6 \text{ cm.}$
- 5 \because The area of the circle = πr^2
 $\therefore 154 = \frac{22}{7} \times r^2 \quad \therefore r^2 = 154 \div \frac{22}{7} = 49$
 $\therefore r = \sqrt{49} = 7 \text{ cm.}$
- 6 \because The area of the circle = πr^2
 $\therefore 616 = \frac{22}{7} \times r^2 \quad \therefore r^2 = 616 \div \frac{22}{7} = 196$
 $\therefore r = \sqrt{196} = 14 \text{ cm.}$
 \therefore Its circumference = $2\pi r = 2 \times \frac{22}{7} \times 14 = 88 \text{ cm.}$
- 7 \because The area of the square = $1 \frac{11}{64} \times \frac{4}{3} = \frac{25}{16} \text{ m}^2$
 \therefore The side length = $\sqrt{\frac{25}{16}} = \frac{5}{4} = 1 \frac{1}{4} \text{ m.}$
- 8 $2 \times$ the width of the rectangle \times the width of the rectangle = 24.5
 \therefore The width of the rectangle \times the width of the rectangle = 12.25
 \therefore The width of the rectangle = $\sqrt{12.25} = 3.5 \text{ cm.}$
 \therefore The length of the rectangle = $2 \times 3.5 = 7 \text{ cm.}$

11 1 zero 2 -1 3 zero

12 $\frac{m^2}{n^2} = \frac{16}{100} \quad \therefore \frac{m}{n} = \pm \sqrt{\frac{16}{100}} = \pm \frac{4}{10}$
 $\therefore \left(\frac{m}{n}\right)^3 = \left(\pm \frac{4}{10}\right)^3 = \pm \frac{64}{1000} = \pm 0.064$

Answers of Exercise 7

- 1
 1 $\because X - 7 = 3 \quad \therefore X - 7 + 7 = 3 + 7$
 $\therefore X = 10 \quad \therefore$ The S.S. = $\{10\}$
- 2 $\because X + 17 = 13 \quad \therefore X + 17 - 17 = 13 - 17$
 $\therefore (13 - 17) \notin \mathbb{R} \quad \therefore$ The S.S. = \emptyset
- 3 $\because 5X = 20 \quad \therefore 5X \times \frac{1}{5} = 20 \times \frac{1}{5}$
 $\therefore X = 4 \quad \therefore$ The S.S. = $\{4\}$

4 $\because \frac{2}{5}X = \frac{1}{5} \quad \therefore \frac{2}{5}X \times \frac{5}{2} = \frac{1}{5} \times \frac{5}{2}$
 $\therefore X = \frac{1}{2} \quad \therefore$ The S.S. = $\left\{\frac{1}{2}\right\}$

5 $\because -4 + y + 4 = 13 + 4$
 $\therefore y = 17 \quad \therefore$ The S.S. = $\{17\}$

6 $\because m - (-3) = 1 \quad \therefore m + 3 - 3 = 1 - 3$
 $\therefore m = -2 \quad \therefore$ The S.S. = $\{-2\}$

7 $\because X - 7 = 0 \quad \therefore X - 7 + 7 = 0 + 7$
 $\therefore X = 7 \quad \therefore$ The S.S. = $\{7\}$

8 $\because y - (-5) = -3 \quad \therefore y + 5 - 5 = -3 - 5$
 $\therefore y = -8 \quad \therefore$ The S.S. = $\{-8\}$

9 $\because X - 6\frac{1}{4} = 12\frac{1}{2}$
 $\therefore X - 6\frac{1}{4} + 6\frac{1}{4} = 12\frac{1}{2} + 6\frac{1}{4}$
 $\therefore X = 18\frac{3}{4} \quad \therefore$ The S.S. = $\left\{18\frac{3}{4}\right\}$

10 $\because 8.91 + X = 11.09$
 $\therefore 8.91 + X - 8.91 = 11.09 - 8.91$
 $\therefore X = 2.18 \quad \therefore$ The S.S. = $\{2.18\}$

2

1 $\because 2X - 1 = 5 \quad \therefore 2X - 1 + 1 = 5 + 1$
 $\therefore 2X = 6 \quad \therefore 2X \times \frac{1}{2} = 6 \times \frac{1}{2}$
 $\therefore X = 3$

2 $\because 8X + 4 = 12 \quad \therefore 8X + 4 - 4 = 12 - 4$
 $\therefore 8X = 8 \quad \therefore 8X \times \frac{1}{8} = 8 \times \frac{1}{8}$
 $\therefore X = 1$

3 $\because 3X - 13 + 13 = 26 + 13 \quad \therefore 3X = 39$
 $\therefore \frac{3X}{3} = \frac{39}{3} \quad \therefore X = 13$

4 $\because 2X + 14 - 14 = 14 - 14 \quad \therefore 2X = \text{zero}$
 $\therefore \frac{2X}{2} = \frac{0}{2} \quad \therefore X = \text{zero}$

5 $\because 8 + 2X - 8 = 14 - 8 \quad \therefore 2X = 6$
 $\therefore \frac{2X}{2} = \frac{6}{2} \quad \therefore X = 3$

6 $\because \frac{5}{6}X - 4 + 4 = 11 + 4 \quad \therefore \frac{5}{6}X = 15$
 $\therefore \frac{5}{6}X \times \frac{6}{5} = 15 \times \frac{6}{5} \quad \therefore X = 18$

7 $\because 8 - 2X - 8 = -2 - 8 \quad \therefore -2X = -10$
 $\therefore \frac{-2X}{-2} = \frac{-10}{-2} \quad \therefore X = 5$

$$\begin{aligned} \text{[8]} \quad & \because 2 - 5x - 2 = 0 - 2 & \therefore -5x = -2 \\ & \therefore -5x \times -\frac{1}{5} = -2 \times -\frac{1}{5} & \therefore x = \frac{2}{5} \end{aligned}$$

$$\begin{aligned} \text{[9]} \quad & \because 2x + 3x + 25 = 5 \\ & \therefore 5x + 25 - 25 = 5 - 25 \\ & \therefore 5x = -20 & \therefore \frac{5x}{5} = \frac{-20}{5} \\ & \therefore x = -4 \end{aligned}$$

$$\begin{aligned} \text{[10]} \quad & \because 6x - 2x + 7 = 4 & \therefore 4x + 7 - 7 = 4 - 7 \\ & \therefore 4x = -3 & \therefore \frac{4x}{4} = \frac{-3}{4} \\ & \text{where } -\frac{3}{4} \text{ is impossible in } \mathbb{Z} \\ & \therefore \text{The equation has no solution in } \mathbb{Z} \end{aligned}$$

3

$$\begin{aligned} \text{[1]} \quad & \because 2x - 6 = 4 & \therefore 2x = 10 \\ & \therefore x = 5 \end{aligned}$$

$$\text{[2]} \quad \because 3x + 10x - 6 = 7 \quad \therefore 13x = 13 \quad \therefore x = 1$$

$$\begin{aligned} \text{[3]} \quad & \because 7x - 14 - 3x - 3 = 3 & \therefore 4x - 17 = 3 \\ & \therefore 4x = 20 & \therefore x = 5 \end{aligned}$$

$$\begin{aligned} \text{[4]} \quad & \because 3x + 6 + 7x - 7 = 12 & \therefore 10x - 1 = 12 \\ & \therefore 10x = 13 & \therefore x = 1.3 \end{aligned}$$

$$\begin{aligned} \text{[5]} \quad & \because 4x - 4 - x - 3 = 0 & \therefore 3x - 7 = 0 \\ & \therefore 3x = 7 & \therefore x = \frac{7}{3} \end{aligned}$$

$$\begin{aligned} \text{[6]} \quad & \because 5x - 10 + 2x + 8 = -16 & \therefore 7x - 2 = -16 \\ & \therefore 7x = -14 & \therefore x = -2 \end{aligned}$$

$$\begin{aligned} \text{[7]} \quad & \because 2x - 6 + 3x - 6 - 4x = -3 \\ & \therefore x - 12 = -3 & \therefore x = 9 \end{aligned}$$

$$\begin{aligned} \text{[8]} \quad & \because 3y + 6y + 18 - 8y + 16 = 60 \\ & \therefore y + 34 = 60 & \therefore y = 26 \end{aligned}$$

4

$$\begin{aligned} \text{[1]} \quad & \because 2x - x = 9 - 5 & \therefore x = 4 \\ & \therefore \text{The S.S.} = \{4\} \end{aligned}$$

$$\begin{aligned} \text{[2]} \quad & \because 5x - 2x = 11 + 4 & \therefore 3x = 15 \\ & \therefore x = 5 & \therefore \text{The S.S.} = \{5\} \end{aligned}$$

$$\begin{aligned} \text{[3]} \quad & \because x + 3x = 18 - 3 & \therefore 4x = 15 \\ & \therefore x = \frac{15}{4} & \therefore \text{The S.S.} = \left\{ \frac{15}{4} \right\} \end{aligned}$$

$$\begin{aligned} \text{[4]} \quad & \because 3x + 5x = 30 - 6 & \therefore 8x = 24 \\ & \therefore x = 3 & \therefore \text{The S.S.} = \{3\} \end{aligned}$$

$$\begin{aligned} \text{[5]} \quad & \because 4x + 4 = 2x - 2 & \therefore 4x - 2x = -2 - 4 \\ & \therefore 2x = -6 & \therefore x = -3 \\ & \therefore \text{The S.S.} = \{-3\} \end{aligned}$$

$$\begin{aligned} \text{[6]} \quad & \because 3x - 6 = 5x - 10 & \therefore -6 + 10 = 5x - 3x \\ & \therefore 4 = 2x & \therefore x = 2 \end{aligned}$$

$$\therefore \text{The S.S.} = \{2\}$$

$$\begin{aligned} \text{[7]} \quad & \because 6a - 2 = 6 - 2a & \therefore 6a + 2a = 6 + 2 \\ & \therefore 8a = 8 & \therefore a = 1 \\ & \therefore \text{The S.S.} = \{1\} \end{aligned}$$

$$\begin{aligned} \text{[8]} \quad & \because 6x - 24 - 2x - 2 = x - 3 \\ & \therefore 4x - 26 = x - 3 & \therefore 4x - x = -3 + 26 \\ & \therefore 3x = 23 & \therefore x = \frac{23}{3} \\ & \therefore \text{The S.S.} = \left\{ \frac{23}{3} \right\} \end{aligned}$$

$$\begin{aligned} \text{[9]} \quad & \because 4(x + 1) = 3(x - 1) & \therefore 4x + 4 = 3x - 3 \\ & \therefore x = -7 & \therefore \text{The S.S.} = \{-7\} \end{aligned}$$

$$\begin{aligned} \text{[10]} \quad & \because 5(1 - 2x) = 3(4 + 4x) \\ & \therefore 5 - 10x = 12 + 12x \\ & \therefore 5 - 12 = 12x + 10x & \therefore -7 = 22x \\ & \therefore x = -\frac{7}{22} & \therefore \text{The S.S.} = \left\{ -\frac{7}{22} \right\} \end{aligned}$$

5	1 2	2 12	3 10	4 14
	5 2	6 9	7 $1\frac{1}{3}$	8 $(x - 5)$ years
	9 $(y + 4)$ years	10 $(x - 5)$ years		
	11 $(x - 6)$ years	12 $\frac{1}{3}x$	13 6x	
	14 $5 - x$	15 $x + 2$		

6	1 (a)	2 (d)	3 (c)	4 (c)
	5 (a)	6 (a)	7 (c)	8 (b)

7

Since the sum of measures of the angles of the triangle = 180°

$$\begin{aligned} \text{[1]} \quad & x + 2x + 2x + 5^\circ = 180^\circ \\ & \therefore 5x + 5^\circ = 180^\circ & \therefore 5x = 175^\circ \\ & \therefore x = 35^\circ \end{aligned}$$

\therefore The measures of the angles of the triangle are $35^\circ, 70^\circ, 75^\circ$

$$\begin{aligned} \text{[2]} \quad & y + 3y + 8y = 180^\circ & \therefore 12y = 180^\circ \\ & \therefore y = 15^\circ & \therefore 3y = 45^\circ, 8y = 120^\circ \\ & \therefore \text{The measures of the angles of the triangle} \\ & \text{are } 15^\circ, 45^\circ, 120^\circ \end{aligned}$$



$$\textcircled{3} a + \frac{1}{5}a + 90^\circ = 180^\circ \quad \therefore \frac{5}{5}a + \frac{1}{5}a = 90^\circ$$

$$\therefore \frac{6}{5}a = 90^\circ$$

$$\therefore a = 90^\circ \times \frac{5}{6} = 75^\circ, \frac{1}{5}a = 15^\circ$$

\therefore The measures of the angles of the triangle are $75^\circ, 15^\circ, 90^\circ$

8

$$\therefore C \in \overline{AB}, \text{ then } X + 4X + 3X + 20^\circ = 180^\circ$$

$$\therefore 8X + 20^\circ = 180^\circ \quad \therefore 8X = 160^\circ \quad \therefore X = 20^\circ$$

$$\therefore m(\angle DCE) = 4X = 4 \times 20^\circ = 80^\circ$$

9

$$\textcircled{1} \therefore \overline{DE} \parallel \overline{BC}, \overline{DB} \text{ is a transversal to them}$$

$$\therefore m(\angle DBC) + m(\angle D) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle DBC) = 180^\circ - 110^\circ = 70^\circ$$

$$\therefore \overline{AG} \parallel \overline{BD}, \overline{AB} \text{ is the transversal}$$

$$\therefore m(\angle A) = m(\angle DBC) \text{ (corresponding angles)}$$

$$\therefore 3X + 10^\circ = 70^\circ \quad \therefore 3X = 60^\circ$$

$$\therefore X = \frac{60^\circ}{3} = 20^\circ$$

$$\textcircled{2} \therefore \overline{DE} \parallel \overline{AB}, \overline{DB} \text{ is a transversal to them}$$

$$\therefore m(\angle D) + m(\angle DBA) = 180^\circ$$

By the same way : $m(\angle A) + m(\angle DBA) = 180^\circ$

$$\therefore m(\angle A) = m(\angle D) \quad \therefore X + 20^\circ = 2X - 30^\circ$$

$$\therefore 2X - X = 20^\circ + 30^\circ \quad \therefore X = 50^\circ$$

10

Let the length of the rectangle = X metres

\therefore The width of the rectangle = $(X - 4)$ metres

$$\therefore \text{the perimeter} = (\text{length} + \text{width}) \times 2$$

$$\therefore 68 = (X + X - 4) \times 2 \quad \therefore 34 = 2X - 4$$

$$\therefore 2X = 38 \quad \therefore X = 38 \div 2 = 19 \text{ metres}$$

\therefore The dimensions of the rectangle are 19 metres , 15 metres

11

Let the width of the rectangle = X cm.

\therefore The length of the rectangle = $(2X - 4)$ cm.

\therefore the perimeter of the rectangle = the perimeter of the square

$$\therefore (\text{length} + \text{width}) \times 2 = \text{side length} \times 4$$

$$\therefore (X + 2X - 4) \times 2 = 7 \times 4$$

$$\therefore (3X - 4) \times 2 = 28 \quad \therefore 3X - 4 = 14$$

$$\therefore 3X = 18 \quad \therefore X = 18 \div 3 = 6 \text{ cm.}$$

\therefore The width of the rectangle = 6 cm.

\therefore the length of the rectangle = $2 \times 6 - 4 = 8$ cm.

$$\textcircled{12} \text{ Let the width of the rectangle} = X \text{ cm.}$$

\therefore the length of the rectangle = $2X$ cm.

$$\therefore 2X - 5 = X + 6 \quad \therefore 2X - X = 5 + 6$$

$$\therefore X = 11$$

\therefore The width of the rectangle = 11 cm.

and the length of the rectangle = 22 cm.

$$\therefore \text{The area of the rectangle} = 11 \times 22 = 242 \text{ cm}^2$$

$$\textcircled{13} \therefore 7X - 2X = 25 \quad \therefore 5X = 25$$

$$\therefore X = \frac{25}{5} = 5$$

\therefore The two numbers are : 10 , 35

$$\textcircled{14} \text{ Let one of the two numbers be } X$$

\therefore The other number = $2X$

$$\therefore X + 2X = 108 \quad \therefore 3X = 108$$

$$\therefore X = \frac{108}{3} = 36$$

\therefore the two numbers are 36 , 72

$$\textcircled{15} \text{ Let the great number be } X$$

\therefore The small number = $X - 5$

$$\therefore X + X - 5 = 21 \quad \therefore 2X - 5 = 21$$

$$\therefore 2X = 26 \quad \therefore X = \frac{26}{2} = 13$$

\therefore The two numbers are 13 , 8

$$\textcircled{16} \text{ Let the number be } X \quad \therefore \text{its triple} = 3X$$

$$\therefore 3X + X = 32 \quad \therefore 4X = 32$$

$$\therefore X = \frac{32}{4} = 8 \quad \therefore \text{The number is } 8$$

$$\textcircled{17} \text{ Let the number be } X \quad \therefore \text{its triple} = 3X$$

$$\therefore 3X - 9 = 6 \quad \therefore 3X = 15$$

$$\therefore X = \frac{15}{3} = 5 \quad \therefore \text{The number is } 5$$

$$\textcircled{18} \text{ Let the small number be } X$$

\therefore The middle number = $X + 1$

\therefore the great number = $X + 2$

$$\begin{aligned}\therefore X + X + 1 + X + 2 &= 213 & \therefore 3X + 3 &= 213 \\ \therefore 3X &= 210 & \therefore X &= \frac{210}{3} = 70 \\ \therefore \text{The numbers are } 70, 71, 72\end{aligned}$$

19 Let the three numbers be $X, X+2, X+4$

$$\begin{aligned}\therefore X + X + 2 + X + 4 &= 966 & \therefore 3X + 6 &= 966 \\ \therefore 3X &= 960 & \therefore X &= 320 \\ \therefore \text{The numbers are } 320, 322, 324\end{aligned}$$

20 Let the three numbers be $X, X+2, X+4$

$$\begin{aligned}\therefore X + X + 2 + X + 4 &= 357 & \therefore 3X + 6 &= 357 \\ \therefore 3X &= 351 & \therefore X &= \frac{351}{3} = 117 \\ \therefore \text{The numbers are } 117, 119, 121\end{aligned}$$

21 Let the age of the son be X years

$$\begin{aligned}\therefore \text{The age of the father} &= 3X \\ \text{after 2 years the age of the son} &= (X+2) \text{ years} \\ \text{the age of the father} &= (3X+2) \text{ years} \\ \therefore X + 2 + 3X + 2 &= 52 & \therefore 4X + 4 &= 52 \\ \therefore 4X &= 48 & \therefore X &= \frac{48}{4} = 12 \\ \therefore \text{The age of the son} &= 12 \text{ years} \\ \therefore \text{the age of the father} &= 36 \text{ years}\end{aligned}$$

22 Let the age of Bassim be X years

$$\begin{aligned}\therefore \text{Amgad's age} &= X + 2 \text{ years} \\ \text{Ayman's age} &= X - 6 \text{ years} \\ \therefore X + X + 2 + X - 6 &= 89 & \therefore 3X - 4 &= 89 \\ \therefore 3X &= 93 & \therefore X &= \frac{93}{3} = 31 \text{ years} \\ \therefore \text{The age of Bassim} &= 31 \text{ years} \\ \therefore \text{the age of Amgad} &= 31 + 2 = 33 \text{ years} \\ \therefore \text{the age of Ayman} &= 31 - 6 = 25 \text{ years}\end{aligned}$$

23 Let the price of one metre of silk = X pounds

$$\begin{aligned}\therefore \text{The price of one metre of wool} &= (X+2) \text{ pounds} \\ \therefore 3(X+2) + 4X &= 671 & \therefore 3X + 6 + 4X &= 671 \\ \therefore 7X &= 665 & \therefore X &= 95 \\ \therefore \text{The price of one metre of silk} &= 95 \text{ pounds} \\ \therefore \text{the price of one metre of wool} &= 97 \text{ pounds}\end{aligned}$$

24 **1** $\therefore 5 + 1 = \frac{6}{X}$ $\therefore 6 = \frac{6}{X}$

$$\therefore X = 1 \quad \therefore \text{The S.S.} = \{1\}$$

2 $\therefore \frac{X}{10} + \frac{X}{5} = -\frac{1}{5} + \frac{3}{5}$ $\therefore \frac{X}{10} + \frac{2X}{10} = \frac{2}{5}$

$$\therefore \frac{3X}{10} = \frac{2}{5} \quad \therefore 3X = \frac{2 \times 10}{5}$$

$$\therefore 3X = 4 \quad \therefore X = \frac{4}{3}$$

$$\therefore \text{The S.S.} = \left\{\frac{4}{3}\right\}$$

25 **1** $\therefore X^2 + 6X + 9 - (X^2 - 4X + 4) = 15$

$$\begin{aligned}\therefore X^2 + 6X + 9 - X^2 + 4X - 4 &= 15 \\ \therefore 10X + 5 &= 15 & \therefore 10X &= 10 \\ \therefore X &= 1 & \therefore \text{The S.S.} &= \{1\}\end{aligned}$$

2 $\therefore 4X^2 + 4X - 3 - (4X^2 - 4X + 1) = 14$

$$\begin{aligned}\therefore 4X^2 + 4X - 3 - 4X^2 + 4X - 1 &= 14 \\ \therefore 8X - 4 &= 14 & \therefore 8X &= 18 \\ \therefore X &= \frac{18}{8} = \frac{9}{4} & \therefore \text{The S.S.} &= \left\{\frac{9}{4}\right\}\end{aligned}$$

26 $\therefore 12X + 3 = 39$ $\therefore 12X = 36$ $\therefore X = 3$

$$\therefore \text{The S.S.} = \{3\}$$

$$\therefore X = 3 \text{ satisfies the equation } aX - 12 = a$$

$$\therefore 3a - 12 = a \quad \therefore 3a - a = 12 \quad \therefore 2a = 12$$

$$\therefore a = 6$$

27 $\therefore a + 1$ is a solution for the equation

$$\begin{aligned}\therefore X = a + 1 \text{ satisfies the equation.} \\ \therefore (a + 1 + a)(a + 1 - a) &= (a + 1)^2 - a(a + 1) + 3 \\ \therefore 2a + 1 &= a^2 + 2a + 1 - a^2 - a + 3 \\ \therefore 2a + 1 &= a + 4 & \therefore 2a - a &= 4 - 1 & \therefore a &= 3\end{aligned}$$

28 Let the age of the boy who was born in 1980 be X

$$\begin{aligned}\therefore \text{The ages of the two other boys are } X - 4, X - 6 \\ \therefore X + X - 4 + X - 6 &= 41 \\ \therefore 3X - 10 &= 41 & \therefore 3X &= 51 & \therefore X &= \frac{51}{3} = 17 \\ \therefore \text{The year in which the sum of their ages became} \\ 41 \text{ years} &= 1980 + 17 = 1997\end{aligned}$$

Answers of Exercise 8

1

1 - 5

2 4

3 7

4 - 9

5 1.5

6 - 0.6

7 $2\frac{1}{2}$

8 - $\frac{1}{3}$



$$2 \quad x+3-3 \leq 6-3 \quad \therefore x \leq 3$$

$$① \text{ The S.S.} = \{3, 2, 1, 0, \dots\}$$



$$② \text{ The S.S.} = \{3, 2, 1, 0\}$$



$$3 \quad ① \quad x+2-2 > 5-2 \quad \therefore x > 3$$

$$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x > 3\}$$

$$② \quad x+4-4 > 1-4 \quad \therefore x > -3$$

$$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x > -3\}$$

$$③ \quad y-5+5 > 7+5 \quad \therefore y > 12$$

$$\therefore \text{The S.S.} = \{y : y \in \mathbb{Q}, y > 12\}$$

$$④ \quad 19-14 < y+14-14 \quad \therefore 5 < y$$

$$\therefore \text{The S.S.} = \{y : y \in \mathbb{Q}, y > 5\}$$

$$⑤ \quad -1+3 \geq x-3+3 \quad \therefore 2 \geq x$$

$$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x \leq 2\}$$

$$⑥ \quad -5\frac{1}{2} - 1\frac{1}{4} > a + 1\frac{1}{4} - 1\frac{1}{4} \quad \therefore -6\frac{3}{4} > a$$

$$\therefore \text{The S.S.} = \{a : a \in \mathbb{Q}, a < -6\frac{3}{4}\}$$

$$⑦ \quad -2x \times -\frac{1}{2} > 12 \times -\frac{1}{2} \quad \therefore x > -6$$

$$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x > -6\}$$

$$⑧ \quad \frac{2}{3}x \times \frac{3}{2} \geq 1 \times \frac{3}{2} \quad \therefore x \geq \frac{3}{2}$$

$$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x \geq \frac{3}{2}\}$$

$$⑨ \quad -\frac{1}{4}x \times -4 \geq \frac{1}{4} \times -4 \quad \therefore x \geq -1$$

$$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x \geq -1\}$$

4

$$① \quad 3x-2+2 < 1+2 \quad \therefore 3x < 3$$

$$\therefore 3x \times \frac{1}{3} < 3 \times \frac{1}{3} \quad \therefore x < 1$$

$$② \quad 2x+3-3 < 9-3 \quad \therefore 2x < 6$$

$$\therefore 2x \times \frac{1}{2} < 6 \times \frac{1}{2} \quad \therefore x < 3$$

$$③ \quad 4x+2-2 \geq -10-2 \quad \therefore 4x \geq -12$$

$$\therefore 4x \times \frac{1}{4} \geq -12 \times \frac{1}{4} \quad \therefore x \geq -3$$

$$④ \quad 3x-2+2 \geq 5+2 \quad \therefore 3x \geq 7$$

$$\therefore 3x \times \frac{1}{3} \geq 7 \times \frac{1}{3} \quad \therefore x \geq \frac{7}{3}$$

$$⑤ \quad 3x-9+9 < 0+9 \quad \therefore 3x < 9$$

$$\therefore 3x \times \frac{1}{3} < 9 \times \frac{1}{3} \quad \therefore x < 3$$

$$⑥ \quad 1+2x-1 \leq -3-1 \quad \therefore 2x \leq -4$$

$$\therefore 2x \times \frac{1}{2} \leq -4 \times \frac{1}{2} \quad \therefore x \leq -2$$

$$⑦ \quad 9-6x-9 < 15-9 \quad \therefore -6x < 6$$

$$\therefore -6x \times -\frac{1}{6} > 6 \times -\frac{1}{6} \quad \therefore x > -1$$

$$⑧ \quad 2-3x-2 \leq 4-2 \quad \therefore -3x \leq 2$$

$$\therefore -3x \times -\frac{1}{3} \geq 2 \times -\frac{1}{3} \quad \therefore x \geq -\frac{2}{3}$$

$$⑨ \quad \frac{3x-2}{5} \times 5 \geq \frac{1}{2} \times 5 \quad \therefore 3x-2 \geq \frac{5}{2}$$

$$\therefore 3x-2+2 \geq \frac{5}{2}+2 \quad \therefore 3x \geq \frac{9}{2}$$

$$\therefore 3x \times \frac{1}{3} \geq \frac{9}{2} \times \frac{1}{3} \quad \therefore x \geq \frac{3}{2}$$

$$⑩ \quad 5x+1-1 \leq 29-1 \quad \therefore 5x \leq 28$$

$$\therefore 5x \times \frac{1}{5} \leq 28 \times \frac{1}{5} \quad \therefore x \leq \frac{28}{5}$$

$$⑪ \quad 4n-2n+2 \geq 0 \quad \therefore 2n+2-2 \geq 0-2$$

$$\therefore 2n \geq -2 \quad \therefore 2n \times \frac{1}{2} \geq -2 \times \frac{1}{2}$$

$$\therefore n \geq -1$$

$$⑫ \quad -3m+6m-24 > 9$$

$$\therefore 3m-24+24 > 9+24 \quad \therefore 3m > 33$$

$$\therefore 3m \times \frac{1}{3} > 33 \times \frac{1}{3} \quad \therefore m > 11$$

5

$$① \quad 6d-5d \leq -3-1 \quad \therefore d \leq -4$$

$$② \quad 6x-5x \geq 14-2 \quad \therefore x \geq 12$$

$$③ \quad -2+8 < 5x-3x \quad \therefore 6 < 2x$$

$$\therefore 3 < x \quad \therefore x > 3$$

$$④ \quad 8 \leq 5x+2x \quad \therefore 8 \leq 7x$$

$$\therefore x \geq \frac{8}{7}$$

$$⑤ \quad 5x+1 \geq 2x+4 \quad \therefore 5x-2x \geq 4-1$$

$$\therefore 3x \geq 3 \quad \therefore x \geq 1$$

$$⑥ \quad 3x+6 < -x+4 \quad \therefore 3x+x < 4-6$$

$$\therefore 4x < -2 \quad \therefore x < -\frac{2}{4}$$

$$\therefore x < -\frac{1}{2}$$

$$⑦ \quad 3x+6 \geq -2x-2 \quad \therefore 3x+2x \geq -2-6$$

$$\therefore 5x \geq -8 \quad \therefore x \geq -\frac{8}{5}$$

$$⑧ \quad 2-3x+15 \geq x+7 \quad \therefore 17-7 \geq x+3x$$

$$\therefore 10 \geq 4x \quad \therefore x \leq \frac{10}{4}$$

$$\therefore x \leq \frac{5}{2}$$

9 $21y - 1 \leq 20y - 1$

$$\therefore 21y - 20y \leq -1 + 1 \quad \therefore y \leq 0$$

10 $3 - 1 \leq 2X - \frac{X}{2} \quad \therefore 2 \leq \frac{3}{2}X$

$$\therefore X \geq \frac{4}{3}$$

6

1 $9 - 1 \leq 4X + 1 - 1 \leq 17 - 1 \quad \therefore 8 \leq 4X \leq 16$

$$\therefore 8 \times \frac{1}{4} \leq 4X \times \frac{1}{4} \leq 16 \times \frac{1}{4} \quad \therefore 2 \leq X \leq 4$$

$$\therefore \text{The S.S.} = \{2, 3, 4\}$$

2 $9 - 2 \leq 3X + 2 - 2 < 12 - 2$

$$\therefore 7 \times \frac{1}{3} \leq 3X \times \frac{1}{3} < 10 \times \frac{1}{3} \quad \therefore \frac{7}{3} \leq X < \frac{10}{3}$$

$$\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, \frac{7}{3} \leq X < \frac{10}{3}\}$$

3 $9 - 6 > X + 6 - 6 > 2 - 6$

$$\therefore 3 > X > -4 \quad \therefore \text{The S.S.} = \{0, 1, 2\}$$

7 $\text{1} > \quad \text{2} < \quad \text{3} z \quad \text{4} X \quad \text{5} 3 > a$

$$\text{6} a > -5 \quad \text{7} < \quad \text{8} > \quad \text{9} >$$

8 $\text{1} (b) \quad \text{2} (d) \quad \text{3} (d) \quad \text{4} (d) \quad \text{5} (b)$

$$\text{6} (d) \quad \text{7} (b) \quad \text{8} (c) \quad \text{9} (c)$$

9

 When $a = 4$ and $b = 3$, then $a > b$
 $c = 8$ and $d = 2$, then $c > d$ then $a - c = 4 - 8 = -4$
and $b - d = 3 - 2 = 1$

 i.e. $a - c < b - d$

So we see that

 If $a > b$ and $c > d$ it is not always correct that $a - c > b - d$

10

1 (✓)

2 (✗)

For example:

 When $X = -3$ and $y = -5$, then $-3 > -5$ but $-3 < 0$

3 (✓)

4 (✗) For example:

 When $y = \frac{1}{2}$, then $y^2 = \frac{1}{4}$ but $\frac{1}{4} < \frac{1}{2}$

5 (✗) For example:

 When $X = 4$, $y = -2$, it is $4 \times -2 < 0$ $-8 < 0$

6 (✗) For example:

 When $X = -4$, $y = -9$, then

$$X + y = -4 + (-9) = -13 \quad -13 < -9$$

7 (✗) For example:

 When $y = -1$, $X = 5$, then $y^2 = 1$ but $1 < 5$

8 (✗) For example:

 When $X = 4$, $y = -2$, then $y^2 = 4$, $Xy = -8$
but $4 > -8$

9 (✗) For example:

 When $X = -2$, $y = -4$, then $Xy = -2 \times -4 = 8$,
 $X^2 = 4$ but $8 > 4$

10 (✗) For example:

 When $X = 2$, $y = 1$, then $X^3 = 8$, $y^2 = 1$
but $8 > 1$

11

 Let the number of shirts be X

$$\therefore 40X + 70 \leq 200 \quad \therefore 40X \leq 200 - 70$$

$$\therefore 40X \leq 130 \quad \therefore X \leq 3.25$$

 \therefore The greatest number of shirts Hany can buy is 3 shirts.

12

$$a \leq 3X - 5 \leq b \quad \therefore a + 5 \leq 3X \leq b + 5$$

$$\therefore \frac{a+5}{3} \leq X \leq \frac{b+5}{3} \quad \therefore 2 \leq X \leq 5$$

$$\therefore \frac{a+5}{3} = 2 \quad \therefore a + 5 = 6 \quad \therefore a = 1$$

$$\therefore \frac{b+5}{3} = 5 \quad \therefore b + 5 = 15 \quad \therefore b = 10$$

13

 1 The greatest possible value of the expression $X + y$
is when $X = 5$, $y = 7 \rightarrow$ the value $= 5 + 7 = 12$

 2 The greatest possible value of the expression $y - X$
is when $y = 7$, $X = -4$
The value $= 7 - (-4) = 11$

 3 The smallest possible value of the expression Xy
is when $X = -4$, $y = 7$
The value $= -4 \times 7 = -28$

 4 The smallest possible value of the expression
 $X^2 + y^2$ is when $X = 0$, $y = 0$
The value $= 0^2 + 0^2 = 0$



Answers of unit two

Answers of Exercise 9

Answer by yourself.

Answers of Exercise 10

First : Problems on experimental probability

Answer by yourself.

Second : Problems on theoretical probability

1

- 1 $\frac{2}{3}$ 2 $\frac{1}{3}$ 3 $\frac{1}{2}$ 4 $\frac{1}{6}$ 5 zero
 6 1 7 $\frac{1}{2}$ 8 $\frac{1}{6}$ 9 $\frac{1}{6}$ 10 $\frac{1}{3}$

2

- 1 zero, 1 2 $\frac{1}{2}$ 3 $\frac{1}{2}$ 4 zero
 5 $\frac{1}{12}$, $\frac{11}{12}$ 6 $\frac{3}{8}$ 7 $\frac{1}{3}$ 8 600

3

- 1 (d) 2 (b) 3 (a) 4 (b)
 5 (b) 6 (c) 7 (b) 8 (c)

4

- 1 \therefore The numbers from 1 to 25 and divisible by 5 are 5, 10, 15, 20, 25 and their number = 5
 \therefore The probability = $\frac{5}{25} = \frac{1}{5}$
 2 \therefore The numbers from 1 to 25 and more than or equal to 20 are 20, 21, 22, 23, 24, 25 its number is 6
 \therefore The probability = $\frac{6}{25}$
 3 \therefore The numbers from 1 to 25 and each of them is a perfect square are 1, 4, 9, 16, 25 and their number is 5
 \therefore The probability = $\frac{5}{25} = \frac{1}{5}$

5

$$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

- 1 The probability of getting an even number
 $= \frac{4}{8} = \frac{1}{2}$
 2 The probability of getting an odd number
 $= \frac{4}{8} = \frac{1}{2}$

- 3 The probability of getting a number greater than or equal to 6 = $\frac{3}{8}$
 4 The probability of getting a number divisible by 3
 $= \frac{2}{8} = \frac{1}{4}$

6

- 1 The probability that the letter S = $\frac{1}{5}$
 2 The probability that the letter E = $\frac{1}{5}$
 3 The probability that the letter R = zero

7

The total number of balls = $5 + 3 + 2 = 10$

- 1 The probability that the ball is yellow = $\frac{3}{10} = 0.3$
 2 The probability that the ball is yellow or red = $\frac{3+5}{10}$
 $= \frac{8}{10} = 0.8$
 3 The probability that the ball is not yellow = $\frac{5+2}{10}$
 $= \frac{7}{10} = 0.7$

8

- 1 The probability that it shows an odd number
 $= \frac{5}{10} = \frac{1}{2}$
 2 The probability that it shows a prime number
 $= \frac{4}{10} = \frac{2}{5}$
 3 The probability that it shows an even number
 $= \frac{5}{10} = \frac{1}{2}$
 4 The probability that it shows an odd number greater than 3 = $\frac{3}{10}$

9

- 1 The probability of appearing an even number less than or equal to 4 = $\frac{2}{6} = \frac{1}{3}$
 2 The probability of appearing a number between 0 and 10 = $\frac{6}{6} = 1$
 3 The probability of appearing a number divisible by 7 = $\frac{0}{6} = \text{zero}$
 4 The probability of appearing a number not divisible by 2 = $\frac{3}{6} = \frac{1}{2}$

10

$$S = \{1, 2, 3, 4, 5, 6\}$$

- 1 The probability of getting a number greater than 6
 $= \frac{0}{6} = \text{zero}$

2 The probability of getting a number satisfying the inequality $1 \leq X \leq 6 = \frac{6}{6} = 1$

3 The probability of getting a number satisfying the inequality $2 < X < 4 = \frac{1}{6}$

11

1 The probability that the card carries a number whose tens digit is even = $\frac{2}{8} = \frac{1}{4}$

2 The probability that the card carries a number whose units digit is odd = $\frac{2}{8} = \frac{1}{4}$

3 The probability that the card carries a number multiple of 4 = $\frac{4}{8} = \frac{1}{2}$

12

1 $S = \{1, 2, 3\}$

2 The probability that the apparent number on the upper face is 2 = $\frac{1}{3}$

3 The probability that the apparent number is odd = $\frac{2}{3}$

13 The number of red marbles = $\frac{2}{5} \times 30 = 12$ marbles

14

\therefore The probability of drawing a red ball = $\frac{1}{4}$

\therefore The probability of drawing a blue ball = $\frac{3}{4}$

\therefore The number of blue balls = $\frac{3}{4} \times 80 = 60$ balls

15

$S = \{22, 32, 52, 33, 23, 53, 55, 25, 35\}$

1 The probability that the tens digit is odd = $\frac{6}{9} = \frac{2}{3}$

2 The probability that the units digit is odd = $\frac{6}{9} = \frac{2}{3}$

3 The probability that the sum of the two digits 7 = $\frac{2}{9}$

4 The probability that the product of the two digits 15 = $\frac{2}{9}$

16

\therefore The number of red marbles = $22 - 12 = 10$ and after drawing two red marbles the rest marbles will be 20 and the red marbles is 8

\therefore The probability that the drawn marble is black = $\frac{12}{20} = \frac{3}{5}$

17

The number of girls = 20, The number of boys = 30

\therefore The probability that the student is a boy = $\frac{30}{50} = \frac{3}{5}$

18 1 (c)

2 (b)

3 (d)

4 (c)

5 (b)

6 (b)

19

The probability that the pointer stops at yellow colour

$$= 1 - \left(\frac{1}{8} + \frac{1}{4} + \frac{3}{8} \right) = 1 - \frac{6}{8} = \frac{2}{8}$$

\therefore The probability that the pointer stops at the yellow or red colour = $\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

20

1 The probability that the student succeeded

$$\text{in math} = \frac{30}{40} = \frac{3}{4}$$

2 The probability that the student succeeded

$$\text{in science} = \frac{24}{40} = \frac{3}{5}$$

3 The probability that the student failed

$$\text{in science} = 1 - \frac{3}{5} = \frac{2}{5}$$

4 \therefore The number of succeeded students in both math and science is 20 students.

\therefore The number of students who succeeded in math only = $30 - 20 = 10$ students

\therefore The number of students who succeeded in science only = $24 - 20 = 4$ students

\therefore The number of students who failed in both math and science = $40 - (20 + 10 + 4) = 6$ students

\therefore The probability that the student failed in both math and science = $\frac{6}{40} = \frac{3}{20}$

21

\therefore The probability that the first player scores a goal

$$= \frac{18}{21} = 0.86$$

The probability that the second player scores a goal

$$= \frac{25}{32} = 0.78$$

$\therefore 0.86 > 0.78$

\therefore The best is choosing the first player because his probability is the greater.

22

1 No, because the product of an odd number by an even number = an even number.

i.e. The probability of getting an even number is the greater.



- 22 Souad, because she wins when the result is an even number and its probability is the greater.

23

The probability of shooting the shaded part

$$= \frac{\text{The area of the shaded rectangle}}{\text{The area of the external rectangle}} = \frac{5 \times 10}{20 \times 10} = \frac{1}{4}$$

24

\therefore The probability of drawing a red ball = $\frac{2}{3}$

\therefore The probability of drawing a white ball

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

\therefore The total number of balls = $3 \times 5 = 15$ balls

25

The probability that the drawn card carries a number less than or equal to 8

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

\therefore The number of cards = $8 \times \frac{3}{2} = 12$ cards

$\therefore n = 12$

Answers of accumulative basic skills

- | | | | | | | | |
|---|--------|---|--------|---|--------|---|--------|
| 1 | 1 (c) | 2 | (b) | 3 | (a) | 4 | (c) |
| | 5 (b) | | 6 (a) | | 7 (b) | | 8 (a) |
| | 9 (c) | | 10 (c) | | 11 (d) | | 12 (d) |
| | 13 (c) | | 14 (a) | | 15 (c) | | |
-
- | | | | | |
|---|------------------|-------|-----------------|-----------------|
| 2 | 1 30.398 | 2 -2 | 3 $\frac{1}{9}$ | 4 11 |
| | 5 $\frac{9x}{8}$ | 6 90 | 7 50 | 8 $\frac{5}{6}$ |
| | 9 10 | 10 40 | 11 25 | 12 15 |
| | 13 1 | 14 3 | 15 15 | |

Guide Answers

of Geometry and
Measurement Exercises





Answers of unit three

Answers of Exercise 1

- 1 1 Given 2 $A \in \overleftrightarrow{CE}$ 3 Given
4 Corresponding angles. 5 Alternate angles

2

Given :

$$m(\angle AMB) = 50^\circ, m(\angle EMD) = 80^\circ,$$

$$m(\angle CMD) = 65^\circ, \overleftrightarrow{MC} \text{ bisects } \angle BMD$$

$$\text{R.T.F. : } m(\angle AME)$$

Proof :

$$\because \overleftrightarrow{MC} \text{ bisects } \angle BMD \text{ (given)}$$

$$\therefore m(\angle BMC) = m(\angle CMD) = 65^\circ$$

$$\therefore m(\angle AMB) + m(\angle BMC) + m(\angle CMD)$$

$$+ m(\angle DME) + m(\angle AME) = 360^\circ$$

$$\therefore m(\angle AME) = 360^\circ - 260^\circ = 100^\circ \quad (\text{The req.})$$

3

$$\text{Given : } \overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}, m(\angle BMC) = 120^\circ,$$

$$\overleftrightarrow{ME} \text{ bisects } \angle AMD$$

$$\text{R.T.F. : } m(\angle EMC)$$

Proof :

$$\because \overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$$

$$\therefore m(\angle BMC) = m(\angle AMD) \text{ (V.O.A.)}$$

$$\therefore m(\angle AMD) = 120^\circ,$$

$$\because \overleftrightarrow{ME} \text{ bisects } \angle AMD$$

$$\therefore m(\angle AME) = m(\angle EMD)$$

$$\therefore m(\angle EMD) = \frac{120^\circ}{2} = 60^\circ$$

$$\therefore M \in \overleftrightarrow{BD}$$

$$\therefore m(\angle BMC) + m(\angle CMD) = 180^\circ$$

$$\therefore m(\angle DMC) = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore m(\angle EMC) = m(\angle EMD) + m(\angle DMC)$$

$$\therefore m(\angle EMC) = 60^\circ + 60^\circ = 120^\circ \quad (\text{The req.})$$

4

Given :

$$AB = AC, BD = CD$$

$$\text{R.T.P. : } \overleftrightarrow{AD} \text{ bisects } \angle BAC$$

$$\text{Proof : } \because \text{In } \triangle ADB, \triangle ADC :$$

$$\left\{ \begin{array}{l} AB = AC \text{ (given)} \\ BD = CD \text{ (given)} \end{array} \right.$$

$$\left\{ \begin{array}{l} \overleftrightarrow{AD} \text{ is a common side} \end{array} \right.$$

$$\therefore \triangle ADB \cong \triangle ADC, \text{ then we deduce that :}$$

$$m(\angle BAD) = m(\angle CAD)$$

$$\therefore \overleftrightarrow{AD} \text{ bisects } \angle BAC \quad (\text{Q.E.D.})$$

5

$$\therefore m(\angle EBD) + m(\angle CBD) + m(\angle ABC)$$

$$+ m(\angle ABE) = 360^\circ$$

$$\therefore m(\angle EBD) + 35^\circ + 110^\circ + 140^\circ = 360^\circ$$

$$\therefore m(\angle EBD) = 360^\circ - 285^\circ = 75^\circ \quad (\text{The req.})$$

6

$$\therefore m(\angle CBF) = m(\angle CBA) + m(\angle ABF)$$

$$\therefore m(\angle CBF) = 40^\circ + 30^\circ = 70^\circ$$

$$\therefore \overleftrightarrow{CE} \cap \overleftrightarrow{FD} = \{B\}$$

$$\therefore m(\angle DBE) = m(\angle CBF) = 70^\circ \text{ (V.O.A.) (The req.)}$$

7

$$\therefore B \in \overleftrightarrow{AC}$$

$$\therefore m(\angle ABE) + m(\angle CBE) = 180^\circ$$

$$\therefore m(\angle ABE) = 180^\circ - 116^\circ = 64^\circ$$

$$\therefore \overleftrightarrow{BD} \text{ bisects } \angle ABE$$

$$\therefore m(\angle ABD) = m(\angle DBE) = 64^\circ \div 2 = 32^\circ \quad (\text{The req.})$$

8

$$\therefore \overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$$

$$\therefore m(\angle CBE) = m(\angle ABD) = 40^\circ \text{ (V.O.A.)}$$

$$\therefore \overleftrightarrow{BE} \text{ bisects } \angle CBF$$

$$\therefore m(\angle FBE) + m(\angle EBC) = 80^\circ$$

$$\therefore m(\angle FBC) = 80^\circ$$

$$\therefore B \in \overleftrightarrow{AC}$$

$$\therefore m(\angle ABF) = 180^\circ - 80^\circ = 100^\circ \quad (\text{The req.})$$

9

$$\therefore m(\angle BEC) + m(\angle AED) + m(\angle AEB)$$

$$+ m(\angle CED) = 360^\circ$$

$$\therefore m(\angle BEC) + m(\angle AED) + 65^\circ + 85^\circ = 360^\circ$$

$$\therefore m(\angle BEC) + m(\angle AED)$$

$$= 360^\circ - (65^\circ + 85^\circ) = 210^\circ$$

$$\therefore m(\angle BEC) = m(\angle AED)$$

$$\therefore m(\angle BEC) = \frac{210^\circ}{2} = 105^\circ \quad (\text{First req.})$$

$$\therefore m(\angle AEB) + m(\angle BEC) = 65^\circ + 105^\circ$$

$$= 170^\circ \neq 180^\circ$$

$$\therefore A, E \text{ and } C \text{ are not on the same straight line.}$$

$$(\text{Second req.})$$

10

$$1 \quad m(\angle ABC) = 180^\circ - 72^\circ = 108^\circ$$

(Two interior angles in the same side of the transversal)

$$2 \quad m(\angle ABC) = m(\angle BAD) = 57^\circ \text{ (Alternate angles)}$$

$$3 \quad m(\angle ABC) = m(\angle EAD) = 63^\circ$$

(Corresponding angles)

11

$$\therefore m(\angle EAC) + m(\angle BAC) + m(\angle EAB) = 360^\circ$$

$$\therefore m(\angle BAC) = 360^\circ - (130^\circ + 90^\circ) = 140^\circ \text{ (First req.)}$$

$$\therefore \overline{AB} \parallel \overline{CD}, \overline{AC} \text{ is a transversal to them}$$

$$\therefore m(\angle C) + m(\angle CAB) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle C) = 180^\circ - 140^\circ = 40^\circ \text{ (Second req.)}$$

12

$$\therefore \overline{AB} \parallel \overline{CD}, \overline{AC} \text{ is a transversal to them}$$

$$\therefore m(\angle ACD) + (\angle A) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle ACD) = 180^\circ - 60^\circ = 120^\circ \quad (1)$$

$$\therefore \overline{AB} \parallel \overline{CD}, \overline{AB} \parallel \overline{EF}$$

$$\therefore \overline{CD} \parallel \overline{EF}, \overline{CE} \text{ is a transversal to them.}$$

$$\therefore m(\angle DCE) + m(\angle E) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle DCE) = 180^\circ - 35^\circ = 145^\circ \quad (2)$$

$$\therefore m(\angle ACD) + m(\angle DCE) + m(\angle ACE) = 360^\circ$$

From (1) and (2)

$$\therefore m(\angle ACE) = 360^\circ - (120^\circ + 145^\circ) = 95^\circ \text{ (The req.)}$$

$$13 \quad \therefore \overline{DE} \parallel \overline{BC}, \overline{AB} \text{ is a transversal to them}$$

$$\therefore m(\angle B) = m(\angle DAB) = 80^\circ \text{ (Alternate angles)}$$

$$\therefore \overline{DE} \parallel \overline{BC}, \overline{AC} \text{ is a transversal to them.}$$

$$\therefore m(\angle C) = m(\angle EAC) = 50^\circ \text{ (Alternate angles)}$$

$$\therefore A \in \overline{DE}$$

$$\therefore m(\angle BAC) = 180^\circ - (50^\circ + 80^\circ) = 50^\circ \text{ (The req.)}$$

14

$$1 \quad \therefore E \in \overline{AB} \quad \therefore m(\angle BEF) = 180^\circ - 122^\circ = 58^\circ$$

$$\therefore m(\angle BEF) = m(\angle EFC) \text{ (Alternate angles)}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (\text{Q.E.D.})$$

$$2 \quad \therefore E \in \overline{AB}$$

$$\therefore m(\angle BEF) = 180^\circ - 100^\circ = 80^\circ$$

$$\therefore m(\angle BEF) = m(\angle DFN) \text{ (Corresponding angles)}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (\text{Q.E.D.})$$

$$3 \quad \therefore m(\angle BEF) = m(\angle AEM) = 132^\circ \quad (\text{V.O.A.})$$

$$\therefore m(\angle BEF) = m(\angle DFN) \text{ (Corresponding angles)}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (\text{Q.E.D.})$$

15

$$\therefore \overline{AB} \parallel \overline{CD}, \overline{AC} \text{ is a transversal to them.}$$

$$\therefore m(\angle ACD) = m(\angle A) = 50^\circ \text{ (Alternate angles)}$$

$$\therefore m(\angle ACE) = 90^\circ$$

$$\therefore m(\angle DCE) = 90^\circ - 50^\circ = 40^\circ$$

$$\therefore m(\angle DCE) = m(\angle E) \text{ (But they are alternate angles)}$$

$$\therefore \overline{CD} \parallel \overline{EF}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad \therefore \overline{AB} \parallel \overline{EF} \quad (\text{Q.E.D.})$$

16

$$\therefore \overline{EF} \parallel \overline{CD}, \overline{EC} \text{ is a transversal to them.}$$

$$\therefore m(\angle ECD) = m(\angle CEF) = 95^\circ \text{ (Alternate angles)}$$

$$\therefore m(\angle ACD) = 95^\circ - 30^\circ = 65^\circ$$

$$\therefore m(\angle ACD) + m(\angle A) = 65^\circ + 115^\circ = 180^\circ$$

(But they are interior angles in the same side of the transversal)

$$\therefore \overline{AB} \parallel \overline{CD} \quad \therefore \overline{CD} \parallel \overline{EF}$$

$$\therefore \overline{AB} \parallel \overline{EF} \quad (\text{Q.E.D.})$$

17

$$\therefore \overline{AD} \cap \overline{BC} = \{M\}$$

$$\therefore m(\angle AMB) = m(\angle DMC) \text{ (V.O.A.)}$$

$$\therefore \text{In } \triangle AMB \text{ and } \triangle DMC$$

$$\begin{cases} AM = DM \\ BM = CM \\ m(\angle AMB) = m(\angle DMC) \end{cases}$$

$$\therefore \text{The two triangles are congruent and we deduce that } AB = CD \quad (\text{Q.E.D.1})$$

$$\therefore m(\angle A) = m(\angle D) \text{ (But they are alternate angles)}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (\text{Q.E.D.2})$$



18

1 Let $\overline{AB} \parallel \overline{CD}$, $\overline{EF} \perp \overline{AB}$ $\therefore \overline{AB} \parallel \overline{CD}$ and \overline{EF}

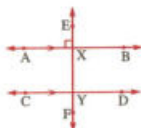
is a transversal to them.

 $\therefore m(\angle XYZ) = m(\angle EXA)$

(Corresponding angles)

 $\therefore m(\angle XYZ) = 90^\circ$ $\therefore \overline{EF} \perp \overline{CD}$

(Q.E.D.)

2 Let the straight line $L \parallel$ the straight line M , the straight line $L \parallel$ the straight line N ,, the straight line K

is a transversal to all of them.

 \therefore The straight line $L \parallel$ the straight line M ,and the straight line K

is a transversal to them.

 $\therefore m(\angle ABC) = m(\angle DEB)$

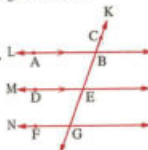
(corresponding angles),

 \therefore The straight line $L \parallel$ the straight line N ,and the straight line K is a transversal to them. $\therefore m(\angle ABC) = m(\angle FGE)$

(Corresponding angles)

 $\therefore m(\angle DEB) = m(\angle FGE)$

(But they are corresponding angles)

 \therefore The straight line $M \parallel$ the straight line N (Q.E.D.)19 $\Delta \triangle ABD$, $\triangle CBD$ in them :

$$\begin{cases} AB = CB \\ AD = CD \\ \overline{BD} \text{ is a common side} \end{cases}$$

 $\therefore \Delta \triangle ABD \cong \Delta \triangle CBD$, then we deduce that : $m(\angle ADB) = m(\angle CDB)$ $\therefore \overline{DB}$ bisects $\angle ADC$

(Q.E.D. 1)

 $\therefore \Delta \triangle ADF$, $\triangle CDF$ in them :

$$\begin{cases} AD = CD \\ \overline{DF} \text{ is a common side} \\ m(\angle ADF) = m(\angle CDF) \end{cases}$$

 $\therefore \Delta \triangle ADF \cong \Delta \triangle CDF$, then we deduce that : $m(\angle AFD) = m(\angle CFD)$ $\therefore F \in \overline{AC}$ $\therefore m(\angle AFD) + m(\angle CFD) = 180^\circ$ $\therefore m(\angle AFD) = m(\angle CFD) = \frac{180^\circ}{2} = 90^\circ$ $\therefore \overline{AC} \perp \overline{DB}$

(Q.E.D. 2)

20

 $\therefore \overline{AE} \cap \overline{DC} = \{F\}$ $\therefore m(\angle AFD) = m(\angle EFC)$

(V.O.A.)

 $\therefore \Delta \triangle AFD$ and $\triangle EFC$ in them :

$$\begin{cases} m(\angle D) = m(\angle FCE) \\ m(\angle AFD) = m(\angle EFC) \\ FD = FC \end{cases}$$

 $\therefore \Delta \triangle AFD \cong \Delta \triangle EFC$ Then we deduce that $CE = AD$ $\therefore ABCD$ is a square $\therefore BC = AD$ $\therefore CE = CB$

(Q.E.D.)

21

 $\Delta \triangle ABD$, $\triangle CBD$ in them :

$$\begin{cases} AD = CB \\ \overline{BD} \text{ is a common side} \\ m(\angle ADB) = m(\angle CBD) \end{cases}$$

 $\therefore \Delta \triangle ABD \cong \Delta \triangle CBD$, then we deduce that : $AB = CD$

(Q.E.D. 1)

 $m(\angle ABD) = m(\angle CBD)$

(And they are two alternate angles)

 $\therefore \overline{AB} \parallel \overline{CD}$

(Q.E.D. 2)

22

 $\Delta \triangle DXB$, $\triangle EYC$ in them :

$$\begin{cases} DB = EC \\ DX = EY \\ m(\angle DXB) = m(\angle EYC) = 90^\circ \end{cases}$$

 $\therefore \Delta \triangle DXB \cong \Delta \triangle EYC$, then we deduce that : $m(\angle B) = m(\angle C)$

(1)

 $\therefore \overline{BC} \parallel \overline{DE}$ and \overline{AB} is a transversal to them $\therefore m(\angle ADE) = m(\angle B)$

(2)

(Corresponding angles)

Similarly : $m(\angle AED) = m(\angle C)$

(3)

(Corresponding angles)

From (1) and (2) and (3):

$$\therefore m(\angle ADE) = m(\angle AED) \quad (\text{Q.E.D.})$$

23

 In $\triangle ABE$ & $\triangle ACD$ in them:

$$\begin{cases} AE = AD \\ m(\angle AEB) = m(\angle ADC) \\ \angle A \text{ is a common angle} \end{cases}$$

$$\therefore \triangle ABE \cong \triangle ACD$$

 Then we deduce that: $BE = CD$ (Q.E.D. 1)

$$\therefore AB = AC \quad \therefore AD + DB = AE + EC$$

$$\therefore AD = AE$$

$$\therefore DB = EC \quad (\text{Q.E.D. 2})$$

24

$$1) \therefore 3x - 5^\circ = 70^\circ \text{ (V.O.A.)}$$

$$\therefore 3x = 70^\circ + 5^\circ = 75^\circ$$

$$\therefore x = \frac{75^\circ}{3} = 25^\circ$$

$$2) \therefore x^2 = 36^\circ + 64^\circ = 100^\circ \text{ (V.O.A.)}$$

$$\therefore x = \pm 10^\circ$$

$$3) \therefore (3x - 8^\circ) + x = 180^\circ \quad \therefore 4x - 8^\circ = 180^\circ$$

$$\therefore 4x = 180^\circ + 8^\circ = 188^\circ$$

$$\therefore x = \frac{188^\circ}{4} = 47^\circ$$

$$4) x = 60^\circ \text{ (Alternate angles),}$$

$$y = 61^\circ \text{ (Corresponding angles)}$$

$$5) \therefore 3x + 90^\circ = 180^\circ$$

$$\therefore 3x = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore x = \frac{90^\circ}{3} = 30^\circ \quad \therefore 4y + 5y = 180^\circ$$

$$\therefore 9y = 180^\circ \quad \therefore y = \frac{180^\circ}{9} = 20^\circ$$

$$6) \therefore m(\angle ABC) = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore x = 60^\circ \text{ (Corresponding angles)}$$

$$\therefore 3y + 6^\circ = x \text{ (Corresponding angles)}$$

$$\therefore 3y = 60^\circ - 6^\circ = 54^\circ$$

$$\therefore y = \frac{54^\circ}{3} = 18^\circ$$

25

 Yes $\triangle ADE \cong \triangle CBF$ (Two angles and side) (Q.E.D. 1)

 Then we deduce that $DE = BF$

$$\therefore AE = FC$$

$$\therefore m(\angle AED) = m(\angle CFB)$$

$$\therefore m(\angle DEF) = m(\angle BFE)$$

 In $\triangle DEF$ & $\triangle BFE$ in them:

$$\begin{cases} DE = BF \\ FE \text{ is a common side} \\ m(\angle DEF) = m(\angle BFE) \end{cases}$$

$$\therefore \triangle DEF \cong \triangle BFE \quad (\text{Q.E.D. 2})$$

 Then we deduce that: $DF = BE$

$$m(\angle BEF) = m(\angle DFE)$$

$$\therefore m(\angle BEA) = m(\angle DFC)$$

 In $\triangle ABE$ & $\triangle CDF$ in them:

$$\begin{cases} AE = FC \\ DF = BE \\ m(\angle BEA) = m(\angle DFC) \end{cases}$$

$$\therefore \triangle ABE \cong \triangle CDF \quad (\text{Q.E.D. 3})$$

26

 Yes $\triangle PAQ \cong \triangle QBO$

"two sides and included angle"

(Q.E.D. 1)

 then we deduce that: $PQ = QO$

 In $\triangle PQR$ & $\triangle OQR$ in them:

$$\begin{cases} PQ = QO \\ QR \text{ is a common side} \\ m(\angle QPR) = m(\angle ROQ) = 90^\circ \end{cases}$$

$$\therefore \triangle PQR \cong \triangle OQR$$

 then we deduce that: $PR = OR$
 $\therefore \triangle PAQ \cong \triangle QBO$

$$\therefore m(\angle 5) = m(\angle 6)$$

$$\therefore m(\angle 5) + m(\angle 4) = 90^\circ$$

$$\therefore m(\angle 6) + m(\angle 4) = 90^\circ$$

$$\therefore m(\angle 4) + m(\angle 1) = 90^\circ$$

$$\therefore m(\angle 6) = m(\angle 1) \quad (1)$$

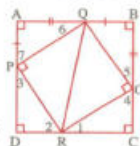
$$\therefore m(\angle 6) + m(\angle 7) = 90^\circ$$

$$\therefore m(\angle 3) + m(\angle 7) = 90^\circ$$

$$\therefore m(\angle 6) = m(\angle 3) \quad (2)$$

From (1) and (2):

$$\therefore m(\angle 1) = m(\angle 3)$$

 In $\triangle PDR$ & $\triangle RCO$:


(Q.E.D. 2)



$$m(\angle 1) = m(\angle 3), m(\angle C) = m(\angle D),$$

$$m(\angle 4) = m(\angle 2)$$

$$\therefore \text{In } \triangle PDR, RCO :$$

$$\begin{cases} m(\angle 3) = m(\angle 1) \\ m(\angle 2) = m(\angle 4) \\ PR = RO \end{cases}$$

$$\therefore \triangle PDR \equiv \triangle RCO$$

(Q.E.D. 3)

Answers of Exercise 2

1

(a) All its sides are equal in length.

(b) All its angles are equal in measure.

$$\text{2 } 360^\circ \quad \text{3 } 540^\circ \quad \text{4 } 720^\circ \quad \text{5 } 900^\circ$$

$$\text{6 } 108^\circ, 128\frac{4}{7}^\circ \quad \text{7 } 360^\circ$$

$$\text{8 } 5 \times 120^\circ \quad \text{9 } 135^\circ$$

2

$$\text{1 (b)} \quad \text{2 (c)} \quad \text{3 (c)} \quad \text{4 (d)}$$

$$\text{5 (d)} \quad \text{6 (c)} \quad \text{7 (c)}$$

3

$$\therefore \text{The number of diagonals of the polygon of } n \text{ sides} = \frac{n(n-3)}{2}$$

1 The number of diagonals of the triangle

$$= \frac{3(3-3)}{2} = \text{zero}$$

2 The number of diagonals of the quadrilateral

$$= \frac{4(4-3)}{2} = 2$$

3 The number of diagonals of the pentagon

$$= \frac{5(5-3)}{2} = 5$$

4

$$\text{Fig. (1): } m(\angle D) = 360^\circ - (90^\circ + 100^\circ + 80^\circ) = 90^\circ$$

$$\text{Fig. (2): } m(\angle D) = 540^\circ - (90^\circ + 90^\circ + 112^\circ + 130^\circ) = 118^\circ$$

5

$$\therefore Z \in \overleftrightarrow{YF}$$

$$\therefore m(\angle LZY) = 180^\circ - 120^\circ = 60^\circ$$

From the quadrilateral XYZL

$$\therefore m(\angle X) = 360^\circ - (70^\circ + 60^\circ + 90^\circ) = 140^\circ \text{ (The req.)}$$

6

\therefore The sum of measures of the interior angles of the triangle = 180°

\therefore In $\triangle ABC$:

$$m(\angle ABC) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$$

$$\therefore \overline{CE} \cap \overline{AD} = \{B\}$$

$$\therefore m(\angle EBD) = m(\angle ABC) = 60^\circ \text{ (V.O.A.)}$$

\therefore The sum of measures of the interior angles of the quadrilateral = 360°

$$\therefore m(\angle E) = 360^\circ - (90^\circ + 130^\circ + 60^\circ) = 80^\circ \text{ (The req.)}$$

7

$\therefore \triangle DFE$ is an equilateral triangle

$$\therefore m(\angle FDE) = \frac{180^\circ}{3} = 60^\circ$$

$$\therefore \overline{AE} \cap \overline{CF} = \{D\}$$

$$\therefore m(\angle ADC) = m(\angle FDE) = 60^\circ \text{ (V.O.A.)}$$

\therefore From the quadrilateral ABCD:

$$\therefore m(\angle B) = 360^\circ - (120^\circ + 60^\circ + 105^\circ) = 75^\circ$$

(The req.)

8

From the quadrilateral ARFE:

$$\therefore m(\angle RAE) = 360^\circ - (120^\circ + 45^\circ + 105^\circ) = 90^\circ$$

$$\therefore \overline{ED} \cap \overline{RB} = \{A\}$$

$$\therefore m(\angle DAB) = m(\angle RAE) = 90^\circ \text{ (V.O.A.)}$$

From the quadrilateral ABCD:

$$m(\angle B) = 360^\circ - (90^\circ + 130^\circ + 80^\circ) = 60^\circ \text{ (The req.)}$$

9

From $\triangle ABC$:

$$m(\angle B) = 180^\circ - (30^\circ + 58^\circ) = 92^\circ$$

$$\therefore \overline{AB} \parallel \overline{DE}, \overline{BD} \text{ is a transversal to them}$$

$$\therefore m(\angle D) = m(\angle B) = 92^\circ \text{ (alternate angles)}$$

$$\therefore \overline{AF} \cap \overline{BD} = \{C\}$$

$$\therefore m(\angle DCF) = m(\angle ACB) = 58^\circ \text{ (V.O.A.)}$$

\therefore From the quadrilateral CFED

$$m(\angle E) = 360^\circ - (58^\circ + 125^\circ + 92^\circ) = 85^\circ \text{ (The req.)}$$

10

From the figure (The quadrilateral ABCD):

$$m(\angle C) = 360^\circ - (120^\circ + 85^\circ + 90^\circ) = 65^\circ$$

$\therefore \overline{BE} \parallel \overline{CD}$, \overline{BC} is a transversal to them

$$\therefore m(\angle C) + m(\angle EBC) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle EBC) = 180^\circ - 65^\circ = 115^\circ$$

$$\therefore m(\angle ABE) = 115^\circ - 85^\circ = 30^\circ \quad (\text{The req.})$$

11

From the hexagon ABCDEF:

$$m(\angle A) + m(\angle C) + m(\angle B) + m(\angle D) + m(\angle E) + m(\angle F) = 720^\circ$$

$$\therefore m(\angle A) + m(\angle C) = 720^\circ - (90^\circ + 115^\circ + 165^\circ + 110^\circ) = 240^\circ$$

$$\therefore m(\angle A) = m(\angle C) = \frac{240^\circ}{2} = 120^\circ$$

$$\therefore X = 120^\circ \quad (\text{The req.})$$

12

From the quadrilateral ABCD:

$$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360^\circ$$

$$\therefore 90^\circ + 3X + 5X + 2X = 360^\circ$$

$$\therefore 10X = 360^\circ - 90^\circ = 270^\circ$$

$$\therefore X = \frac{270^\circ}{10} = 27^\circ \quad (\text{The req.})$$

13

From the quadrilateral ABCD:

$$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360^\circ$$

$$\therefore m(\angle B) + m(\angle D) = 360^\circ - (95^\circ + 85^\circ) = 180^\circ$$

$$\therefore m(\angle B) = \frac{1}{2} m(\angle D)$$

$$\therefore m(\angle D) + \frac{1}{2} m(\angle D) = 180^\circ$$

$$\therefore \frac{3}{2} m(\angle D) = 180^\circ \quad \therefore m(\angle D) = \frac{180^\circ \times 2}{3} = 120^\circ$$

$$\therefore m(\angle B) = 60^\circ \quad (\text{The req.})$$

14

$$\text{[1]} X = \frac{(5-2) \times 180^\circ}{5} = 108^\circ$$

$$\text{[2]} y = \frac{(8-2) \times 180^\circ}{8} = 135^\circ, X = 180^\circ - 135^\circ = 45^\circ$$

$$\text{[3]} 2X = \frac{(6-2) \times 180^\circ}{6} = 120^\circ$$

$$\therefore X = \frac{120^\circ}{2} = 60^\circ \quad \therefore y = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore z = \frac{180^\circ}{3} = 60^\circ$$

$$\text{[4]} y = \frac{(8-2) \times 180^\circ}{8} = 135^\circ \quad \therefore X = \frac{135^\circ}{2} = 67.5^\circ$$

$$\therefore z = 180^\circ - 135^\circ = 45^\circ, a = 360^\circ - 135^\circ = 225^\circ$$

15

[1] \therefore The figure is a pentagon

$$\therefore a + 2a + 2a + a + 2a = 540^\circ$$

$$\therefore 8a = 540^\circ \quad \therefore a = 67.5^\circ$$

[2] \therefore The figure is a pentagon

$$\therefore a + a + a + 90^\circ + 90^\circ = 540^\circ$$

$$\therefore 3a + 180^\circ = 540^\circ$$

$$\therefore 3a = 540^\circ - 180^\circ = 360^\circ \quad \therefore a = 120^\circ$$

[3] $b = 180^\circ - (90^\circ + 42^\circ) = 48^\circ$,

$$a = 90^\circ \text{ (alternate angle),}$$

$$c = 180^\circ - (90^\circ + 38^\circ) = 52^\circ$$

[4] $a = 180^\circ - (2 \times 62^\circ) = 56^\circ$,

$$b = \frac{180^\circ - 56^\circ}{2} = 62^\circ$$

16

From the quadrilateral ABCD:

$$m(\angle DCB) = 360^\circ - (110^\circ + 130^\circ + 60^\circ) = 60^\circ$$

$$\therefore C \in \overline{BE}$$

$$\therefore m(\angle DCE) = 180^\circ - 60^\circ = 120^\circ$$

$$\therefore \overline{CF} \text{ bisects } \angle DCE$$

$$\therefore m(\angle FCE) = \frac{120^\circ}{2} = 60^\circ$$

$$\therefore m(\angle FCE) = m(\angle B) = 60^\circ$$

(And they are two corresponding angles)

$$\therefore \overline{CF} \parallel \overline{AB} \quad (\text{Q.E.D.})$$

17

From $\triangle XYB$:

$$m(\angle XBY) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$$

\therefore The figure ABCDEF is a regular hexagon

$$\therefore m(\angle ABC) = \frac{(6-2) \times 180^\circ}{6} = 120^\circ$$

$$\therefore B \in \overline{YC}$$

$$\therefore m(\angle ABY) = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore m(\angle ABY) = m(\angle YBX) + m(\angle XBA)$$

$$\therefore 60^\circ = 30^\circ + m(\angle XBA)$$

$$\therefore m(\angle XBA) = 30^\circ$$

$$\therefore \overline{BX} \text{ bisects } \angle ABY \quad (\text{Q.E.D.})$$



18

Let the measures of the interior angles of the pentagon be $3x, 3x, 2x, 3x, 4x$

\therefore The sum of the measures of the interior angles

$$\text{of the pentagon} = (5 - 2) \times 180^\circ = 540^\circ$$

$$\therefore 3x + 3x + 2x + 3x + 4x = 540^\circ$$

$$\therefore 15x = 540^\circ \quad \therefore x = \frac{540^\circ}{15} = 36^\circ$$

\therefore The greatest measure $= 4 \times 36^\circ = 144^\circ$ (The req.)

19

Let the number of the sides of the polygon be n

\therefore The measure of the exterior angle of the polygon $= 30^\circ$

\therefore The measure of the interior angle of the polygon $= 180^\circ - 30^\circ = 150^\circ$

$$\therefore \frac{(n-2) \times 180^\circ}{n} = 150^\circ \quad \therefore 150^\circ n = 180^\circ n - 360^\circ$$

$$\therefore 360^\circ = 180^\circ n - 150^\circ n \quad \therefore 360^\circ = 30^\circ n$$

$$\therefore n = \frac{360^\circ}{30^\circ} = 12$$

The sum of the measures of the interior angles

$$= (12 - 2) \times 180^\circ = 1800^\circ \quad (\text{The req.})$$

20

Let the number of sides of the polygon be n

$$\therefore \frac{(n-2) \times 180^\circ}{n} = 100^\circ$$

$$\therefore 100^\circ n = 180^\circ n - 360^\circ$$

$$\therefore 360^\circ = 180^\circ n - 100^\circ n \quad \therefore 360^\circ = 80^\circ n$$

$$\therefore n = \frac{360^\circ}{80^\circ} = 4.5 \notin \mathbb{Z}$$

\therefore There is no regular polygon of interior angles each of them is of measure 100° (The req.)

21

\therefore The sum of measures of the interior angles of the polygon of 9 sides

$$= (9 - 2) \times 180^\circ = 1260^\circ$$

\therefore The measure of the remained angle $= 1260^\circ - 1140^\circ = 120^\circ$ (First req.)

\therefore The measure of the interior angle of a regular

$$\text{polygon of 9 sides} = \frac{(9-2) \times 180^\circ}{9} = 140^\circ$$

\therefore The polygon has an interior angle of measure 120°
 $\therefore 120^\circ \neq 140^\circ$

\therefore The polygon is not regular. (Second req.)

22

[1] The sum of the measures of the interior angles of the polygon of 15 sides $= (15 - 2) \times 180^\circ = 2340^\circ$

[2] The sum of the measures of the interior and exterior angles at five vertices $= 5 \times 180^\circ = 900^\circ$

\therefore The sum of the measures of the interior angles at these vertices $= 900^\circ - 200^\circ = 700^\circ$

\therefore The sum of the measures of the ten remained interior angles $= 2340^\circ - 700^\circ = 1640^\circ$ (The req.)

23

In ΔAMD :

$$\therefore m(\angle AMD) = 71^\circ$$

$$\therefore m(\angle MAD) + m(\angle MDA) = 180^\circ - 71^\circ = 109^\circ$$

$\therefore \overline{AM}$ bisects $\angle BAD$, \overline{DM} bisects $\angle ADC$

$$\begin{aligned} \therefore m(\angle BAD) + m(\angle ADC) \\ &= 2[m(\angle MAD) + m(\angle MDA)] \\ &= 2 \times 109^\circ = 218^\circ \end{aligned}$$

\therefore The sum of measures of the interior angles of the quadrilateral $= 360^\circ$

$$\therefore m(\angle B) + m(\angle C) = 360^\circ - 218^\circ = 142^\circ \quad (\text{Q.E.D.})$$

24

$\therefore \overline{AE} \parallel \overline{BC}$, \overline{AB} is a transversal to them

$$\therefore m(\angle A) + m(\angle B) = 180^\circ \quad (1)$$

(Two interior angles in the same side of the transversal)

From the pentagon $ABCDE$:

$$\begin{aligned} \therefore m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) \\ + m(\angle E) = 540^\circ \end{aligned}$$

$$\therefore m(\angle A) + m(\angle B) = 180^\circ \quad \text{"from (1)"} \quad (2)$$

$$\begin{aligned} \therefore m(\angle C) + m(\angle D) + m(\angle E) \\ = 540^\circ - 180^\circ = 360^\circ \end{aligned}$$

$$\therefore m(\angle C) = m(\angle D) = m(\angle E) = \frac{360^\circ}{3} = 120^\circ$$

$$\therefore m(\angle A) = 120^\circ$$

$$\therefore m(\angle B) = 180^\circ - 120^\circ = 60^\circ \quad (\text{The req.})$$

Answers of Exercise 3

1

- 1 parallel and equal in length
 2 equal in measure 3 supplementary
 4 bisect each other 5 a trapezium
 6 each two opposite sides are parallel
 7 130° 8 120°

2

- 1 $6 + 2$ 2 $105^\circ, 75^\circ, 75^\circ$
 3 16

3

- \therefore ABCD is a parallelogram
 \therefore The two diagonals bisect each other
 \therefore MA = MC = 2.5 cm, AB = CD = 2 cm.
 \therefore MB = $\frac{1}{2}$ BD = $\frac{3.6}{2}$ = 1.8 cm.
 \therefore The perimeter of \triangle AMB
 = $2 + 2.5 + 1.8 = 6.3$ cm. (The req.)

4

- From \triangle XYZ :
 $m(\angle YXZ) = 180^\circ - (118^\circ + 27^\circ) = 35^\circ$ (First req.)
 \therefore XYZL is a parallelogram
 $\therefore \overline{XY} \parallel \overline{LZ}$, \overline{XZ} is a transversal
 $\therefore m(\angle LXZ) = m(\angle YXZ) = 35^\circ$ (alternate angles)
 (Second req.)
 $\therefore \overline{XL} \parallel \overline{YZ}$, \overline{XZ} is a transversal
 $\therefore m(\angle LXZ) = m(\angle XZY) = 27^\circ$ (alternate angles)
 (Third req.)
 $\therefore m(\angle L) = m(\angle Y) = 118^\circ$ (Fourth req.)

5

- In the figure ABCD :
 \therefore MA = MC (Given), MB = MD (Given)
 \therefore Its diagonals bisect each other.
 \therefore The figure ABCD is a parallelogram.
 (First req.)

In \triangle MBA :

$\therefore m(\angle AMB) = 110^\circ$, $m(\angle MBA) = 25^\circ$

- $\therefore m(\angle MAB) = 180^\circ - (110^\circ + 25^\circ) = 45^\circ$
 \therefore the figure ABCD is a parallelogram.
 $\therefore \overline{AB} \parallel \overline{CD}$
 $\therefore \overline{CA}$ is a transversal to them.
 $\therefore m(\angle ACD) = m(\angle CAB)$
 = 45° (alternate angles) (Second req.)

6

- $\therefore \overline{AD} \parallel \overline{BC}$, \overline{AB} is a transversal
 $\therefore m(\angle B) = m(\angle EAB)$
 = 110° (alternate angles)
 $\therefore m(\angle B) + m(\angle C) = 110^\circ + 70^\circ = 180^\circ$
 and they are interior angles on the same side of the transversal
 $\therefore \overline{AB} \parallel \overline{CD}$
 $\therefore \overline{AD} \parallel \overline{BC}$
 \therefore ABCD is a parallelogram (Q.E.D.)

7

- \therefore ABCD is a parallelogram
 $\therefore m(\angle B) + m(\angle C) = 180^\circ$
 $\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ$
 From \triangle DHC :
 $m(\angle HDC) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$ (The req.)

8

- \therefore ABCD is a parallelogram
 $\therefore m(\angle A) + m(\angle ABC) = 180^\circ$
 $\therefore m(\angle ABC) = 180^\circ - 60^\circ = 120^\circ$
 $\therefore m(\angle ABO) = m(\angle ABC) - m(\angle OBC)$
 = $120^\circ - 40^\circ = 80^\circ$ (The req.)

9

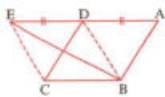
- $\therefore M \in \overline{AC}$
 $\therefore m(\angle BMC) = 180^\circ - 70^\circ = 110^\circ$
 \therefore In \triangle BMC :
 $m(\angle MCB) = 180^\circ - (110^\circ + 40^\circ) = 30^\circ$
 $\therefore m(\angle MCB) = m(\angle MAD)$
 and they are alternate angles
 $\therefore \overline{AD} \parallel \overline{BC}$
 $\therefore \overline{AB} \parallel \overline{DC}$
 \therefore ABCD is a parallelogram. (Q.E.D.)



10

Construction :Draw \overline{BD} , \overline{CE} **Proof :** \because $ABCD$ is a parallelogram. $\therefore AD = BC$ $\because AD = DE \quad \therefore DE = BC$ (1) $\because \overline{AD} \parallel \overline{BC}$ (Because $ABCD$ is a parallelogram) $\therefore \overline{DE} \parallel \overline{BC}$ (2)

From (1) and (2) :

 $\therefore DECB$ is a parallelogram. \therefore The two diagonals bisect each other. $\therefore \overline{DC}$ and \overline{BE} bisect each other. (Q.E.D.)

11

 $\therefore m(\angle ABE) = m(\angle C)$

and they are corresponding angles

 $\therefore \overline{AB} \parallel \overline{CD}$ (1) $\because m(\angle A) = m(\angle ABE)$ and they are alternate angles $\therefore \overline{AD} \parallel \overline{BC}$ (2)

From (1) & (2) :

 $\therefore ABCD$ is a parallelogram (Q.E.D.)

12

 $\because ABCD$ is a parallelogram $\therefore AB = DC$ $\therefore \frac{1}{2} AB = \frac{1}{2} DC$ $\because H$ is the midpoint of \overline{AB} , O is the midpoint of \overline{CD} $\therefore HB = DO$ (1) $\because \overline{AB} \parallel \overline{CD}$ i.e. $\therefore \overline{HB} \parallel \overline{DO}$ (2)

From (1) & (2) :

 $\therefore HBOD$ is a parallelogram (Q.E.D.)

13

 $\because ABCD$ is a parallelogram $\therefore AB = DC$ $\because \triangle DHC$ is equilateral $\therefore DC = HC \quad \therefore AB = HC$ (First req.) $\because \triangle DHC$ is equilateral $\therefore m(\angle C) = 60^\circ$ $\therefore m(\angle B) + m(\angle C) = 180^\circ$ (Two consecutive angles in $\square ABCD$) $\therefore m(\angle B) + 60^\circ = 180^\circ$ $\therefore m(\angle B) = 180^\circ - 60^\circ = 120^\circ$ (Second req.) $\because \overline{AD} \parallel \overline{BC}$, \overline{DH} is a transversal $\therefore m(\angle HDA) = m(\angle DHC)$ (alternate angles) $\because m(\angle DHC) = 60^\circ$ $\therefore m(\angle HDA) = 60^\circ$ (Third req.)

14

 $\because \overline{DH}$ bisects $\angle ADC \quad \therefore m(\angle ADH) = 64^\circ$ $\because ABCD$ is a parallelogram $\therefore \overline{AD} \parallel \overline{BC}$, \overline{DH} is a transversal $\therefore m(\angle ADH) + m(\angle DHB) = 180^\circ$ $\therefore m(\angle DHB) = 180^\circ - 64^\circ = 116^\circ$ (First req.) $\because ABCD$ is a parallelogram $\therefore m(\angle ABC) = m(\angle ADC) = 2 m(\angle HDC)$ $= 2 \times 64^\circ = 128^\circ$ (Second req.)

15

 $\because E \in \overline{BC}$ $\therefore m(\angle AEC) = 180^\circ - 70^\circ = 110^\circ$ $\because AECD$ is a quadrilateral. $\therefore m(\angle EAD) = 360^\circ - (110^\circ + 115^\circ + 65^\circ) = 70^\circ$ $\therefore m(\angle BAD) = 70^\circ + 45^\circ = 115^\circ$ $\therefore m(\angle BAD) = m(\angle C)$ (1) \therefore in $\triangle ABE$: $\therefore m(\angle B) = 180^\circ - (45^\circ + 70^\circ) = 65^\circ$ $\therefore m(\angle B) = m(\angle D)$ (2)

From (1) and (2) :

 $\therefore ABCD$ is a parallelogram. (Q.E.D.)

16

 $\because ABCD$ is a parallelogram $\therefore m(\angle ABC) = m(\angle ADC) = 115^\circ$ $\because HBCO$ is a parallelogram $\therefore m(\angle HBC) = m(\angle O) = 50^\circ$ $\therefore m(\angle ABH) = m(\angle ABC) - m(\angle HBC)$ $= 115^\circ - 50^\circ = 65^\circ$ (The req.)

17

\therefore ABCD is a parallelogram
 $\therefore m(\angle DAB) + m(\angle ABC) = 180^\circ$
 $\therefore x^\circ + 2x^\circ + 5x^\circ = 180^\circ$
 $\therefore 8x^\circ = 180^\circ \quad \therefore x^\circ = 22.5^\circ$
 $\therefore m(\angle BCD) = m(\angle BAD)$
 $= 3x^\circ = 3 \times 22.5^\circ = 67.5^\circ$ (First req.)
 $m(\angle ADC) = m(\angle ABC) = 5x^\circ = 5 \times 22.5^\circ = 112.5^\circ$
 (Second req.)

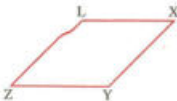
18 **1** (a) **2** (c) **3** (c) **4** (d) **5** (b)

19

\therefore ABCD is a parallelogram.
 $\therefore AB = CD$
 \therefore E and F are the midpoints of \overline{AB} and \overline{CD} respectively.
 $\therefore BE = DF$ (1)
 $\therefore \overline{AB} \parallel \overline{CD}$
 $\therefore \overline{BE} \parallel \overline{DF}$ (2)
 From (1) and (2):
 \therefore The figure BEDF is a parallelogram.
 $\therefore \overline{ED} \parallel \overline{BF}$ (3) (Q.E.D. 1)
 By the same way we can prove that:
 AECF is a parallelogram.
 $\therefore \overline{AF} \parallel \overline{EC}$ (4)
 \therefore from (3) and (4):
 $\therefore \overline{EM} \parallel \overline{NF}, \overline{EN} \parallel \overline{MF}$
 \therefore FMEN is a parallelogram. (Q.E.D. 2)

20

\therefore XYZL is a parallelogram
 $\therefore m(\angle X) + m(\angle Y) = 180^\circ$
 $\therefore m(\angle Y) = 3m(\angle X)$
 $\therefore m(\angle X) + 3m(\angle X) = 180^\circ$
 $\therefore 4m(\angle X) = 180^\circ$
 $\therefore m(\angle X) = \frac{180^\circ}{4} = 45^\circ$
 $\therefore m(\angle Y) = 3m(\angle X) = 3 \times 45^\circ = 135^\circ$
 $\therefore m(\angle Z) = m(\angle X) = 45^\circ$
 $\therefore m(\angle L) = m(\angle Y) = 135^\circ$ (The req.)


Answers of Exercise 4
1

- | | |
|---|---------------------------|
| 1 a rhombus. | 2 equal in length |
| 3 a square. | 4 a rhombus |
| 5 a parallelogram | 6 parallelogram |
| 7 parallelogram | 8 rhombus |
| 9 a square | 10 a square |
| 11 a square | 12 a parallelogram |
| 13 $\overline{AC} \perp \overline{BD}$ | |
| 14 side length $\times 4$, (length + width) $\times 2$,
side length $\times 4$ 15 10.5 | |

2

- | | | | |
|--------------|--------------|--------------|--------------|
| 1 (b) | 2 (a) | 3 (c) | 4 (b) |
| 5 (b) | 6 (b) | 7 (a) | 8 (c) |

3

- | | | |
|------------|------------|------------|
| 1 2 | 2 3 | 3 8 |
|------------|------------|------------|

4

- | | | |
|-------------|----------------------|----------------------|
| 1 16 | 2 135° | 3 110° |
|-------------|----------------------|----------------------|

5

- Fig. (1): $m(\angle ADB) = 40^\circ$, $m(\angle CDB) = 80^\circ$,
 $m(\angle A) = 60^\circ$, $m(\angle C) = 60^\circ$
 Fig. (2): $m(\angle MFE) = 45^\circ$, $m(\angle L) = 90^\circ$,
 $m(\angle LMF) = 45^\circ$, $m(\angle FME) = 45^\circ$,
 $m(\angle E) = 90^\circ$
 Fig. (3): $m(\angle XZY) = 35^\circ$, $m(\angle ZXH) = 35^\circ$,
 $m(\angle HZX) = 35^\circ$, $m(\angle H) = 110^\circ$
 Fig. (4): $m(\angle NWK) = 60^\circ$, $m(\angle WMK) = 30^\circ$,
 $m(\angle KMN) = 60^\circ$, $m(\angle MNW) = 60^\circ$,
 $m(\angle MZN) = 60^\circ$, $m(\angle WNK) = 30^\circ$,
 $m(\angle NZK) = 120^\circ$

6

- \therefore ABCD is a rectangle $\therefore m(\angle B) = 90^\circ$
 From $\triangle ABH$:
 $m(\angle AHB) = 180^\circ - (44^\circ + 90^\circ) = 46^\circ$
 $\therefore H \in \overline{BC}$
 $\therefore m(\angle AHB) + m(\angle AHD) + m(\angle DHC) = 180^\circ$
 $\therefore 46^\circ + m(\angle AHD) + 46^\circ = 180^\circ$
 $\therefore m(\angle AHD) = 88^\circ$ (The req.)

7

- \therefore ABCD is a rhombus and \overline{BD} is a diagonal in it.
 $\therefore m(\angle ABC) = 2m(\angle ABD) = 2 \times 62^\circ = 124^\circ$
 $\therefore m(\angle A) = 180^\circ - 124^\circ = 56^\circ$ (The req.)



8

\therefore XYZL is a rhombus

$\therefore m(\angle XYL) = m(\angle XLY)$

$\therefore \overline{DH} \parallel \overline{YX}$, \overline{YL} is a transversal

$\therefore m(\angle HDL) = m(\angle XYL)$ (corresponding angles)

$\therefore m(\angle HDL) = m(\angle HLD)$ (Q.E.D.)

9

\therefore ABCD is a rectangle $\therefore AD = BC$ (1)

\therefore OBCH is a parallelogram $\therefore OH = BC$ (2)

From (1) + (2): $\therefore AD = OH$

by subtracting AH from both sides

$\therefore AD - AH = OH - AH$

$\therefore HD = AO$ (Q.E.D.)

10

\therefore ABCD is a square.

$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore E \in \overline{BC}$

$\therefore \overline{AD} \parallel \overline{CE}$

$\therefore \overline{AC} \parallel \overline{DE}$

\therefore ACED is a parallelogram. (First req.)

\therefore ABCD is a square and \overline{AC} is a diagonal in it.

$\therefore m(\angle DAC) = 45^\circ$

\therefore ACED is a parallelogram.

$\therefore m(\angle ACE) = 180^\circ - 45^\circ = 135^\circ$ (Second req.)

11

\therefore ABCD is a parallelogram.

$\therefore AD = BC$, $\overline{AD} \parallel \overline{BC}$

$\therefore EB = BC$, $E \in \overline{CB}$ (Given)

$\therefore AD = EB$, $\overline{AD} \parallel \overline{EB}$

\therefore The figure AEBD is a parallelogram.

$\therefore DE = DC$ (Given)

$\therefore AB = DC$ (Properties of parallelogram)

$\therefore DE = AB$

\therefore The two diagonals of the parallelogram AEBD are equal in length.

\therefore The figure AEBD is a rectangle. (Q.E.D.)

12

\therefore ABCD is a rectangle

$\therefore \overline{AB} \parallel \overline{DC}$

\therefore ABYX is a square

$\therefore \overline{AB} \parallel \overline{XY}$

$\therefore \overline{XY} \parallel \overline{DC}$, \overline{YD} is a transversal

$\therefore m(\angle XYD) = m(\angle YDC) = 52^\circ$ (alternate angles)

\therefore ABYX is a square, \overline{AY} is a diagonal

$\therefore m(\angle AYX) = 45^\circ$

$\therefore m(\angle AYD) = m(\angle AYX) + m(\angle XYD)$

$= 45^\circ + 52^\circ = 97^\circ$ (The req.)

13

\therefore ABCD is a square and \overline{BD} is a diagonal in it.

$\therefore m(\angle BDC) = 45^\circ$, $m(\angle C) = 90^\circ$

\therefore In the quadrilateral DEFC:

$m(\angle EFC) = 360^\circ - (45^\circ + 90^\circ + 102^\circ) = 123^\circ$

$\therefore y = 123^\circ$

$\therefore F \in \overline{BC}$

$\therefore m(\angle AFB) = 180^\circ - 123^\circ = 57^\circ$

\therefore In the right-angled triangle ABF at B:

$m(\angle BAF) = 180^\circ - (90^\circ + 57^\circ) = 33^\circ$

$\therefore x = 33^\circ$ (The req.)

14

\therefore ABCD is a square.

$\therefore AM = CM$

$\therefore AE = CF$ and by subtracting $\therefore EM = FM$

$\therefore BM = MD$ (properties of the square)

\therefore The figure EBFD is a parallelogram.

\therefore ABCD is a square.

$\therefore \overline{AC} \perp \overline{BD}$

$\therefore \overline{EF} \perp \overline{BD}$

\therefore The parallelogram EBFD is a rhombus. (Q.E.D.)

15

1 square

2 parallelogram

3 rhombus

4 rectangle

16

1 All

2 Some

3 All

4 Some

5 All

6 Some

Answers of Exercise 5

1

1 180°

2 the measures of its nonadjacent interior angles.

3 right-angled

4 obtuse-angled

5 90°

6 obtuse

7 60°

2 **1** (a) **2** (b) **3** (a) **4** (c) **5** (c) **6** (b)

3

Fig. (1) : $m(\angle C) = 180^\circ - (70^\circ + 65^\circ) = 45^\circ$

Fig. (2) : $m(\angle A) = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$

Fig. (3) : $\therefore m(\angle ACB) = 52^\circ$ (V.O.A.)

$\therefore m(\angle A) = 180^\circ - (90^\circ + 52^\circ) = 38^\circ$

Fig. (4) : $m(\angle ACD) = 67^\circ + 42^\circ = 109^\circ$

Fig. (5) : $m(\angle B) = 120^\circ - 60^\circ = 60^\circ$

Fig. (6) : $m(\angle A) = 125^\circ - 35^\circ = 90^\circ$

Fig. (7) : $m(\angle A) = \frac{100^\circ}{2} = 50^\circ$

Fig. (8) : $m(\angle DAB) = 60^\circ + 60^\circ = 120^\circ$

Fig. (9) : $m(\angle C) = \frac{180^\circ - 30^\circ}{2} = 75^\circ$

$\therefore m(\angle ABD) = 30^\circ + 75^\circ = 105^\circ$

Fig. (10) : $m(\angle XAC) = m(\angle ABC) + m(\angle ACB)$
 $= (180^\circ - 110^\circ) + (180^\circ - 130^\circ)$
 $= 70^\circ + 50^\circ = 120^\circ$

Fig. (11) : $m(\angle ACB) = 180^\circ - 120^\circ = 60^\circ$

$\therefore m(\angle A) = 100^\circ - 60^\circ = 40^\circ$

Fig. (12) : $m(\angle ABC) = 180^\circ - 150^\circ = 30^\circ$

$\therefore m(\angle ACD) = 25^\circ + 30^\circ = 55^\circ$

Fig. (13) : $m(\angle BCE) = 72^\circ + 48^\circ = 120^\circ$

$m(\angle ACB) = 180^\circ - 120^\circ = 60^\circ$

$m(\angle BAC) = 120^\circ - 53^\circ = 67^\circ$

Fig. (14) : $m(\angle ABC) = 115^\circ - 50^\circ = 65^\circ$

$\therefore m(\angle EBG) = 65^\circ$ (V.O.A.)

Fig. (15) : $\therefore m(\angle ACB) = 42^\circ$ (alternate angles)

$\therefore m(\angle BAC) = 180^\circ - (42^\circ + 63^\circ) = 75^\circ$

Fig. (16) : $\therefore m(\angle ABC) = m(\angle ACB)$

$= \frac{180^\circ - 70^\circ}{2} = 55^\circ$

$\therefore m(\angle ABD) = 180^\circ - (73^\circ + 55^\circ) = 52^\circ$

$\therefore m(\angle ACE) = 180^\circ - (82^\circ + 55^\circ) = 43^\circ$

4

$\therefore \overrightarrow{BD} \parallel \overrightarrow{CA}$, \overrightarrow{AB} is a transversal to them

$\therefore m(\angle A) = m(\angle ABD) = 75^\circ$ (alternate angles)

In $\triangle ABC$:

$m(\angle ABC) = 180^\circ - (75^\circ + 45^\circ) = 60^\circ$ (The req.)

5

$\therefore \angle CAD$ is an exterior angle of $\triangle ABC$

$\therefore m(\angle DAC) = m(\angle ACB) + m(\angle B)$

$\therefore 112^\circ = m(\angle ACB) + 58^\circ$

$\therefore m(\angle ACB) = 54^\circ$

$\therefore \overrightarrow{CE}$ bisects $\angle ACB$

$\therefore m(\angle ECB) = \frac{54^\circ}{2} = 27^\circ$

$\therefore \angle AEC$ is an exterior angle of $\triangle BCE$

$\therefore m(\angle AEC) = 58^\circ + 27^\circ = 85^\circ$ (The req.)

6

$\therefore \overrightarrow{DE} \parallel \overrightarrow{BC}$, \overrightarrow{DB} is a transversal to them

$\therefore m(\angle B) + m(\angle D) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle B) = 180^\circ - 100^\circ = 80^\circ$

\therefore In $\triangle ABC$:

$m(\angle BAC) = 180^\circ - (80^\circ + 30^\circ) = 70^\circ$ (The req.)

7

$\therefore \overrightarrow{FR} \parallel \overrightarrow{BC}$, \overrightarrow{FB} is a transversal to them

$\therefore m(\angle B) + m(\angle F) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle B) = 180^\circ - 135^\circ = 45^\circ$

$\therefore \overrightarrow{DE} \parallel \overrightarrow{BC}$ and \overrightarrow{DC} is a transversal to them

$\therefore m(\angle C) + m(\angle D) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ$

\therefore In $\triangle ABC$:

$m(\angle BAC) = 180^\circ - (45^\circ + 60^\circ) = 75^\circ$ (The req.)

8

$\therefore \overrightarrow{DE} \parallel \overrightarrow{BC}$, \overrightarrow{DB} is a transversal to them.

$\therefore m(\angle ADE) = m(\angle B) = 60^\circ$ (corresponding angles)

\therefore In $\triangle ADE$

$m(\angle AED) = 180^\circ - (80^\circ + 60^\circ) = 40^\circ$

$m(\angle DEC) = 180^\circ - 40^\circ = 140^\circ$ (The req.)

9

In $\triangle ABC$:

$m(\angle C) = 180^\circ - (64^\circ + 52^\circ) = 64^\circ$

$\therefore m(\angle DEC) + m(\angle C) = 116^\circ + 64^\circ = 180^\circ$

(but they are interior angles in the same side of the transversal)

$\therefore \overrightarrow{DE} \parallel \overrightarrow{BC}$

(Q.E.D.)



10

$$\therefore \overline{AY} \cap \overline{BE} = \{C\}$$

$$\therefore m(\angle XCF) = m(\angle ACB) = 42^\circ \quad (\text{V.O.A.})$$

$$\text{Similarly : } m(\angle XFC) = m(\angle YFZ) = 53^\circ$$

$$\therefore \text{In } \triangle XFC :$$

$$m(\angle FXC) = 180^\circ - (42^\circ + 53^\circ) = 85^\circ$$

$$\therefore \overline{EB} \cap \overline{DF} = \{X\}$$

$$\therefore m(\angle DXE) = m(\angle FXC) = 85^\circ \quad (\text{V.O.A.}) \quad (\text{First req.})$$

$$\therefore X \in \overline{EC}$$

$$\therefore m(\angle DXC) = 180^\circ - 85^\circ = 95^\circ \quad (\text{Second req.})$$

$$\therefore \overline{DF} \cap \overline{EC} = \{X\}$$

$$\therefore m(\angle EXF) = m(\angle DXC) = 95^\circ \quad (\text{V.O.A.}) \quad (\text{Third req.})$$

11

$$\therefore \overline{ED} \parallel \overline{BF}, \overline{AE} \text{ is a transversal to them}$$

$$\therefore m(\angle E) + m(\angle CAF) = 180^\circ$$

(two interior angles in the same side of the transversal)

$$m(\angle E) = 180^\circ - 110^\circ = 70^\circ$$

$$\therefore \text{In } \triangle DCE :$$

$$m(\angle D) = 180^\circ - (60^\circ + 70^\circ) = 50^\circ$$

$$\therefore \overline{ED} \parallel \overline{BA}, \overline{DB} \text{ is a transversal to them}$$

$$\therefore m(\angle B) = m(\angle D) = 50^\circ \quad (\text{alternate angles})$$

$$, m(\angle ACB) = m(\angle DCE) = 60^\circ \quad (\text{V.O.A.})$$

$$\therefore \text{In } \triangle ABC :$$

$$m(\angle BAC) = 180^\circ - (50^\circ + 60^\circ) = 70^\circ \quad (\text{The req.})$$

12

$$\text{In } \triangle ABC :$$

$$m(\angle ABC) + m(\angle ACB) = 180^\circ - 80^\circ = 100^\circ$$

$$\therefore \overline{BM} \text{ bisects } \angle ABC, \overline{CM} \text{ bisects } \angle ACB$$

$$\therefore m(\angle MBC) + m(\angle MCB) = \frac{1}{2} \times 100^\circ = 50^\circ$$

$$\therefore \text{In } \triangle MBC :$$

$$m(\angle BMC) = 180^\circ - 50^\circ = 130^\circ$$

$$\therefore m(\angle EMD) = m(\angle BMC) = 130^\circ \quad (\text{V.O.A.}) \quad (\text{The req.})$$

13

$$\therefore \angle ADC \text{ is an exterior angle of } \triangle ABD$$

$$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD) \\ = 35^\circ + 25^\circ = 60^\circ$$

$$\therefore \angle XAC \text{ is an exterior angle of } \triangle ADC$$

$$\therefore m(\angle XAC) = m(\angle ADC) + m(\angle C) \\ = 60^\circ + 60^\circ = 120^\circ \quad (\text{The req.})$$

14

$$\therefore A \in \overline{BF}$$

$$\therefore m(\angle BAC) + m(\angle CAF) = 180^\circ$$

$$\therefore m(\angle BAC) = 180^\circ - 110^\circ = 70^\circ$$

$$\text{Similarly : } m(\angle EAB) = 180^\circ - 130^\circ = 50^\circ$$

$$\therefore \overline{AE} \parallel \overline{BC}, \overline{AB} \text{ is a transversal to them}$$

$$\therefore m(\angle B) = m(\angle EAB) = 50^\circ \quad (\text{alternate angles})$$

$$\therefore \angle ACD \text{ is an exterior angle of } \triangle ABC$$

$$\therefore m(\angle ACD) = m(\angle BAC) + m(\angle B) \\ = 70^\circ + 50^\circ = 120^\circ \quad (\text{The req.})$$

15

$$\therefore \overline{AC} \parallel \overline{DF}, \overline{FC} \text{ is a transversal to them.}$$

$$\therefore m(\angle ACB) = m(\angle F) \quad (\text{corresponding angles})$$

$$\therefore \text{In } \triangle ABC, \triangle DEF :$$

$$\therefore m(\angle B) = m(\angle DEF) = 90^\circ \quad (\text{given})$$

$$, m(\angle ACB) = m(\angle F) \quad (\text{by proof})$$

$$\therefore m(\angle A) = m(\angle D) \quad (\text{Q.E.D.})$$

16

$$\therefore \overline{AD} \text{ bisects } \angle BAC$$

$$\therefore m(\angle BAD) = m(\angle CAD),$$

$$\therefore m(\angle B) = m(\angle C) \quad \therefore m(\angle ADB) = m(\angle ADC)$$

$$\therefore \triangle ABD, \triangle ACD \text{ in them :}$$

$$\left\{ \begin{array}{l} m(\angle BAD) = m(\angle CAD) \\ m(\angle ADB) = m(\angle ADC) \end{array} \right.$$

$$\overline{AD} \text{ is a common side}$$

$$\therefore \triangle ABD \cong \triangle ACD, \text{ then we deduce that : } AB = AC \quad (\text{Q.E.D.})$$

$$17 \therefore \overline{AB} \parallel \overline{ED}, \overline{AD} \text{ is a transversal}$$

$$\therefore m(\angle BAC) = m(\angle ADE) \quad (\text{alternate angles})$$

$$\therefore \text{In } \triangle ABC \text{ and } \triangle DEA :$$

$$\therefore m(\angle B) = m(\angle E), m(\angle BAC) = m(\angle ADE)$$

$$\therefore m(\angle ACB) = m(\angle DAE)$$

(and they are alternate angles).

$$\therefore \overline{BC} \parallel \overline{AE} \quad (\text{Q.E.D.})$$

18

In $\triangle XYZ$ and $\triangle XML$

$\therefore m(\angle XML) = m(\angle Y)$, $\angle X$ is a common angle.

$\therefore m(\angle XLM) = m(\angle Z) = 52^\circ$ (The req.)

19 $\therefore \angle DAC$ is an exterior angle of $\triangle ABC$

$\therefore m(\angle DAC) = m(\angle B) + m(\angle C)$

$\therefore m(\angle B) = m(\angle C)$

$\therefore \frac{1}{2} m(\angle DAC) = m(\angle B)$

$\therefore m(\angle DAE) = m(\angle B)$

(and they are corresponding angles).

$\therefore \overline{AE} \parallel \overline{BC}$ (Q.E.D.)

20 $\therefore m(\angle 1) = m(\angle A)$, $m(\angle 2) = m(\angle C)$

$\therefore m(\angle 1) + m(\angle 2) = m(\angle A) + m(\angle C)$

$\therefore m(\angle ABC) = m(\angle A) + m(\angle C)$

$\therefore \angle ABC$ is a right angle. (Q.E.D.)

21 In $\triangle ABC$:

$\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$

$\therefore 2m(\angle C) + 4m(\angle C) + m(\angle C) = 180^\circ$

$\therefore 7m(\angle C) = 180^\circ \quad \therefore m(\angle C) = \frac{180^\circ}{7}$

$\therefore m(\angle B) = 4 \times \frac{180^\circ}{7} = 102\frac{6}{7}$

$\therefore \angle B$ is an obtuse angle. (Q.E.D.)

22

In $\triangle ABC$:

$\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$

$\therefore 4x + 2x + 2^\circ + 28^\circ = 180^\circ$

$\therefore 6x + 30^\circ = 180^\circ$

$\therefore 6x = 150^\circ \quad \therefore x = 25^\circ$

$\therefore m(\angle A) = 4 \times 25^\circ = 100^\circ$

$\therefore m(\angle B) = 2 \times 25^\circ + 2^\circ = 52^\circ$ (The req.)

Answers of Exercise 6

1

[1] bisects the third side.

[2] parallel to

[3] half the length of the third side

[4] 3

[5] 90°

[6] 12

[7] 2.5

[8] 3

[9] $3, 45^\circ$

2

In $\triangle ABC$:

$\therefore E$ is the midpoint of \overline{AB} , $\overline{EX} \parallel \overline{BC}$

$\therefore X$ is the midpoint of \overline{AC}

\therefore in $\triangle ACD$:

$\therefore X$ is the midpoint of \overline{AC} , $\overline{XY} \parallel \overline{CD}$

$\therefore Y$ is the midpoint of \overline{AD}

$\therefore AY = \frac{6}{2} = 3$ cm. (The req.)

3

In $\triangle DBC$:

$\therefore X$ is the midpoint of \overline{DC} , $\overline{XY} \parallel \overline{BC}$

$\therefore Y$ is the midpoint of \overline{DB}

\therefore in $\triangle ADB$:

$\therefore Y$ is the midpoint of \overline{DB} , Z is the midpoint of \overline{AB}

$\therefore \overline{ZY} \parallel \overline{AD}$

$\therefore \overline{DB}$ is a transversal to them.

$\therefore m(\angle ZYB) = m(\angle ADB) = 40^\circ$

(corresponding angles) (The req.)

4

[1] The perimeter of the parallelogram

$ABCD = 2(12 + 8) = 40$ cm.

[2] $\therefore ABCD$ is a parallelogram, M is the point of intersection of its diagonals

$\therefore M$ is the midpoint of \overline{BD}

In $\triangle ABD$:

$\therefore M$ is the midpoint of \overline{BD} , $\overline{MO} \parallel \overline{AD}$

$\therefore O$ is the midpoint of \overline{AB}

$\therefore AO = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4$ cm. (The req.)

5

$\therefore ABCD$ is a parallelogram, M is the point of intersection of its diagonals

$\therefore M$ is the midpoint of \overline{BD} , $\therefore \overline{MX} \parallel \overline{AB}$

$\therefore X$ is the midpoint of \overline{AD} (First req.)

$\therefore MX = \frac{1}{2} AB$

$\therefore AB = 2MX = 2 \times 5 = 10$ cm.

$\therefore DC = AB = 10$ cm. (Second req.)



6

∴ ABCD is a parallelogram.

$$\therefore \overline{AB} \parallel \overline{DC} \quad \therefore \overline{OC} \parallel \overline{AB}$$

In $\triangle ABH$: ∵ C is the midpoint of \overline{BH} , $\overline{CO} \parallel \overline{AB}$

∴ O is the midpoint of \overline{AH}

$$\therefore AO = OH \quad (\text{Q.E.D.})$$

7

∵ D, E are the midpoints of \overline{AB} , \overline{AC} respectively.

$$\therefore \overline{DE} \parallel \overline{BC} \quad \therefore \overline{AX} \parallel \overline{BC}$$

$$\therefore \overline{DE} \parallel \overline{AX}$$

$$\therefore Y \in \overline{DE} \quad \therefore \overline{EY} \parallel \overline{AX}$$

In $\triangle ACX$:

∵ $\overline{EY} \parallel \overline{AX}$, E is the midpoint of \overline{AC}

$$\therefore Y \text{ is the midpoint of } \overline{XC} \quad (\text{Q.E.D.})$$

8

In $\triangle ACD$:

∵ Z is the midpoint of \overline{AD} , $\overline{ZY} \parallel \overline{CD}$

∴ Y is the midpoint of \overline{AC}

$$\therefore AY = YC = 4 \text{ cm.} \quad (\text{First req.})$$

∴ in $\triangle ABC$:

∵ X is the midpoint of \overline{AB}

∴ Y is the midpoint of \overline{AC}

$$\therefore XY = \frac{1}{2} BC = \frac{1}{2} \times 6 = 3 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle AXY = 5 + 3 + 4 = 12 \text{ cm.} \quad (\text{Second req.})$$

9

In $\triangle ABC$:

∵ D is the midpoint of \overline{AB} , E is the midpoint of \overline{BC}

$$\therefore DE = \frac{1}{2} AC = 3.5 \text{ cm.}$$

∴ E is the midpoint of \overline{BC} , F is the midpoint of \overline{AC}

$$\therefore EF = \frac{1}{2} AB = 2.5 \text{ cm.}$$

∴ F is the midpoint of \overline{AC} , D is the midpoint of \overline{AB}

$$\therefore DF = \frac{1}{2} BC = 4 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle DEF = 3.5 + 2.5 + 4 = 10 \text{ cm.} \quad (\text{The req.})$$

10

In $\triangle XYZ$:

∵ H is the midpoint of \overline{XY} , G is the midpoint of \overline{XZ}

$$\therefore HG = \frac{1}{2} YZ \quad \therefore YZ = 2 HG \quad (1)$$

∵ H is the midpoint of \overline{XY}

∴ O is the midpoint of \overline{YZ}

$$\therefore HO = \frac{1}{2} XZ \quad \therefore XZ = 2 HO \quad (2)$$

∵ O is the midpoint of \overline{YZ} , G is the midpoint of \overline{XZ}

$$\therefore OG = \frac{1}{2} XY \quad \therefore XY = 2 OG \quad (3)$$

∴ The perimeter of $\triangle XYZ = XY + YZ + XZ$

$$\begin{aligned} \therefore \text{From (1), (2), (3): The perimeter of } \triangle XYZ \\ = 2 OG + 2 HG + 2 HO = 2(OG + HG + HO) \\ = 2 \text{ the perimeter of } \triangle HOG = 2 \times 18 = 36 \text{ cm.} \end{aligned}$$

(The req.)

11

In $\triangle ABC$:

∵ D is the midpoint of \overline{AB} , F is the midpoint of \overline{AC}

$$\therefore DF = \frac{1}{2} BC = 6 \text{ cm.}$$

Similarly: ∵ D is the midpoint of \overline{AB}

∴ E is the midpoint of \overline{BC}

$$\therefore DE = \frac{1}{2} AC = 5 \text{ cm.}$$

$$\therefore \text{The perimeter of the figure DECF} \\ = 6 + 5 + 6 + 5 = 22 \text{ cm.} \quad (\text{The req.})$$

12

In $\triangle DBC$:

∵ H is the midpoint of \overline{BD}

∴ O is the midpoint of \overline{DC}

$$\therefore \overline{HO} \parallel \overline{BC}$$

$$\therefore \overline{AD} \parallel \overline{BC} \quad \therefore \overline{HO} \parallel \overline{AD} \quad (1)$$

$$\therefore HO = \frac{1}{2} BC$$

$$\therefore AD = \frac{1}{2} BC \quad \therefore HO = AD \quad (2)$$

From (1), (2):

$$\therefore \text{AHOD is a parallelogram.} \quad (\text{Q.E.D.})$$

13

In $\triangle ABC$:

∵ D is the midpoint of \overline{AB} , E is the midpoint of \overline{AC}

$$\therefore \overline{DE} \parallel \overline{BC}, \therefore F \in \overline{CB}$$

$$\therefore \overline{DE} \parallel \overline{BF} \quad (1)$$

$$\because DE = \frac{1}{2} BC, BF = \frac{1}{2} BC$$

$$\therefore DE = BF \quad (2)$$

From (1) and (2) :

\therefore BEDF is a parallelogram. (Q.E.D.)

14

In $\triangle ABC$:

\therefore D is the midpoint of \overline{AB} , E is the midpoint of \overline{AC}

$$\therefore \overline{DE} \parallel \overline{BC}, DE = \frac{1}{2} BC = 6 \text{ cm.}$$

\therefore in $\triangle FDE$:

\therefore X is the midpoint of \overline{DF} , $\overline{XY} \parallel \overline{DE}$

\therefore Y is the midpoint of \overline{EF}

$$\therefore XY = \frac{1}{2} DE = 3 \text{ cm.} \quad (\text{The req.})$$

15

In $\triangle ABH$:

\therefore X is the midpoint of \overline{AB} , $\overline{XY} \parallel \overline{BH}$

\therefore Y is the midpoint of \overline{AH}

\therefore ABCD is a parallelogram

$\therefore \overline{AB} \parallel \overline{CD}$

In $\triangle ABH$: \therefore Y is the midpoint of \overline{AH} , $\overline{YC} \parallel \overline{AB}$

\therefore C is the midpoint of \overline{BH} (Q.E.D.)

16

In $\triangle ABC$:

\therefore D is the midpoint of \overline{AB} , $\overline{DH} \parallel \overline{BC}$

\therefore H is the midpoint of \overline{AC}

$\therefore AH = HC$

$\therefore HC = CO$

$$\therefore AH = HC = CO = \frac{1}{3} AO \quad (\text{Q.E.D.1})$$

\therefore in $\triangle DHO$:

\therefore C is the midpoint of \overline{HO} , $\overline{CX} \parallel \overline{DH}$

\therefore X is the midpoint of \overline{DO}

$$\therefore OX = XD \quad (\text{Q.E.D.2})$$

17

In $\triangle BCO$:

\therefore D is the midpoint of \overline{CB} , $\overline{DZ} \parallel \overline{OC}$

\therefore Z is the midpoint of \overline{BO}

$$\therefore OZ = ZB \quad (1)$$

\therefore In $\triangle AZD$: \therefore H is the midpoint of \overline{AD} , $\overline{HO} \parallel \overline{DZ}$

\therefore O is the midpoint of \overline{AZ}

$$\therefore AO = OZ \quad (2)$$

$$\text{From (1), (2) : } \therefore AO = OZ = ZB \quad (\text{Q.E.D.})$$

18

$\therefore \overline{AD} \parallel \overline{BC}, \overline{HX} \parallel \overline{BC}$

$\therefore \overline{AD} \parallel \overline{BC} \parallel \overline{HX}, \therefore AH = HB$

$\therefore DX = XC$

\therefore X is the midpoint of \overline{DC}

\therefore in $\triangle DBC$:

\therefore X is the midpoint of \overline{DC} , $\overline{XY} \parallel \overline{BD}$

\therefore Y is the midpoint of \overline{BC} (Q.E.D.)

19

Construction : Draw \overline{XZ}

Proof :

$\therefore \overline{AD} \parallel \overline{EY} \parallel \overline{BC}, AE = EB$

$\therefore DX = XB, DY = YC$

\therefore In $\triangle DBC$:

\therefore Y is the midpoint of \overline{DC} , $\overline{YZ} \parallel \overline{BD}$

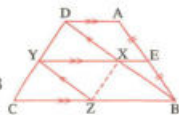
\therefore Z is the midpoint of \overline{BC}

\therefore X is the midpoint of \overline{BD}

$\therefore \overline{XZ} \parallel \overline{CD}, \therefore \overline{XD} \parallel \overline{ZY}$

\therefore XDYZ is a parallelogram.

$$\therefore XD = YZ \quad (\text{Q.E.D.})$$



20

Construction : Draw $\overline{AF} \parallel \overline{DX}$

Proof :

$\therefore \overline{AF} \parallel \overline{DX} \parallel \overline{EY} \parallel \overline{BC}$

$\therefore AD = DE = EB = 3 \text{ cm.}$

$$\therefore AX = XY = YC = \frac{8}{3}$$

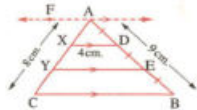
From $\triangle AXY$: \therefore D is the midpoint of \overline{AE}

\therefore X is the midpoint of \overline{AY}

$$\therefore DX = \frac{1}{2} EY \quad \therefore EY = 2 DX = 8 \text{ cm.}$$

\therefore The perimeter of the shape DEYX

$$= 4 + 3 + 8 + \frac{8}{3} = 15 + 2\frac{2}{3} = 17\frac{2}{3} \text{ cm.} \quad (\text{The req.})$$





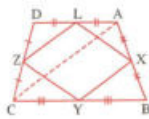
21

Construction : Draw \overline{AC} **Proof :** In $\triangle ABC$: $\therefore X$ is the midpoint of \overline{AB} $\therefore Y$ is the midpoint of \overline{BC} $\therefore \overline{XY} \parallel \overline{AC}, XY = \frac{1}{2} AC$ \therefore in $\triangle ADC$: $\therefore L$ is the midpoint of \overline{AD} $\therefore Z$ is the midpoint of \overline{CD} $\therefore \overline{LZ} \parallel \overline{AC}, LZ = \frac{1}{2} AC$

From (1) and (2) :

 $\therefore \overline{XY} \parallel \overline{LZ}, XY = LZ$ $\therefore XYZL$ is a parallelogram.

(Q.E.D.)



(1)

(2)

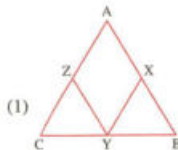
22

In $\triangle ABC$: $\therefore X$ is the midpoint of \overline{AB} $\therefore Y$ is the midpoint of \overline{BC} $\therefore XY = \frac{1}{2} AC = AZ$ $\therefore \overline{XY} \parallel \overline{AC}$ $\therefore \overline{XY} \parallel \overline{AZ}$

From (1), (2) :

 $\therefore AXYZ$ is a parallelogram $\therefore AX = \frac{1}{2} AB, AZ = \frac{1}{2} AC$ $\therefore AC = AB$ $\therefore AZ = AX$ $\therefore AXYZ$ is a rhombus

(Q.E.D.)



(1)

(2)

23

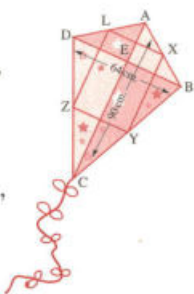
In $\triangle ABC$: $\therefore F$ is the midpoint of \overline{AB} , E is the midpoint of \overline{BC} $\therefore FE = \frac{1}{2} AC$ $\therefore x = \frac{1}{2} (3x - 6)$ $\therefore 2x = 3x - 6$ $\therefore x = 6 \text{ cm. (First req.)}$ $\therefore AB = 2 \times 6 + 1 = 13 \text{ cm.}$ $\therefore D$ is the midpoint of \overline{AC} , E is the midpoint of \overline{BC} $\therefore DE = \frac{1}{2} AB$ $\therefore y = \frac{1}{2} AB$ $\therefore y = 6.5 \text{ cm.}$

(Second req.)

24

In $\triangle ABD$: $\therefore X$ is the midpoint of \overline{AB} , L is the midpoint of \overline{AD} $\therefore XL = \frac{1}{2} BD = 32 \text{ cm.}$ \therefore in $\triangle ADC$: $\therefore L$ is the midpoint of \overline{AD} , Z is the midpoint of \overline{CD} $\therefore LZ = \frac{1}{2} AC = 45 \text{ cm.}$ In $\triangle BCD$: $\therefore Y$ is the midpoint of \overline{BC} , Z is the midpoint of \overline{CD} $\therefore YZ = \frac{1}{2} BD = 32 \text{ cm.}$ In $\triangle ABC$: $\therefore X$ is the midpoint of \overline{AB} , Y is the midpoint of \overline{BC} $\therefore XY = \frac{1}{2} AC = 45 \text{ cm.}$ \therefore The length of the stripe = $32 + 45 + 32 + 45 = 154 \text{ cm.}$

(The req.)



25

 $\therefore \angle BAO$ is an exterior angle of $\triangle ABC$ $\therefore m(\angle BAO) = m(\angle ABC) + m(\angle ACB)$ $\therefore m(\angle ABC) = m(\angle ACB)$ $\therefore m(\angle BAO) = 2m(\angle ABC)$ $\therefore \frac{1}{2} m(\angle BAO) = m(\angle ABC)$ $\therefore m(\angle HAB) = m(\angle ABC)$ and they are alternate angles $\therefore \overline{BC} \parallel \overline{AH}$ \therefore In $\triangle AHD$: $\therefore C$ is the midpoint of \overline{AD} , $\overline{CB} \parallel \overline{AH}$ $\therefore B$ is the midpoint of \overline{DH} $\therefore DB = BH$

(Q.E.D.)

Answers of Exercise 7

1

Fig. (1) : In $\triangle ABC$: $\therefore m(\angle A) = 90^\circ$ $\therefore (BC)^2 = (AB)^2 + (AC)^2 = 64 + 36 = 100$ $\therefore BC = \sqrt{100} = 10 \text{ cm.}$ Fig. (2) : In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$ $\therefore (AC)^2 = (AB)^2 + (BC)^2$ $= 144 + 25 = 169$ $\therefore AC = \sqrt{169} = 13 \text{ cm.}$

Fig. (3) : In ΔABC : $\therefore m(\angle C) = 90^\circ$

$$\begin{aligned}\therefore (AB)^2 &= (AC)^2 + (BC)^2 \\ &= 256 + 144 = 400 \\ \therefore AB &= \sqrt{400} = 20 \text{ cm.}\end{aligned}$$

 Fig. (4) : In ΔABC : $\therefore m(\angle B) = 90^\circ$

$$\begin{aligned}\therefore (BC)^2 &= (AC)^2 - (AB)^2 \\ &= 225 - 81 = 144 \\ \therefore BC &= \sqrt{144} = 12 \text{ cm.}\end{aligned}$$

 Fig. (5) : In ΔABC : $\therefore m(\angle B) = 90^\circ$

$$\begin{aligned}\therefore (BC)^2 &= (AC)^2 - (AB)^2 \\ &= 625 - 225 = 400 \\ \therefore BC &= \sqrt{400} = 20 \text{ cm.}\end{aligned}$$

 Fig. (6) : In ΔABC : $\therefore m(\angle A) = 90^\circ$

$$\begin{aligned}\therefore (AB)^2 &= (BC)^2 - (AC)^2 \\ &= 625 - 576 = 49 \\ \therefore AB &= \sqrt{49} = 7 \text{ cm.}\end{aligned}$$

2
 $\therefore ABCD$ is a square.

 $\therefore DC = 4 \text{ cm, } \therefore m(\angle DCB) = 90^\circ$

$$\begin{aligned}\therefore \text{In } \Delta DCE : (DE)^2 &= (DC)^2 + (CE)^2 \\ &= (4)^2 + (3)^2 = 16 + 9 = 25\end{aligned}$$

 $\therefore DE = 5 \text{ cm.}$ (The req.)

3

 In ΔADC : $\therefore m(\angle ADC) = 90^\circ$

$$\begin{aligned}\therefore (AD)^2 &= (AC)^2 - (DC)^2 = 400 - 256 = 144 \\ \therefore AD &= \sqrt{144} = 12 \text{ cm.}\end{aligned}$$
 (First req.)

 $\therefore \text{in } \Delta ABD : \therefore m(\angle ADB) = 90^\circ$

$$\begin{aligned}\therefore (AB)^2 &= (AD)^2 + (BD)^2 = 144 + 81 = 225 \\ \therefore AB &= \sqrt{225} = 15 \text{ cm.}\end{aligned}$$
 (Second req.)

$$\begin{aligned}\therefore \text{The area of } \Delta ABC &= \frac{1}{2} BC \times AD = \frac{1}{2} \times 25 \times 12 \\ &= 150 \text{ cm}^2\end{aligned}$$
 (Third req.)

4

 In ΔABD : $\therefore m(\angle ADB) = 90^\circ$

$$\begin{aligned}\therefore (BD)^2 &= (AB)^2 - (AD)^2 = 676 - 576 = 100 \\ \therefore BD &= \sqrt{100} = 10 \text{ cm.}\end{aligned}$$

 $\therefore \text{in } \Delta ADC : \therefore m(\angle ADC) = 90^\circ$

$$\therefore (CD)^2 = (AC)^2 - (AD)^2 = 900 - 576 = 324$$

$$\therefore CD = \sqrt{324} = 18 \text{ cm.}$$

$$\therefore BC = BD + DC = 10 + 18 = 28 \text{ cm.}$$
 (First req.)

$$\begin{aligned}\therefore \text{The area of } \Delta ABC &= \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 28 \times 24 \\ &= 336 \text{ cm}^2\end{aligned}$$
 (Second req.)

5

 In ΔXYZ : $\therefore m(\angle Y) = 90^\circ$

$$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = 49 + 576 = 625$$

$$\therefore XZ = \sqrt{625} = 25 \text{ cm.}$$
 (First req.)

 $\therefore \text{in } \Delta XLZ : \therefore m(\angle L) = 90^\circ$

$$\therefore (LZ)^2 = (XZ)^2 - (XL)^2 = 625 - 225 = 400$$

$$\therefore LZ = \sqrt{400} = 20 \text{ cm.}$$
 (Second req.)

6

 In ΔABC : $\therefore m(\angle B) = 90^\circ$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 81 + 144 = 225$$

$$\therefore AC = \sqrt{225} = 15 \text{ cm.}$$
 (First req.)

 $\therefore \text{in } \Delta ACD : \therefore m(\angle ACD) = 90^\circ$

$$\therefore (AD)^2 = (AC)^2 + (CD)^2 = 225 + 400 = 625$$

$$\therefore AD = \sqrt{625} = 25 \text{ cm.}$$
 (Second req.)

 \therefore the perimeter of the figure $ABCD$

$$= 9 + 12 + 20 + 25 = 66 \text{ cm.}$$
 (Third req.)

 \therefore the area of the figure $ABCD$
 $=$ The area of ΔABC + The area of ΔACD

$$\begin{aligned}&= \frac{1}{2} \times 12 \times 9 + \frac{1}{2} \times 20 \times 15 = 54 + 150 = 204 \text{ cm}^2 \\ &\text{(Fourth req.)}\end{aligned}$$

7

 In ΔABC : $\therefore m(\angle B) = 90^\circ$

$$\therefore (BC)^2 = (AC)^2 - (AB)^2 = 100 - 64 = 36$$

$$\therefore BC = \sqrt{36} = 6 \text{ cm.}$$
 (First req.)

 $\therefore \text{in } \Delta ABD : \therefore m(\angle B) = 90^\circ$

$$\therefore (BD)^2 = (AD)^2 - (AB)^2 = 289 - 64 = 225$$

$$\therefore BD = \sqrt{225} = 15 \text{ cm.}$$
 (Second req.)

 $\therefore CD = BD - BC$

$$\therefore CD = 15 - 6 = 9 \text{ cm.}$$
 (Third req.)

8

 In ΔABD : $\therefore m(\angle ABD) = 90^\circ$

$$\therefore (BD)^2 = (AD)^2 - (AB)^2 = 225 - 81 = 144$$

$$\therefore BD = \sqrt{144} = 12 \text{ cm.}$$



$\therefore \overline{AB} \parallel \overline{CD}$, \overline{BD} is a transversal

$$\therefore m(\angle BDC) = m(\angle ABD) = 90^\circ$$

$$\therefore \text{In } \triangle BDC : m(\angle BDC) = 90^\circ$$

$$\therefore (BC)^2 = (BD)^2 + (DC)^2 = 144 + 25 = 169$$

$$\therefore BC = \sqrt{169} = 13 \text{ cm.} \quad (\text{The req.})$$

9

$\therefore \overline{AD} \parallel \overline{EC}$, $m(\angle AEB) = m(\angle C) = 90^\circ$

and they are corresponding angles

$\therefore \overline{AE} \parallel \overline{DC}$ \therefore The figure AECD is a parallelogram

$$\therefore m(\angle C) = 90^\circ$$

\therefore The figure AECD is a rectangle

$$\therefore EC = AD = 9 \text{ cm.} \quad \therefore BE = 17 - 9 = 8 \text{ cm.}$$

$$\therefore \text{In } \triangle AEB : m(\angle AEB) = 90^\circ$$

$$\therefore (AE)^2 = (AB)^2 - (BE)^2 = 289 - 64 = 225$$

$$\therefore AE = \sqrt{225} = 15 \text{ cm.}$$

$$\therefore DC = AE = 15 \text{ cm.} \quad (\text{First req.})$$

The area of the trapezium ABCD

= The area of $\triangle ABE$ + The area of the rectangle AECD

$$= \frac{1}{2} \times 8 \times 15 + 9 \times 15 = 195 \text{ cm}^2 \quad (\text{Second req.})$$

10

In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 36 + 64 = 100$$

$$\therefore AC = \sqrt{100} = 10 \text{ cm.}$$

$\therefore \overline{AB} \parallel \overline{DE}$, \overline{BD} is a transversal

$$\therefore m(\angle D) = m(\angle B) \quad (\text{alternate angles})$$

In $\triangle ABC$, $\triangle EDC$

$$\begin{cases} m(\angle D) = m(\angle B) \\ m(\angle ACB) = m(\angle ECD) \\ BC = DC \end{cases} \quad (\text{V.O.A.})$$

$$\therefore \triangle ABC \cong \triangle EDC$$

$$\therefore CE = CA = 10 \text{ cm.} \quad (\text{The req.})$$

11

In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 16 + 9 = 25$$

$$\therefore AC = \sqrt{25} = 5 \text{ cm.}$$

\therefore The area of $\triangle ABC = \frac{1}{2} \times BC \times AB$

$$= \frac{1}{2} \times AC \times BD$$

$$\therefore \frac{1}{2} \times 3 \times 4 = \frac{1}{2} \times 5 \times BD \quad \therefore 6 = \frac{5}{2} \times BD$$

$$\therefore BD = 6 \div \frac{5}{2} = 2.4 \text{ cm.} \quad (\text{The req.})$$

12

In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$

$$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$$

$$\therefore AB = \sqrt{256} = 16 \text{ cm.}$$

$$\therefore BD = 9 \text{ cm.} \quad \therefore AD = 16 - 9 = 7 \text{ cm.} \quad (\text{First req.})$$

$$\therefore AE = 2 BC = 2 \times 12 = 24 \text{ cm.}$$

$\therefore \overline{EA} \parallel \overline{BC}$, \overline{AB} is a transversal

$$\therefore m(\angle EAB) = m(\angle B) = 90^\circ \quad (\text{alternate angles})$$

$$\therefore \text{In } \triangle EAD : \therefore m(\angle EAD) = 90^\circ$$

$$\therefore (ED)^2 = (EA)^2 + (AD)^2 = 576 + 49 = 625$$

$$\therefore ED = \sqrt{625} = 25 \text{ cm.} \quad (\text{Second req.})$$

13 1 The sum of areas of the squares on the sides of the right angle.

$$\boxed{2} \ 15$$

$$\boxed{3} \ 15$$

$$\boxed{4} \ 144$$

$$\boxed{5} \ 10$$

$$\boxed{6} \ 13$$

$$\boxed{7} \ 3$$

$$\boxed{8} \ 169$$

$$\boxed{9} \ 4$$

14

$$\boxed{1} \ (d)$$

$$\boxed{2} \ (b)$$

$$\boxed{3} \ (c)$$

$$\boxed{4} \ (c)$$

15

In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$

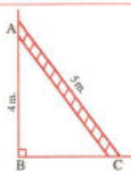
$$\therefore (BC)^2 = (AC)^2 - (AB)^2$$

$$= 25 - 16 = 9$$

$$\therefore BC = \sqrt{9} = 3 \text{ m.}$$

$$\therefore \text{The distance} = 3 \text{ m.}$$

(The req.)



16

In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$

$$\therefore (AB)^2 = (AC)^2 - (BC)^2$$

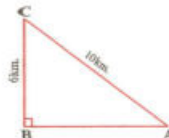
$$= 100 - 36 = 64$$

$$\therefore AB = \sqrt{64} = 8 \text{ km.}$$

$$\therefore \text{If Mina takes the two roads, the distance will be} = 6 + 8 = 14 \text{ km.}$$

$$\therefore \text{If he takes the main road, the distance will be} = 10 \text{ km.}$$

$$\therefore \text{The distance he saves} = 14 - 10 = 4 \text{ km.} \quad (\text{The req.})$$



17

 In $\triangle ABC$: $\because m(\angle B) = 90^\circ$

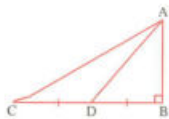
$$\begin{aligned}\therefore (AC)^2 &= (AB)^2 + (BC)^2 \\ &= (AB)^2 + (2BD)^2 \\ &= (AB)^2 + 4(BD)^2\end{aligned}$$

 in $\triangle ABD$: $\because m(\angle B) = 90^\circ$

$$\therefore (AD)^2 = (AB)^2 + (BD)^2 \quad (2)$$

By subtracting (2) from (1) :

$$\therefore (AC)^2 - (AD)^2 = 3(BD)^2 \quad (\text{Q.E.D.})$$



18

$$\text{Let } r_1 = \frac{1}{2}AC, r_2 = \frac{1}{2}AB, r_3 = \frac{1}{2}BC$$

 In $\triangle ABC$: $\because m(\angle B) = 90^\circ$

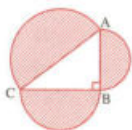
$$\therefore (AC)^2 = (AB)^2 + (BC)^2$$

$$\therefore \left(\frac{1}{2}AC\right)^2 = \left(\frac{1}{2}AB\right)^2 + \left(\frac{1}{2}BC\right)^2$$

$$\therefore r_1^2 = r_2^2 + r_3^2 \text{ multiplying by } \frac{1}{2}\pi$$

$$\therefore \frac{1}{2}\pi r_1^2 = \frac{1}{2}\pi r_2^2 + \frac{1}{2}\pi r_3^2$$

\therefore The area of the semicircle drawn on the hypotenuse equals the sum of areas of the two semicircles drawn on the two sides of the right angle. (Q.E.D.)



Answers of Exercise 8

1 1 translation 2 rotation 3 reflection

2 1 reflection 2 rotation 3 translation

3 1 translation 2 rotation 3 reflection

4 1 translation 2 reflection 3 rotation

4 rotation 5 reflection

5

$$1 (X, y) \longrightarrow (-X, y)$$

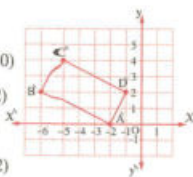
$$\therefore A(2, 0) \longrightarrow \hat{A}(-2, 0)$$

$$, B(6, 2) \longrightarrow \hat{B}(-6, 2)$$

$$, C(5, 4) \longrightarrow \hat{C}(-5, 4)$$

$$, D(1, 2) \longrightarrow \hat{D}(-1, 2)$$

(The type is reflection)



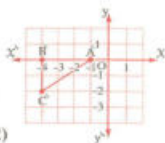
$$2 (X, y) \longrightarrow (-X, -y)$$

$$\therefore A(1, 0) \longrightarrow \hat{A}(-1, 0)$$

$$, B(4, 0) \longrightarrow \hat{B}(-4, 0)$$

$$, C(4, 2) \longrightarrow \hat{C}(-4, -2)$$

(The type is rotation)



$$3 (X, y) \longrightarrow (X + 2, y + 3)$$

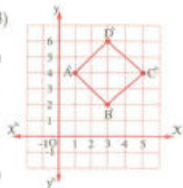
$$\therefore A(-1, 1) \longrightarrow \hat{A}(1, 4)$$

$$, B(1, -1) \longrightarrow \hat{B}(3, 2)$$

$$, C(3, 1) \longrightarrow \hat{C}(5, 4)$$

$$, D(1, 3) \longrightarrow \hat{D}(3, 6)$$

(The type is translation)



$$4 (X, y) \longrightarrow (y, -X)$$

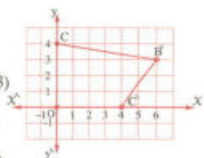
$$\therefore A(-4, 0) \longrightarrow C(0, 4)$$

$$, B(-3, 6) \longrightarrow \hat{B}(6, 3)$$

$$, C(0, 4) \longrightarrow \hat{C}(4, 0)$$

$$, O(0, 0) \longrightarrow O(0, 0)$$

(The type is rotation)



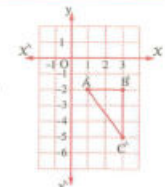
6

$$1 (X, y) \longrightarrow (X, -y)$$

$$\therefore A(1, 2) \longrightarrow \hat{A}(1, -2)$$

$$, B(3, 2) \longrightarrow \hat{B}(3, -2)$$

$$, C(3, 5) \longrightarrow \hat{C}(3, -5)$$

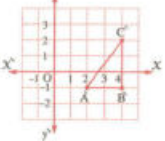


$$2 (X, y) \longrightarrow (X + 1, y - 3)$$

$$\therefore A(1, 2) \longrightarrow \hat{A}(2, -1)$$

$$, B(3, 2) \longrightarrow \hat{B}(4, -1)$$

$$, C(3, 5) \longrightarrow \hat{C}(4, 2)$$

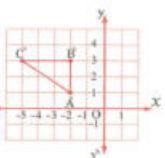


$$3 (X, y) \longrightarrow (-y, X)$$

$$\therefore A(1, 2) \longrightarrow \hat{A}(-2, 1)$$

$$, B(3, 2) \longrightarrow \hat{B}(-2, 3)$$

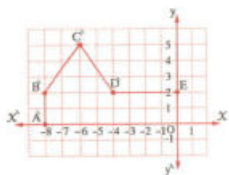
$$, C(3, 5) \longrightarrow \hat{C}(-5, 3)$$





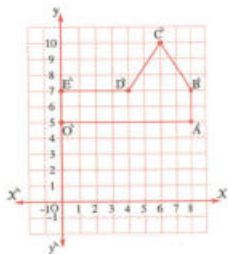
7

$$1 \quad (X, y) \longrightarrow (-X, y)$$



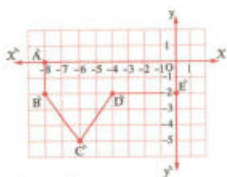
The type is reflection.

$$2 \quad (X, y) \longrightarrow (X, y + 5)$$



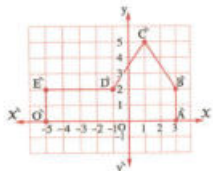
The type is translation.

$$3 \quad (X, y) \longrightarrow (-X, -y)$$



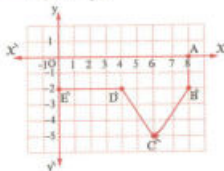
The type is rotation.

$$4 \quad (X, y) \longrightarrow (X - 5, y)$$



The type is translation.

$$5 \quad (X, y) \longrightarrow (X, -y)$$



The type is reflection.

8

$$\therefore (X, y) \longrightarrow (-y, X)$$

$$\therefore A(X, y) \longrightarrow \hat{A}(1, -1)$$

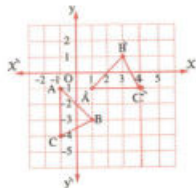
$$\therefore A(-1, -1)$$

$$, B(X, y) \longrightarrow \hat{B}(3, 1)$$

$$\therefore B(1, -3)$$

$$, C(X, y) \longrightarrow \hat{C}(4, -1)$$

$$\therefore C(-1, -4)$$

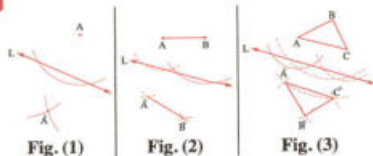


The type is rotation.

Answers of Exercise 9

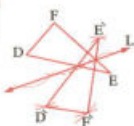
First : Problems on reflection in the plane :

1

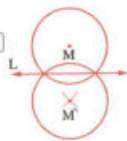


2

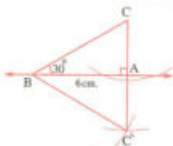
1



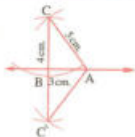
2



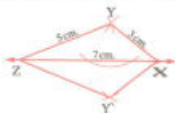
3



4

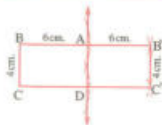


5



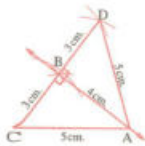
6

The resulting figure is a rectangle and its perimeter = $2 \times (12 + 4) = 32$ cm.



7

- 1 The perimeter of $\triangle ACD$ = 16 cm.
2 The area of $\triangle ACD$ = $\frac{1}{2} \times 6 \times 4 = 12$ cm²



8

- 1 The point D 2 \overline{BM} 3 $\triangle DLM$
4 $\triangle AXM$ 5 $\triangle DMC$ 6 $\triangle BMA$
7 The square DZML, the square ALMX
8 The square DCBA 9 \overline{XZ} 10 \overline{BD}

9

- 1 \overrightarrow{AE} , \overrightarrow{BF} , \overrightarrow{CD} 2 \overrightarrow{AE} 3 \overline{CF} , \overline{BD}
4 $\triangle AMF$, $\triangle AMF$, measures of the angles
5 $\triangle AMC$
6 $\triangle AMC$, $\triangle BMA$, lengths of the line segments

10

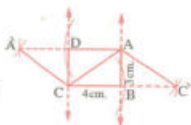
- 1 the lengths of line segments, the measures of angles, parallelism, betweenness
2 the axis of symmetry of the figure.
3 (a) 3 (b) 1 (c) zero
(d) zero (e) 2 (f) 2
(g) 4 (h) zero
(i) 1 (j) an infinite number

11

- \therefore the straight line L is the axis of symmetry of the figure ABCDE
 $\therefore m(\angle A) = m(\angle E) = 130^\circ$
 $\therefore m(\angle D) = m(\angle B) = 110^\circ$
from the pentagon ABCDE
 $\therefore m(\angle BCD) = 540^\circ - (130^\circ + 130^\circ + 110^\circ + 110^\circ) = 60^\circ$ (The req.)

12

- \therefore A is the image of itself by reflection in \overline{AB}
 \therefore B is the image of itself by reflection in \overline{AB}
 \therefore C is the image of C by reflection in \overline{AB}
 $\therefore \triangle ABC$ is the image of $\triangle ABC$ by reflection in \overline{AB}
 $\therefore m(\angle CAB) = m(\angle CAB)$
 $\therefore m(\angle CAC) = 2m(\angle CAB)$ (Q.E.D.1) (1)
similarly we can prove that:
 $m(\angle ACA) = 2m(\angle ACD)$ (2)
 $\therefore ABCD$ is a rectangle $\therefore \overline{AB} \parallel \overline{CD}$
 $\therefore m(\angle BAC) = m(\angle ACD)$ (alternate angles) (3)
from (1), (2), (3):
 $\therefore m(\angle CAC) = m(\angle ACA)$
but they are alternate angles
 $\therefore \overline{AC} \parallel \overline{AC}$ (Q.E.D.2)





Second : Problems on reflection in the Cartesian plane :

1

$$1) A(2, 4) \xrightarrow{\text{its image in } X\text{-axis}} \tilde{A}(2, -4)$$

$$B(4, 0) \xrightarrow{\text{its image in } X\text{-axis}} \tilde{B}(4, 0)$$

$$C(0, -2) \xrightarrow{\text{its image in } X\text{-axis}} \tilde{C}(0, 2)$$

$$D(-2, 1) \xrightarrow{\text{its image in } X\text{-axis}} \tilde{D}(-2, -1)$$

$$E(-3, -2) \xrightarrow{\text{its image in } X\text{-axis}} \tilde{E}(-3, 2)$$

$$2) A(2, 4) \xrightarrow{\text{its image in } y\text{-axis}} \tilde{A}(-2, 4)$$

$$B(4, 0) \xrightarrow{\text{its image in } y\text{-axis}} \tilde{B}(-4, 0)$$

$$C(0, -2) \xrightarrow{\text{its image in } y\text{-axis}} \tilde{C}(0, 2)$$

$$D(-2, 1) \xrightarrow{\text{its image in } y\text{-axis}} \tilde{D}(2, 1)$$

$$E(-3, -2) \xrightarrow{\text{its image in } y\text{-axis}} \tilde{E}(3, -2)$$

2

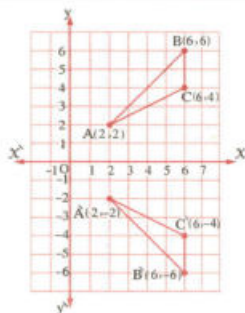


Fig. (1)

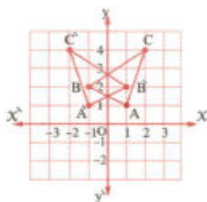


Fig. (2)

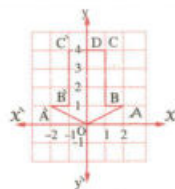


Fig. (3)

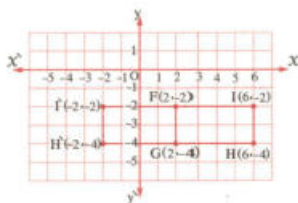
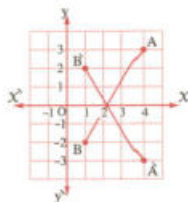


Fig. (4)

3

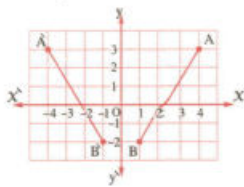
$$1) A(4, 3) \xrightarrow{\text{its image in } X\text{-axis}} \tilde{A}(4, -3)$$

$$B(1, -2) \xrightarrow{\text{its image in } X\text{-axis}} \tilde{B}(1, 2)$$



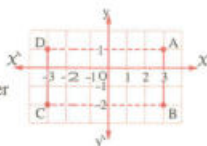
$$2) A(4, 3) \xrightarrow{\text{its image in } y\text{-axis}} \tilde{A}(-4, 3)$$

$$B(1, -2) \xrightarrow{\text{its image in } y\text{-axis}} \tilde{B}(-1, -2)$$

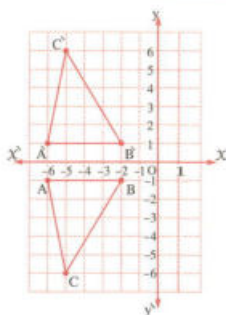


4

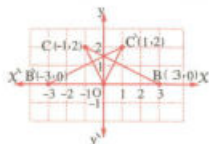
The figure ABCD is
a rectangle and its perimeter
 $= (6 + 3) \times 2$
 $= 18$ length units



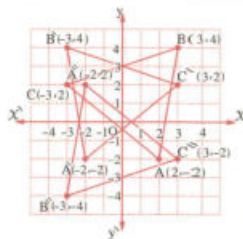
5



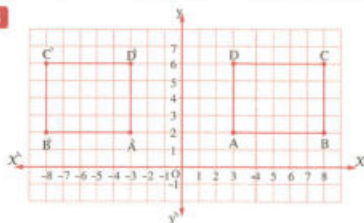
6



7



8



9

We notice that

$$AB = \hat{A}B, BC = \hat{B}C$$

$$, CD = \hat{C}D$$

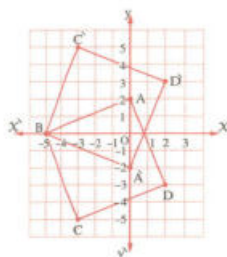
$$, AD = \hat{A}D$$

The area of the

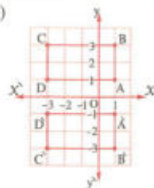
square ABCD

= The area of the

square $\hat{A}\hat{B}\hat{C}\hat{D}$



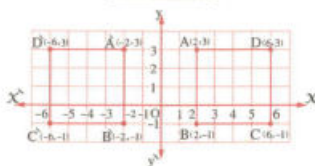
10 $D = (-3, 1)$



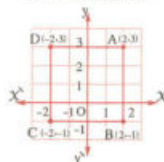
11

There are two cases to draw the square.

The first case



The second case



In the second case we notice that the image of the square ABCD by reflection in y-axis is DCBA

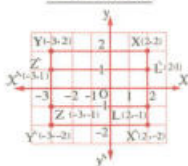
i.e. The same square.



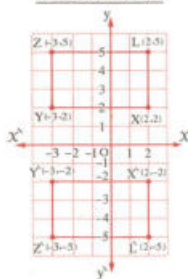
12

There are two cases to draw the rectangle

The first case



The second case



13

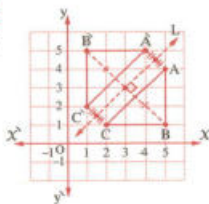
The point	Its image by reflection in the X -axis	Its image by reflection in the y -axis
$(3, -2)$	$(3, 2)$	$(-3, -2)$
$(1, -2)$	$(1, 2)$	$(-1, -2)$
$(2, 4)$	$(2, -4)$	$(-2, 4)$
$(0, 5)$	$(0, -5)$	$(0, 5)$
$(3, 0)$	$(3, 0)$	$(-3, 0)$
$(0, 0)$	$(0, 0)$	$(0, 0)$

14

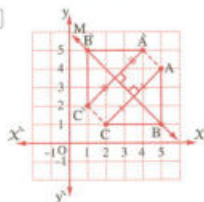
- 1 $(1, -3)$ 2 $(2, 5)$ 3 X -axis
 4 y -axis 5 y -axis 6 X -axis
 7 $(-2, -1)$ 8 $(-2, 3)$ 9 $A(2, 3)$

15

1



2



Answers of Exercise 10

First : Problems on reflection in the plane :

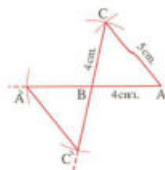
1

- 1 (c) 2 (b) 3 (d) 4 (c)

2

- 1 the point C 2 the point Z 3 \overline{CY}
 4 \overline{MX} 5 \overline{DM} 6 \overline{BX}
 7 $\triangle CYM$ 8 $\triangle DZM$ 9 $\triangle CMD$
 10 the square CZMY

3



4

Fig. (1)

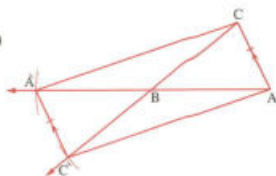
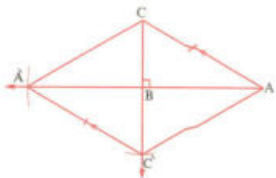
 $\therefore \hat{A}$ is the image of A by reflection in the point B \hat{C} is the image of C by reflection in the point B $\therefore \overline{AC}$ is the image of \overline{AC} by reflection in the point B $\therefore \overline{AC} \parallel \overline{\hat{A}\hat{C}}, AC = \hat{A}\hat{C}$ \therefore The quadrilateral $AC\hat{A}\hat{C}$ is a parallelogram.

Fig. (2)

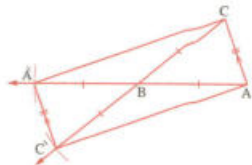


As we said previously the quadrilateral $ACAC$ is a parallelogram.

$$\therefore \overline{AA'} \perp \overline{CC'}$$

\therefore The quadrilateral $ACAC$ is a rhombus.

Fig. (3)



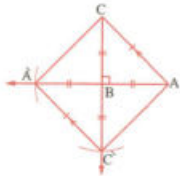
As we mentioned before the quadrilateral $ACAC$ is a parallelogram.

$$\therefore AB = \hat{A}B, CB = \hat{C}B, AB = BC$$

$$\therefore \hat{A}\hat{A} = \hat{C}\hat{C}$$

\therefore The quadrilateral $ACAC$ is a rectangle.

Fig. (4)



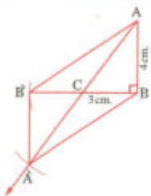
As we mentioned before $ACAC$ is a parallelogram.

$$\therefore AB = \hat{B}\hat{A}, CB = \hat{B}\hat{C}, AB = BC$$

$$\therefore \hat{A}\hat{A} = \hat{C}\hat{C}, \overline{AA'} \perp \overline{CC'}$$

\therefore The quadrilateral $ACAC$ is a square.

5



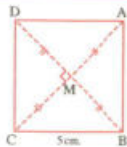
$\therefore \hat{A}$ is the image of A by reflection in C

$\therefore \hat{B}$ is the image of B by reflection in C

$$\therefore AB = \hat{A}\hat{B}, \overline{AB} \parallel \overline{\hat{A}\hat{B}}$$

\therefore The quadrilateral $AB\hat{A}\hat{B}$ is a parallelogram. (Q.E.D.)

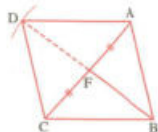
6



The image of the square ABCD by reflection in the point M is the square CDAB

We notice that we got the same square.

7

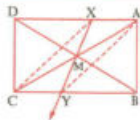


The figure ABCD is a parallelogram.

1 A right-angled triangle at B.

2 An isosceles triangle ($AB = BC$)

8



\therefore ABCD is a rectangle

$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore m(\angle XAM) = m(\angle YCM) \quad (\text{alternate angles})$$

$\Delta \hat{A}MX, CMY$ in them :

$$\begin{cases} m(\angle XAM) = m(\angle YCM) \\ m(\angle AMX) = m(\angle CMY) \\ AM = CM \end{cases} \quad (\text{V.O.A.})$$

$\therefore \Delta \hat{A}MX \cong \Delta CMY$ then we deduce that $XM = MY$

$$\therefore Y \in \overline{XM}$$

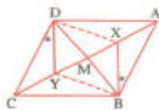
\therefore Y is the image of X by reflection in the point M (Q.E.D. 1)

$$\therefore AM = CM, MX = MY$$

\therefore The figure $AXCY$ is a parallelogram. (Q.E.D. 2)



9



$\triangle ABX$ & $\triangle CDY$ in them

$$AB = CD$$

$$m(\angle BAX) = m(\angle DCY) \text{ (alternate angles)}$$

$$m(\angle ABX) = m(\angle CDY)$$

$\therefore \triangle ABX \cong \triangle CDY$ then we deduce that $AX = CY$

$$\therefore AM = CM \quad \therefore AM - AX = CM - CY$$

$$\therefore XM = YM$$

$\therefore Y$ is the image of X by reflection in M

$$\therefore AM = CM$$

$\therefore C$ is the image of A by reflection in M

$$\therefore BM = DM$$

$\therefore D$ is the image of B by reflection in M

$\therefore \triangle ABX$ is the image of $\triangle CDY$ by reflection in M

(Q.E.D. 1)

$$\therefore XM = YM, BM = DM$$

\therefore The figure $XBYD$ is a parallelogram. (Q.E.D. 2)

Second : Problems on reflection in the Cartesian plane :

1

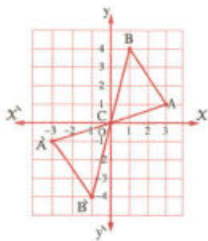
$$\text{① (c)}$$

$$\text{② (c)}$$

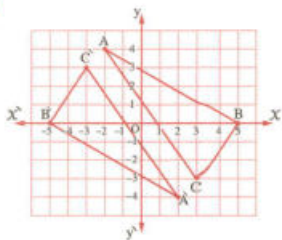
$$\text{③ (c)}$$

$$\text{④ (b)}$$

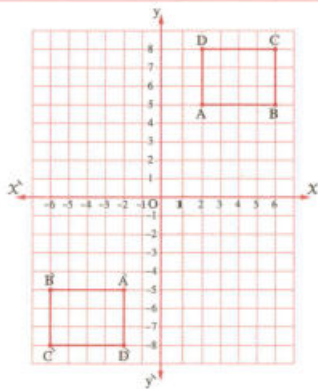
2



3



4



5

$\therefore \overline{CD}$ is the image of \overline{BA} by reflection in M

$$\therefore BA = CD \quad \therefore 2x + 5 = x + 9 \quad \therefore x = 4$$

\therefore the length of $\overline{CD} = 4 + 9 = 13$ cm. (First req.)

$\therefore \overline{CD}$ is the image of \overline{BA} by reflection in M

$$\therefore BM = MC, AM = MD$$

$\therefore ACDB$ is a parallelogram.

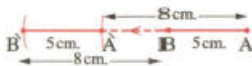
$$\therefore m(\angle BAD) = m(\angle CDA) \text{ (alternate angles)}$$

$$\therefore 2y = 60^\circ \quad \therefore y = 30^\circ \text{ (Second req.)}$$

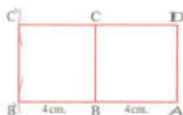
Answers of Exercise 11

First : Problems on translation in the plane :

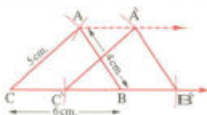
1



2

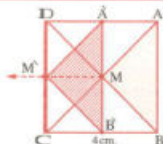


3

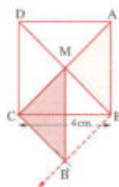


4

- 1 $\triangle M\hat{A}B$ is the image of $\triangle MAB$ by the translation of distance 2 cm, in the direction of \overrightarrow{AD}



- 2 $\triangle MC\hat{B}$ is the image of $\triangle AMB$ by the translation of distance AM in the direction of \overrightarrow{AM}



5

- 1 $\triangle DBE$

- 2 BE, \overrightarrow{BE}

6

- 1 M

- 2 \overrightarrow{BM}

- 3 $\triangle ABM$

- 4 AM in the direction \overrightarrow{AM}

7

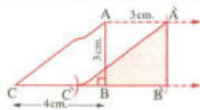
- 1 \overline{MN}

- 2 the square $FBLG$

- 3 4 cm, \overrightarrow{CY}

8

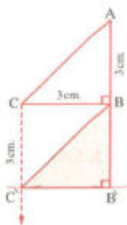
$\triangle A\hat{B}\hat{C}$ is the image of $\triangle ABC$ by the translation of distance 3 cm, in the direction of \overrightarrow{CB}



$\therefore \hat{A}$ is the image of A, \hat{C} is the image of C
 $\therefore \overline{A\hat{C}}$ is the image of \overline{AC} by the translation of distance 3 cm, in the direction of \overrightarrow{CB}
 $\therefore \overline{A\hat{C}} \parallel \overline{AC}, \hat{A}\hat{C} = AC$
 $\therefore A\hat{A}\hat{C}\hat{C}$ is a parallelogram. (Q.E.D.)

9

$\triangle B\hat{B}\hat{C}$ is the image of $\triangle ABC$ by the translation of distance 3 cm, in the direction of \overrightarrow{AB}



$\therefore \hat{B}$ is the image of B, \hat{C} is the image of C by the translation of distance 3 cm, in the direction of \overrightarrow{AB}

$\therefore \overline{B\hat{C}} \parallel \overline{BC}, \hat{B}\hat{C} = BC$

$\therefore B\hat{B}\hat{C}\hat{C}$ is a parallelogram.

$\therefore m(\angle ABC) = 90^\circ$

$\therefore m(\angle B\hat{B}\hat{C}) = 90^\circ$

"the translation reserves the measures of angles"

$\therefore B\hat{B}\hat{C}\hat{C}$ is a rectangle.

$\therefore AB = BC = 3 \text{ cm}$

\therefore the length of the image of \overline{AB} = the length of the image of $\overline{BC} = 3 \text{ cm}$.

"Translation reserves the lengths of line segments"

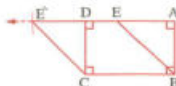
$\therefore B\hat{B} = \hat{B}\hat{C}$

$\therefore B\hat{B}\hat{C}\hat{C}$ is a square.

(Q.E.D.)

10

$\triangle D\hat{C}\hat{E}$ is the image of $\triangle ABE$ by the translation of distance AD in the direction of \overrightarrow{AD}



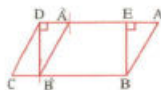
$\therefore \overline{C\hat{E}}$ is the image of \overline{BE} by translation of distance AD in the direction of \overrightarrow{AD}

$\therefore \overline{C\hat{E}} \parallel \overline{BE}, C\hat{E} = BE$

\therefore The figure $BC\hat{E}\hat{E}$ is a parallelogram. (Q.E.D.)

11

$\triangle A\hat{B}\hat{D}$ is the image of $\triangle ABE$ by the translation of distance ED in the direction of \overrightarrow{AD}



$\therefore \overline{B\hat{D}}$ is the image of \overline{BE} by translation of distance ED in the direction of \overrightarrow{AD}

$\therefore \overline{B\hat{D}} \parallel \overline{BE}, \hat{B}\hat{D} = BE$

$\therefore EB\hat{B}\hat{D}$ is a parallelogram.

$\therefore m(\angle A\hat{D}\hat{B}) = m(\angle AEB) = 90^\circ$

"The translation reserves the measures of angles"

\therefore The figure $EB\hat{B}\hat{D}$ is a rectangle.

(Q.E.D.)


Second : Problems on translation in the Cartesian plane :

1

$$1 \quad (4, 6) \quad 2 \quad (6, 0) \quad 3 \quad (-1, -1)$$

$$4 \quad (-4, -5) \quad 5 \quad (3, 1)$$

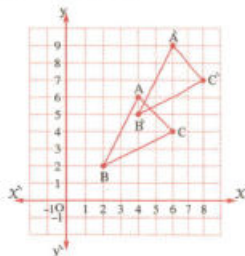
$$2 \quad 1 \quad (b) \quad 2 \quad (a) \quad 3 \quad (b) \quad 4 \quad (d) \quad 5 \quad (c) \quad 6 \quad (a)$$

3

$$1 \quad A(4, 6) \longrightarrow \tilde{A}(6, 9)$$

$$B(2, 2) \longrightarrow \tilde{B}(4, 5)$$

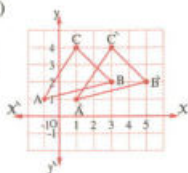
$$C(6, 4) \longrightarrow \tilde{C}(8, 7)$$



$$2 \quad A(-1, 1) \longrightarrow \tilde{A}(1, 1)$$

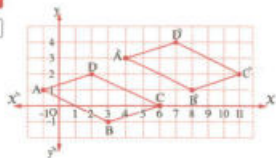
$$B(3, 2) \longrightarrow \tilde{B}(5, 2)$$

$$C(1, 4) \longrightarrow \tilde{C}(3, 4)$$

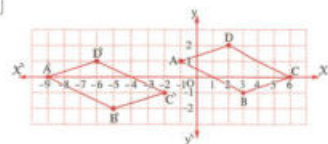


4

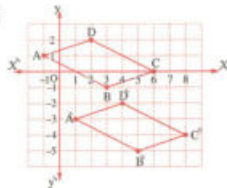
1



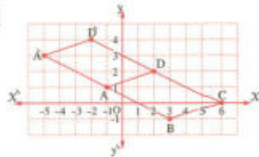
2



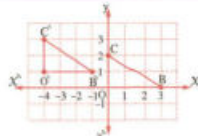
3



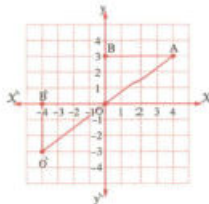
4



5



6



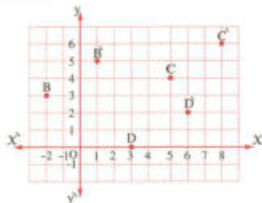
7

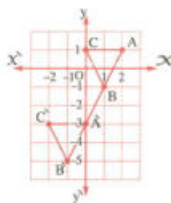
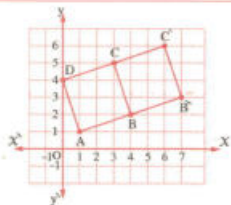
$$(X, y) \longrightarrow (X + 3, y + 2)$$

1 The image of $B(-2, 3)$ by translation LM is $\tilde{B}(1, 5)$

2 The image of $C(5, 4)$ by translation LM is $\tilde{C}(8, 6)$

3 The image of $D(3, 0)$ by translation LM is $\tilde{D}(6, 2)$



8

9
1


2 The mapping rule is $(X, y) \longrightarrow (X + 3, y + 1)$

10 $(0, 0)$

11

$\therefore \hat{A}(2, 2)$ is the image of the point $A(1, 1)$

\therefore The mapping rule of this translation

$$\text{is } (X, y) \longrightarrow (X + 1, y + 1)$$

\therefore The image of $O(0, 0)$ is $\hat{O}(1, 1)$

, the image of $B(-1, 3)$ is $\hat{B}(0, 4)$

, the image of $C(-3, 5)$ is $\hat{C}(-2, 6)$

12 $(X, y) \longrightarrow (X + 4, y - 3)$

13

$$(X, y) \longrightarrow (X + 2, y - 1)$$

1 The image of $C(1, -1)$ is $\hat{C}(3, -2)$

2 Let $D(X, y)$ $\therefore X + 2 = 2$

$$\therefore X = 0, y - 1 = 1 \quad \therefore y = 2 \quad \therefore D(0, 2)$$

14

$$\text{Let } A(X, y) \quad \therefore X - 1 = 3$$

$$\therefore X = 4 \quad y - 4 = -3 \quad \therefore y = 1$$

$$\therefore A(4, 1)$$

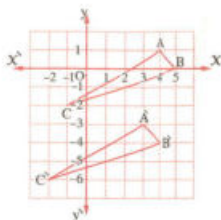
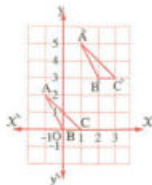
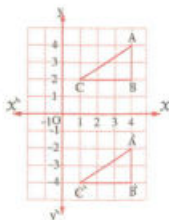

15

16

Fig. (1) Translation $(X, y) \longrightarrow (X - 5, y - 3)$

Fig. (2) Reflection and the reflection axis is X -axis

Fig. (3) Reflection and the reflection axis is y -axis

Fig. (4) Translation $(X, y) \longrightarrow (X - 5, y + 3)$

17

18

$\therefore A(2, 1)$ is the image of B by reflection

in X -axis followed by reflection in y -axis

$$\therefore B(-2, -1)$$

\therefore The mapping rule of translation that makes the point $A(2, 1)$ the image of the point $B(-2, -1)$

$$\text{is } (X, y) \longrightarrow (X + 4, y + 2)$$

15

- $\therefore \overline{XZ} \parallel \overline{YZ} + \overline{XY}$ is a transversal to them.
 $\therefore m(\angle ZXY) = m(\angle Y) = 30^\circ$ (alternate angles)
 $\therefore m(\angle YXZ) = m(\angle YXZ) + m(\angle ZXY)$
 $= 90^\circ + 30^\circ = 120^\circ$
 \therefore The measure of the angle of rotation $= 120^\circ$
 (First req.)
 $\therefore \hat{Z}$ is the image of Z and \hat{Y} is the image of Y by the same rotation and X is the image of itself.
 $\therefore \Delta \hat{Y}\hat{Z}X$ is the image of ΔYZX by rotation
 $R(X, 120^\circ)$
 $\therefore \hat{Z}X = ZX = 5 \text{ cm.}$ (Second req.)

Second : Problems on rotation in the Cartesian plane :

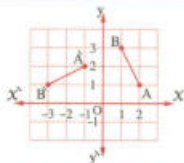
1

- | | |
|-------------------|--------------------|
| 1 (3, 2), (-2, 3) | 2 (0, -1), (-1, 0) |
| 3 -90° | 4 (4, 1) |
| 5 (-5, 2) | 6 (7, -3) |
| 7 (1, 1) | 8 (y, x) |
| 9 $\pm 360^\circ$ | 10 zero |

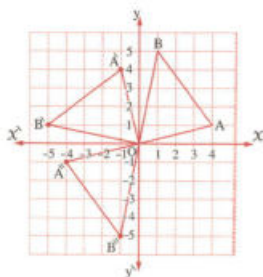
2

- (2, 1), (-2, -1), (-2, 1), (-1, 3),
 (1, 2), (-1, -2), (-2, 1), (2, -1)

3



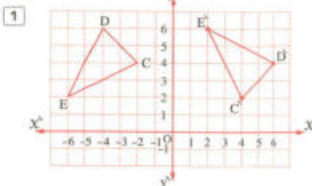
4



- 1 $\Delta \hat{A}\hat{B}\hat{O}$ is the image of ΔABO by rotation about O with an angle of measure 90°

- 2 $\Delta \hat{A}\hat{B}\hat{O}$ is the image of ΔABO by rotation about O with an angle of measure 180°

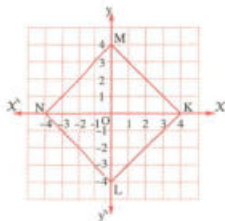
5



- $\Delta \hat{D}\hat{C}\hat{E}$ is the image of ΔDCE by rotation
 about O with an angle of measure -90°

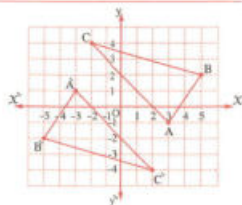
2

- The square
 $MNLK$ is the
 image of the
 square $KMNL$ by
 rotation about O
 with an angle of
 measure 90°

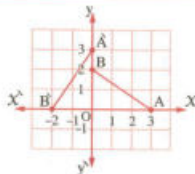


6

- $\Delta \hat{A}\hat{B}\hat{C}$ is the
 image of ΔABC
 by rotation about
 the origin point
 with an angle of
 measure 180°



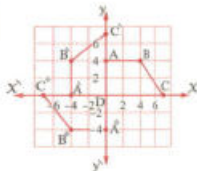
7



- $\therefore \Delta \hat{A}\hat{O}\hat{B}$ is the image of ΔAOB by rotation about
 O with an angle of measure 90°



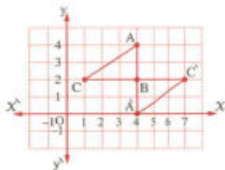
8



- The figure $\hat{A}\hat{B}\hat{C}\hat{D}$ is the image of the figure ABCD by rotation about the origin point where :
 $(X, y) \longrightarrow (-y, X)$
- The figure $\hat{A}\hat{B}\hat{C}\hat{D}$ is the image of the figure ABCD by rotation about the origin with an angle of measure -180°

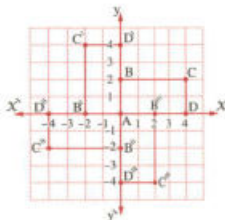
9 $C = (5, 4)$, $\hat{C} = (-5, -4)$

10



$\Delta \hat{A}\hat{B}\hat{C}$ is the image of ΔABC by rotation about B with an angle of measure 180°

11 [a]



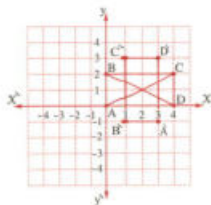
- The rectangle $\hat{A}\hat{B}\hat{C}\hat{D}$ is the image of the rectangle ABCD by rotation about the origin with an angle of measure 90°
- The rectangle $\hat{A}\hat{B}\hat{C}\hat{D}$ is the image of the rectangle ABCD by rotation about the origin with an angle of measure 180°

- The rectangle $\hat{A}\hat{B}\hat{C}\hat{D}$ is the image of the rectangle ABCD by rotation about the origin with an angle of measure 270°

[b] $(2, 1)$

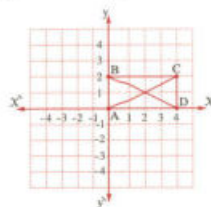
[c]

1



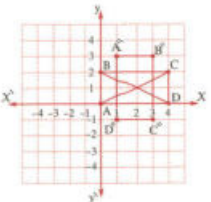
The rectangle $\hat{A}\hat{B}\hat{C}\hat{D}$ is the image of the rectangle ABCD by rotation about the centre of the rectangle with an angle of measure 90°

2



The rectangle CDAB is the image of the rectangle ABCD by rotation about the centre of the rectangle with an angle of measure 180°

3



The rectangle $\hat{A}\hat{B}\hat{C}\hat{D}$ is the image of the rectangle ABCD by rotation about the centre of the rectangle with an angle of measure 270°

12

ΔABC is a right-angled triangle at A from Pythagoras $(AC)^2 = (BC)^2 - (AB)^2 = 25 - 9 = 16$

$\therefore AC = 4 \text{ cm.}$

$\therefore \triangle \hat{C}\hat{A}\hat{B}$ is the image of $\triangle CAB$ by the given rotation

$\therefore \triangle \hat{C}\hat{A}\hat{B} \cong \triangle CAB$

i.e. $\hat{AC} = AC = 4 \text{ cm.}$, then $\hat{AA} = 8 \text{ cm.}$

$\hat{AB} = AB = 3 \text{ cm.}$

\therefore the area of $\triangle \hat{A}\hat{A}\hat{B}$

$$= \frac{1}{2} \hat{AA} \times \hat{AB} = \frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2 \quad (\text{The req.})$$

Answers of accumulative basic skills

1 1 (b) 2 (c) 3 (d) 4 (d)

5 (b) 6 (c) 7 (d) 8 (a)

9 (c) 10 (b) 11 (d) 12 (b)

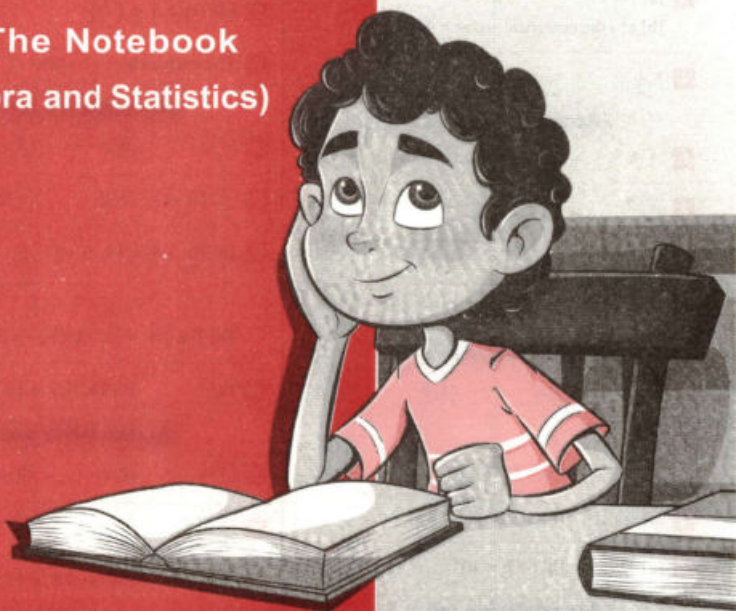
2 1 125 2 2 3 an acute 4 60°

5 115.5 6 50 7 6 8 360°

9 18 10 37.5 % 11 16 12 28

Guide Answers

Of The Notebook
(Algebra and Statistics)



Answers of the accumulative tests on Algebra and Statistics

Accumulative test 1

1 1 c 2 b 3 b 4 b

2 1 1 2 $\frac{3}{8}$ 3 1 4 3

3 $-\frac{2}{9}$

4 $\frac{1}{24}$

Accumulative test 2

1 1 b 2 d 3 b 4 d

2 1 3 2 6 3 2^{11} 4 $-\frac{1}{9}$

3 [a] 144
[b] a^2 , the numerical value = 4

4 $8\frac{1}{2}$

Accumulative test 3

1 1 b 2 c 3 c 4 a

2 1 $\frac{9}{4}$ 2 $\frac{y}{x}$ 3 25 4 $\frac{4}{9}$

3 [a] $\frac{1}{10}$ [b] $\frac{1}{9}$

4 $\frac{1}{x^2}$, the numerical value = 4

Accumulative test 4

1 1 a 2 c 3 d 4 d

2 1 4^{10} 2 -4
3 $\frac{9}{25}$ 4 3×10^{-3}

3 [a] 3×10^8 [b] 6×10^5

4 [a] 729 [b] 6.12×10^3

Accumulative test 5

1 1 d 2 b 3 b 4 b

2 1 20 2 -4 3 11 4 x^8

3 [a] 22 [b] 4

4 [a] $\frac{1}{36}$ [b] 25

Accumulative test 6

1 1 d 2 c 3 a 4 c

2 1 10 2 3^8 3 16 4 zero

3 [a] $\frac{2}{5}$ [b] 1

4 9 cm.

Accumulative test 7

1 1 c 2 a 3 c 4 d

2 1 \emptyset 2 $\{0\}$ 3 11 4 $x - 4$

3 [a] The S.S. = $\{\frac{1}{3}\}$ [b] 49

4 The integers are: -1, zero, 1

Accumulative test 8

1 1 c 2 d 3 c 4 b

2 1 $\{0, 1\}$ 2 \emptyset 3 2^{39} 4 2

3 [a] The S.S. = $\{4, 5, 6, \dots\}$



[b] The S.S. = $\{x : x \in \mathbb{Q}, x \geq -1\}$

4 [a] 1 [b] The S.S. = $\{3\}$

Accumulative test 9

1 1 b 2 c 3 c 4 b

2 1 0.2 2 $\frac{1}{2}$ 3 1 4 -5

3 [a] zero [b] The S.S. = $\{5\}$

4 1 $\frac{3}{4}$ 2 $\frac{3}{5}$ 3 $\frac{2}{5}$ 4 $\frac{3}{20}$



Answers of monthly tests on Algebra and Statistics

Answers of March tests

Answers of Test 1

1

1 c

2 b

3 d

2

1 36

2 3

3 -4

3 $2 \times 0.4 \times \frac{1}{2} + (-2)^2 = 0.4 + 4 = 4.4$

4

$$\frac{b^3 + (-5)}{b^{-2} + 6} = \frac{b^{-2}}{b^4} = b^{-2-4} = b^{-6} = \frac{1}{b^6}$$

at $b = 2$, then $\frac{1}{2^6} = \frac{1}{64}$

Answers of Test 2

1

1 d

2 d

3 c

2

1 1

2 0

3 3

3 6×10^5

4
$$\frac{(2^2)^{n+1} \times 3^{n-1}}{(2^2 \times 3)^n} = \frac{2^{2n+2} \times 3^{n-1}}{2^{2n} \times 3^n}$$

$$= 2^{2n+2-2n} \times 3^{n-1-n}$$

$$= 2^2 \times 3^{-1} = \frac{4}{3}$$

Answers of April tests

Answers of Test 1

1

1 b

2 b

3 c

2

1 $\frac{2}{3}$

2 12

3 >

3

$$\because 2 - 3X - 2 \leq 7 - 2$$

$$\therefore -3X \leq 5$$

$$\therefore -3X \times \frac{-1}{3} \geq 5 \times \frac{-1}{3}$$

$$\therefore X \geq \frac{-5}{3}$$

$$\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X \geq \frac{-5}{3}\}$$

4

$$\left(\frac{5}{2}\right)^2 \times \frac{2}{5} \times 2 = \frac{25}{4} \times \frac{4}{5} = 5$$

Answers of Test 2

1

1 b

2 c

3 b

2

1 $\{0, 1, 2, \dots\}$

2 3

3 $6 - X$

3

$$\because 2 + 3X - 2 = 4 - 2$$

$$\therefore 3X = 2$$

$$\therefore 3X \times \frac{1}{3} = 2 \times \frac{1}{3}$$

$$\therefore X = \frac{2}{3}$$

$$\therefore \text{The S.S.} = \left\{\frac{2}{3}\right\}$$

4

$$\text{Let the numbers be : } X, X+1, X+2$$

$$\therefore X + X + 1 + X + 2 = 42$$

$$\therefore 3X + 3 = 42$$

$$\therefore 3X + 3 - 3 = 42 - 3$$

$$\therefore 3X = 39$$

$$\therefore 3X \times \frac{1}{3} = 39 \times \frac{1}{3}$$

$$\therefore X = 13$$

$$\therefore \text{The numbers are : } 13, 14, 15$$

Answers of important questions on Algebra and Statistics

Unit one

First Answers of multiple choice questions

- | | | | | |
|--------|--------|--------|--------|--------|
| 1 (b) | 2 (b) | 3 (d) | 4 (a) | 5 (c) |
| 6 (d) | 7 (c) | 8 (b) | 9 (c) | 10 (c) |
| 11 (c) | 12 (a) | 13 (b) | 14 (b) | 15 (b) |
| 16 (a) | 17 (b) | 18 (b) | 19 (b) | 20 (c) |
| 21 (c) | 22 (c) | 23 (b) | 24 (b) | 25 (d) |
| 26 (b) | 27 (c) | 28 (a) | 29 (c) | 30 (c) |
| 31 (c) | 32 (d) | 33 (d) | 34 (b) | 35 (c) |
| 36 (c) | 37 (a) | 38 (a) | 39 (a) | 40 (b) |
| 41 (d) | 42 (b) | 43 (c) | 44 (d) | 45 (a) |

Second Answers of complete questions

- | | | |
|--------------------------|--------------------|-------------------------|
| 1 1 | 2 $\frac{9}{4}$ | 3 -2 |
| 4 $-\frac{1}{3}$ | 5 $\frac{4}{25}$ | 6 $6561x^3$ |
| 7 zero | 8 3 | 9 4 |
| 10 1 | 11 $\frac{y}{x}$ | 12 $1+a^5$ |
| 13 18 | 14 7 | 15 1 |
| 16 7.21×10^{-6} | 17 7×10^6 | 18 5.346×10^5 |
| 19 3.7×10^6 | 20 3.7 | 21 4.5×10^{-5} |
| 22 3 | 23 14 | 24 6 |
| 25 zero | 26 $-\frac{3}{4}$ | 27 $\frac{6}{5}$ |
| 28 4 | 29 10 | 30 0.6 |
| 31 50 | 32 24 | 33 \emptyset |
| 34 15 | 35 x | 36 \emptyset |
| 37 $\{3, 4\}$ | 38 $x-3$ | |

Third Answers of the essay questions

- 1 $5^{6+2-7} = 5^1 = 5$
- 2 $a^{5+8-3-2-4} = a^4$

$$3 \quad (3^{4-2} \times 7^{2-3})^{-1} = (3^2 \times 7^{-1})^{-1} = 3^{-2} \times 7 = \frac{7}{9}$$

4

$$\frac{(10)^2 \times (10^{-2})^3}{(10)^{-3}} = \frac{(10)^2 \times (10)^{-6}}{(10)^{-3}} = (10)^{2-6+3} = (10)^{-1} = \frac{1}{10}$$

$$5 \quad -\frac{27}{125} \times \left(-\frac{25}{27}\right) = \frac{1}{5}$$

$$6 \quad \frac{1}{4} \times \frac{-1}{8} = -\frac{1}{32}$$

$$7 \quad (x+y)^{-2} = \left(\frac{1}{2} + \frac{1}{3}\right)^{-2} = \left(\frac{5}{6}\right)^{-2} = \left(\frac{6}{5}\right)^2 = \frac{36}{25}$$

8

$$\left(\frac{a}{b^2}\right)^2 = \left(\frac{\frac{3}{4}}{\left(\frac{-1}{2}\right)^2}\right)^2 = \left(\frac{\frac{3}{4}}{\frac{1}{4}}\right)^2 = \left(\frac{3}{4} \times 4\right)^2 = 3^2 = 9$$

$$9 \quad x^{-2 \times -3} \div x^{-1 \times 2} = x^{-6} \div x^{-2} = x^{-6-(-2)} = x^{-4} = \frac{1}{x^4}$$

$$10 \quad x^2 y^2 z^2 = \left(\frac{3}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{-4}{3}\right)^2 = \frac{9}{4} \times \frac{1}{4} \times \frac{16}{9} = 1$$

$$11 \quad \left(\frac{-2}{5}\right)^x + \left(\frac{2}{5}\right)^y = \left(\frac{-2}{5}\right)^4 + \left(\frac{2}{5}\right)^3 = \frac{16}{625} + \frac{8}{125} = \frac{16}{625} + \frac{40}{625} = \frac{56}{625}$$

$$12 \quad \because (-2)^{82} = (2)^{82}, (-2)^{83} = -(2)^{83} \\ \therefore (-2)^{82} \text{ is greater.}$$

$$13 \quad \because \left(\frac{x}{y}\right)^2 = 36 \quad \therefore \frac{x}{y} = \pm \sqrt{36} = \pm 6 \\ \therefore \left(\frac{x}{y}\right)^3 = (\pm 6)^3 = \pm 216$$

$$14 \quad 10^3 (2.3 + 6.3 \times 10) = 10^3 (2.3 + 63) = 65.3 \times 10^3 = 6.53 \times 10^4$$

$$15 \quad (4.4 \times 10^3) \times (2 \times 10^5) = 8.8 \times 10^8$$

$$16 \quad (3.6 \times 10^8) \div (1.8 \times 10^3) = \frac{3.6}{1.8} \times \frac{10^8}{10^3} = 2 \times 10^5$$

$$17 \quad 5.812 \times 10^{11}$$

$$18 \quad 1.4 \times 10^{-3}$$

$$19 \quad 10^4 (5.4 + 3.7) = 9.1 \times 10^4$$



20 7×10^{-8}

21 $8 \div 8 + 5 = 1 + 5 = 6$

22 $12 \times 4 \div 24 + 9 = 48 \div 24 + 9 = 2 + 9 = 11$

23 $\frac{1}{8} \times \frac{9}{4} \div \frac{9}{16} = \frac{9}{32} \times \frac{16}{9} = \frac{1}{2}$

24 $3 + [5 + 2 \times 2] = 3 + [5 + 4] = 3 + 9 = 12$

25 $12 \times 4 \div 24 \times \frac{1}{2} = 48 \div 24 \times \frac{1}{2} = 2 \times \frac{1}{2} = 1$

26 $x^2 + y^2 = \left(\frac{-2}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{4}{25} + \frac{9}{25} = \frac{13}{25}$

27 $\sqrt{2x+y} = \sqrt{2 \times 9 + 7} = \sqrt{25} = 5$

28 $\sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^{\text{zero}} \times \left(\frac{-2}{7}\right)^2 = \frac{7}{2} \times 1 \times \frac{4}{49} = \frac{2}{7}$

29 $\sqrt{\frac{25}{4}} + \frac{1}{5} \sqrt{25} = \frac{5}{2} + \frac{1}{5} \times 5 = \frac{5}{2} + 1 = \frac{7}{2}$

30 $\left(\frac{-3}{7}\right)^{\text{zero}} \times \left(\frac{-2}{5}\right)^2 \times \frac{5}{2} = 1 \times \frac{2^2}{5^2} \times \frac{5}{2}$
 $= \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$

31 $\frac{1}{25} + \frac{24}{25} - \frac{15}{3} = \frac{25}{25} - 5 = 1 - 5 = -4$

32 $\left|\frac{5xy}{6}\right|$

33 Let the number be x \therefore its triple = $3x$

$\therefore 3x + x = 28$ $\therefore 4x = 28$

$\therefore x = \frac{28}{4} = 7$ \therefore The number is 7

34 Let the smaller number be x

\therefore the another number = $x + 4$

$\therefore x + x + 4 = 26$ $\therefore 2x + 4 = 26$

$\therefore 2x = 22$ $\therefore x = 11$

\therefore The two numbers are 11 and 15

35 $\therefore 2x - 3 = 5$ $\therefore 2x = 8$

$\therefore x = 4$ \therefore The S.S. = $\{4\}$

36 $\therefore 2x - 10 = 12$ $\therefore 2x = 22$

$\therefore x = 11$ \therefore The S.S. = $\{11\}$

37 $\therefore 3x - 3 = 12$ $\therefore 3x = 15$

$\therefore x = 5$ \therefore The S.S. = $\{5\}$

38 $\therefore 5x - 2x = 11 + 4$ $\therefore 3x = 15$

$\therefore x = 5$ \therefore The S.S. = $\{5\}$

39 $\therefore 5x - 8 + 8 \geq 7 + 8$ $\therefore 5x \geq 15$

$\therefore 5x \times \frac{1}{5} \geq 15 \times \frac{1}{5}$ $\therefore x \geq 3$

\therefore The S.S. = $\{x : x \in \mathbb{Q}, x \geq 3\}$

40 $\therefore 3x - 1 + 1 > 5 + 1$ $\therefore 3x > 6$

$\therefore 3x \times \frac{1}{3} > 6 \times \frac{1}{3}$ $\therefore x > 2$

\therefore The S.S. = $\{x : x \in \mathbb{Q}, x > 2\}$

41 $\therefore 3 - 2x - 3 \leq 7 - 3$

$\therefore -2x \leq 4$

$\therefore -2x \times \frac{-1}{2} \geq 4 \times \frac{-1}{2}$ $\therefore x \geq -2$

\therefore The S.S. = $\{x : x \in \mathbb{Q}, x \geq -2\}$

42 $\therefore 9x + 1 \leq 8x + 1$ $\therefore 9x - 8x \leq 1 - 1$

$\therefore x \leq 0$

\therefore The S.S. = $\{x : x \in \mathbb{Q}, x \leq 0\}$

Unit two

First Answers of multiple choice questions

- 1 (a) 2 (d) 3 (a) 4 (c)
5 (b) 6 (b) 7 (a) 8 (a)

Second Answers of complete questions

- 1 $\frac{1}{2}$ 2 $\frac{1}{6}$ 3 80 %
4 $\frac{3}{8}$ 5 $\frac{5}{12}$ 6 zero
7 $\frac{1}{6}$ 8 zero, 1

Third Answers of the essay questions

- 1 The total number of balls = $4 + 5 + 6 = 15$ balls
 ① The probability of getting a red ball = $\frac{5}{15} = \frac{1}{3}$
 ② The probability of getting a white or red ball
 = $\frac{4+5}{15} = \frac{9}{15} = \frac{3}{5}$
- 2 ① The probability of appearing a prime number
 = $\frac{3}{6} = \frac{1}{2}$
 ② The probability of appearing a multiple of the number 3 = $\frac{2}{6} = \frac{1}{3}$
 ③ The probability of appearing a number greater than 7 = $\frac{0}{6} = \text{zero}$
- 3 $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$
 ① The probability of getting an odd number
 = $\frac{4}{8} = \frac{1}{2}$
 ② The probability of getting a number divisible by 3 = $\frac{2}{8} = \frac{1}{4}$
 ③ The probability of getting a number less than 9 = $\frac{8}{8} = 1$
- 4 ① The probability that the drawn card carries a number divisible by 5 = $\frac{5}{25} = \frac{1}{5}$
 ② The probability that the drawn card carries a number $\geq 20 = \frac{6}{25}$
 ③ The probability that drawn card carries a perfect square number = $\frac{5}{25} = \frac{1}{5}$
 ④ The probability that the drawn card carries an odd number greater than 13 and less than $25 = \frac{5}{25} = \frac{1}{5}$

- 5 \therefore The probability of drawing a green ball
 = $\frac{\text{The number of green balls}}{\text{The total number of balls}}$
 $\therefore \frac{1}{6} = \frac{\text{The total number of balls}}{2}$
 \therefore The total number of balls = $2 \times 6 = 12$ balls
 \therefore The number of red balls = $12 - (2 + 4) = 6$ balls

- 6 ① The probability that the chosen card shows an even number = $\frac{5}{10} = \frac{1}{2}$
 ② The probability that the chosen card shows an odd number greater than 3 = $\frac{3}{10}$

- 7 ① $S = \{1, 2, 3, 4, 5, 6\}$
 ② The probability of getting number 7
 = $\frac{0}{6} = \text{zero}$
 ③ The probability of getting an odd number
 = $\frac{3}{6} = \frac{1}{2}$
 ④ The probability of getting a prime number
 = $\frac{3}{6} = \frac{1}{2}$
 ⑤ The probability of getting a number less than 3 = $\frac{2}{6} = \frac{1}{3}$

- 8 ① $S = \{1, 2, 3, 4, 5, 6\}$
 ② The probability of getting a number greater than 6 = $\frac{0}{6} = \text{zero}$
 ③ \therefore The number that satisfying the inequality : $2 < X < 4$ is 3
 \therefore The probability = $\frac{1}{6}$



Answers of School book models on Algebra and Statistics

Model 1

1

1 -2 2 2 3 $\frac{7}{12}$ 4 3.5×10^{-3} 5 1

2

1 (b) 2 (a) 3 (a) 4 (a) 5 (a) 6 (a)

3

[a] $1 \times \frac{4}{25} \times \sqrt{\frac{25}{4}} = 1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$

[b] $3ab + 8a + (4b)$
 $= 3 \times 4 \times (-2) + 8 \times 4 \div (4 \times -2)$
 $= -24 + 32 \div (-8)$
 $= -24 - 4 = -28$

4

[a] $\therefore 3X + 1 = 25$ $\therefore 3X = 24$
 $\therefore X = 8$ \therefore The S.S. = $\{8\}$

[b] $\frac{8 \times 8^{-3}}{8^{-4}} = 8^{1-3+4} = 8^2 = 64$

5

[a] 1 The probability of change it before travelled
 50 thousand km. = $\frac{80}{800} = \frac{1}{10}$

2 The probability of change it after travelled more
 than 100 thousand km. = $\frac{600}{800} = \frac{3}{4}$

[b] $\therefore 2X + 5 < 16$ $\therefore 2X < 11$
 $\therefore X < \frac{11}{2}$
 \therefore The S.S. = $\{X: X \in \mathbb{Q}, X < \frac{11}{2}\}$

Model 2

1

1 1 2 $\frac{4}{7}$ 3 zero
 4 $13 + 21$ 5 510 students

2

1 (a) 2 (d) 3 (d) 4 (b) 5 (c) 6 (d)

3

[a] $\therefore 5X - 2X = 30$ $\therefore 3X = 30$ $\therefore X = 10$
 \therefore The two numbers are : 20, 50

[b] $\frac{5^{-4} \times 5^7}{5^3} = 5^{-4+7-3} = 5^0 = 1$

4

[a] 1 $\therefore 3X + 7 = 13$ $\therefore 3X = 6$

$\therefore X = 2$ \therefore The S.S. = $\{2\}$

2 $\therefore 2X + 15 < 19$ $\therefore 2X < 4$

$\therefore X < 2$

\therefore The S.S. = $\{X: X \in \mathbb{Q}, X < 2\}$

[b] $\frac{1}{9} + \frac{8}{9} - 1 = 1 - 1 = \text{zero}$

5

[a] 1 The probability of getting a prime even
 number = $\frac{1}{6}$

2 The probability of getting an odd number less
 than 4 = $\frac{2}{6} = \frac{1}{3}$

[b] $\left(\frac{y}{x^2}\right)^{-2} = \left[\frac{-\frac{3}{4}}{\left(\frac{-1}{2}\right)^2}\right]^{-2} = \left(\frac{-\frac{3}{4}}{\frac{1}{4}}\right)^{-2} = (-3)^{-2}$
 $= \frac{1}{(-3)^2} = \frac{1}{9}$

Model examination for the merge students

1

1 a 2 b 3 a
 4 b 5 b

2

1 4 2 $\frac{1}{2}$ 3 zero
 4 $\frac{2}{5}$ 5 42

3

1 $12 \times 2^2 + 24 + 3^2 = 12 \times 4 + 24 + 9$
 $= 48 + 24 + 9 = 2 + 9 = 11$
 2 $\frac{8+20-4}{8-4} = \frac{28-4}{4} = \frac{24}{4} = 6$

4

1 ✓ 2 ✗ 3 ✓
 4 ✗ 5 ✗

5

1 $\{8, 6, 4, 2\}$ 2 $\frac{1}{2}$
 3 $\{8, 7\}$ 4 1 5 $\frac{1}{8}$

**Answers of the schools examinations
on Algebra and Statistics**

1 Cairo

- 1** [1] b [2] a [3] c
[4] b [5] d [6] a
- 2** [1] 4 [2] 11 [3] 9
[4] -2 [5] 1 [6] -6

- 3**
[a] $\frac{1}{9} + \frac{8}{9} - 1 = \frac{9}{9} - 1 = 0$
[b] [1] $\because 3X + 2 = 11 \therefore 3X = 9$
 $\therefore X = 3 \therefore \text{The S.S.} = \{3\}$
[2] $\because 2X + 15 < 19 \therefore 2X < 4 \therefore X < 2$
 $\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X < 2\}$

- 4**
[a] Let the numbers be : $X, X + 1, X + 2$
 $\therefore X + X + 1 + X + 2 = 63$
 $\therefore 3X + 3 = 63 \therefore 3X = 60$
 $\therefore X = 20$
 $\therefore \text{The numbers are : } 20, 21, 22$

[b] $\frac{5^{-4} \times 5^7}{5^2} = 5^{-4+7-2} = 5$

- 5**
[a] $9^X = (3^2)^X = (3^X)^2 = (2)^2 = 4$
[b] [1] The probability that the drawn ball is white
 $= \frac{0}{10} = 0$
[2] The probability that the drawn
ball is not blue $= \frac{6}{10} = \frac{3}{5}$

2 Cairo

- 1** [1] b [2] d [3] a
[4] c [5] a [6] c
- 2** [1] 3.5×10^{-3} [2] 7 [3] $\frac{7}{4}$
[4] 0.2 [5] 2 [6] second

- 3**
[a] $\because 2X + 4 < 16 \therefore 2X < 12 \therefore X < 6$
 $\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X < 6\}$
[b] $\frac{5^{-4} \times 5^7}{5^2} = 5^{-4+7-2} = 5$

- 4**
[a] $1 \times \frac{4}{25} \times \sqrt{\frac{25}{4}} = 1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$
[b] $X^2 Y^2 = \left(\frac{2}{3}\right)^2 \times \left(\frac{3}{4}\right)^2 = \frac{4}{9} \times \frac{9}{16} = \frac{1}{4}$

- 5**
[a] [1] The probability that the drawn
ball is white $= \frac{0}{10} = 0$
[2] The probability that the drawn
ball is not blue $= \frac{6}{10} = \frac{3}{5}$
[b] $\because 3X + 1 = 25 \therefore 3X = 24$
 $\therefore X = 8 \therefore \text{The S.S.} = \{8\}$

3 Giza

- 1** [1] c [2] b [3] d
[4] a [5] d [6] b
- 2** [1] $-\frac{4}{9}$ [2] 0 [3] $\frac{3}{a}$
[4] 29 [5] 8 [6] 6

- 3**
[a] [1] $\because 8X + 4 = 12 \therefore 8X = 8$
 $\therefore X = 1 \therefore \text{The S.S.} = \{1\}$
[2] $\because 3X - 1 \leq 3 \therefore 3X \leq 4$
 $\therefore X \leq \frac{4}{3}$
 $\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X \leq \frac{4}{3}\}$
[b] $12 + (9 - 2) \times 3^2 = 12 + 7 \times 3^2 = 12 + 7 \times 9$
 $= 12 + 63 = 75$

- 4**
[a] $(XYZ)^2 = \left(\frac{1}{2} \times \frac{2}{3} \times \frac{-3}{2}\right)^2 = \left(\frac{-1}{2}\right)^2 = \frac{1}{4}$
[b] $\frac{X^3 \times X^{-2}}{X^{-5} \times X} = X^{3-2+5-1} = X^5$
when $X = -2$
 $\therefore X^5 = (-2)^5 = -32$



5

- [1] The probability of appearance of an even prime number = $\frac{1}{6}$
- [2] The probability of appearance of number 5 = $\frac{1}{6}$
- [3] The probability of appearance of a number ≤ 6 = $\frac{6}{6} = 1$

4

Giza

1

[1] 1

[2] zero

[3] 12

[4] 1

[5] -4

[6] -4

2

[1] a

[2] d

[3] c

[4] c

[5] d

[6] d

3

- [a] $\because 3X - 2 > X + 4 \quad \therefore 3X - X > 4 + 2$
 $\therefore 2X > 6 \quad \therefore X > 3$
 $\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X > 3\}$
- [b] $a^2 b^3 + b^2 c = \left(\frac{-1}{2}\right)^2 \times (2)^3 + (2)^2 \times \frac{3}{4}$
 $= \frac{1}{4} \times 8 + 4 \times \frac{3}{4} = 2 + 3 = 5$

4

- [1] $\frac{16}{25} \times \frac{5}{4} \times 1 = \frac{4}{5}$
- [2] $\frac{(-3)^6 \times (3)^{-3}}{(3)^5 \times (3)^{-4}} = \frac{(3)^6 \times (3)^{-3}}{(3)^5 \times (3)^{-4}} = 3^{6-3-5+4} = 3^2 = 9$

5

- [a] $\because 2(X+4) = 15 \quad \therefore 2X + 8 = 15$
 $\therefore 2X = 7 \quad \therefore X = \frac{7}{2}$
 $\therefore \text{The S.S.} = \left\{\frac{7}{2}\right\}$
- [b] [1] The probability of drawing a card carrying an odd number greater than 10 = $\frac{0}{10} = 0$
- [2] The probability of drawing a card carrying an even number less than 10 = $\frac{4}{10} = \frac{2}{5}$
- [3] The probability of drawing a card carrying a prime number = $\frac{4}{10} = \frac{2}{5}$

5

Alexandria

1

[1] b

[2] b

[3] c

[4] b

[5] d

[6] a

2

[1] 10

[2] 3

[3] 0

[4] 3

[5] 25

[6] 0

3

- [a] $\because 3X - 5 = 7 \quad \therefore 3X = 12$
 $\therefore X = 4 \quad \therefore \text{The S.S.} = \{4\}$
- [b] $\frac{5^{-4} \times 5^7}{5^3} = 5^{-4+7-3} = 5^0 = 1$

4

- [a] $1 \times \frac{16}{9} \times \frac{9}{4} = 4$
- [b] $\because 2X + 3 \geq 13 \quad \therefore 2X \geq 10 \quad \therefore X \geq 5$
 $\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X \geq 5\}$

5

- [a] $a^2 b^2 = \left(\frac{-1}{2}\right)^2 \times \left(\frac{2}{3}\right)^2 = \frac{1}{4} \times \frac{4}{9} = \frac{1}{9}$
- [b] [1] The probability that the drawn ball is yellow = $\frac{3}{12} = \frac{1}{4}$
- [2] The probability that the drawn ball is red = $\frac{5}{12}$
- [3] The probability that the drawn ball is not white = $\frac{5+3}{12} = \frac{8}{12} = \frac{2}{3}$

6

El-Kalyoubia

1

[1] c

[2] b

[3] c

[4] b

[5] c

[6] d

2

[1] 15

[2] $\frac{1}{8}$

[3] 8

[4] -4

[5] 7

[6] 2

3

- [a] $\because 3X - 5 = X + 7 \quad \therefore 3X - X = 7 + 5$
 $\therefore 2X = 12 \quad \therefore X = 6$
 $\therefore \text{The S.S.} = \{6\}$
- [b] $\frac{4}{3} \times \frac{9}{4} \times 1 = 3$

4

- [a] $\frac{3^{-1} \times 3^7}{(-3)^5} = \frac{3^{-1} \times 3^7}{-(3)^5} = -3^{-1+7-5} = -3$
- [b] $\because 4X - 7 \geq 5 \quad \therefore 4X \geq 12$
 $\therefore X \geq 3$
 $\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X \geq 3\}$

5 $\frac{x^{-2}}{y^2} = \frac{\left(\frac{-3}{2}\right)^2}{\left(\frac{1}{2}\right)^2} = \frac{\frac{9}{4}}{\frac{1}{4}} = 9$

[b] 1 The probability that the drawn ball is black $= \frac{6}{15} = \frac{2}{5}$

2 The probability that the drawn ball is not red $= \frac{5+6}{15} = \frac{11}{15}$

7 El-Sharkia

1 **1** a **2** d **3** b
4 d **5** b **6** a

2 **1** > **2** 1 **3** 0.2
4 0 **5** 5.3×10^{-5} **6** -2

3
1 $\therefore 3X + 2 = 8 \quad \therefore 3X = 6$
 $\therefore X = 2 \quad \therefore \text{The S.S.} = \{2\}$
2 $\therefore 3 - 2X \leq 7 \quad \therefore -2X \leq 4$
 $\therefore X \geq -2$
 $\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X \geq -2\}$

4
[a] $1 \times \frac{-2}{5} \times \sqrt{\frac{25}{4}} = 1 \times \frac{-2}{5} \times \frac{5}{2} = -1$
[b] $\frac{5^{-4} \times 5^7}{5^3} = 5^{-4+7-3} = 5^0 = 1$

5
[a] 1 The probability of getting a blue ball $= \frac{6}{15} = \frac{2}{5}$
2 The probability of getting a white or red ball $= \frac{4+5}{15} = \frac{9}{15} = \frac{3}{5}$
3 The probability of getting a green ball $= \frac{0}{15} = 0$
[b] $X^2 y^2 = (Xy)^2 = \left(\frac{3}{4} \times \frac{4}{3}\right)^2 = 1^2 = 1$

8 El-Gharbia

1 **1** b **2** b **3** d
4 b **5** a **6** c

2 **1** 1 **2** $-\frac{4}{25}$ **3** -4
4 8 **5** 510 **6** 2

3 **[a]** $\left(\frac{y^2}{x}\right)^2 = \left[\frac{\left(\frac{3}{4}\right)^2}{\left(\frac{-1}{2}\right)}\right]^2 = \left[\frac{\frac{9}{16}}{\frac{-1}{2}}\right]^2 = \left[-\frac{9}{8}\right]^2 = \frac{81}{64}$

[b] $1 \times \frac{4}{25} \times \sqrt{\frac{25}{4}} = 1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$

4 **[a]** $\left(\frac{9^3 \times 9}{9^5}\right)^{-3} = (9^{3+1-5})^{-3} = (9^{-1})^{-3} = 9^3 = 729$

[b] Let the two numbers be : $X, 2X$
 $\therefore X + 2X = 108$
 $\therefore 3X = 108 \quad \therefore X = 36$
 $\therefore \text{The two numbers are : } 36, 72$

5
[a] 1 $\therefore 2X + 15 < 19$
 $\therefore 2X < 4 \quad \therefore X < 2$
 $\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X < 2\}$
2 $\therefore 3X + 1 = 25 \quad \therefore 3X = 24$
 $\therefore X = 8 \quad \therefore \text{The S.S.} = \{8\}$
[b] 1 The probability that the drawn ball is white $= \frac{5}{16}$
2 The probability that the drawn ball is red $= \frac{7}{16}$
3 The probability that the drawn ball is not white $= \frac{4+7}{16} = \frac{11}{16}$

9 Ismailia

1 **1** 1 **2** $\frac{5}{7}$ **3** 0
4 13, 21 **5** 37 **6** 4.8×10^{-3}

2 **1** a **2** b **3** a
4 d **5** d **6** b

3
[a] $\therefore X + 4 > 1 \quad \therefore X > -3$
 $\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X > -3\}$
[b] $\frac{8 \times 8^{-3}}{8^{-4}} = 8^{1-3+4} = 8^2 = 64$

4
[a] $1 \times \frac{4}{25} \times \sqrt{\frac{25}{4}} = 1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$
[b] 5.812×10^{11}



5

[a] ① The probability that the chosen card shows an odd number = $\frac{5}{10} = \frac{1}{2}$

② The probability that the chosen card shows a prime number = $\frac{4}{10} = \frac{2}{5}$

[b] $x^2 - y = \left(-\frac{1}{2}\right)^2 - \left(-\frac{3}{4}\right) = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

10 El-Beheira

1

① d

② c

③ b

④ c

⑤ d

⑥ c

2

① -4

② fourth

③ 0

④ $\frac{1}{3}$

⑤ 4

⑥ $\frac{25}{16}$

3

[a] $\therefore 5x - 1 = 3x + 13$

$$\therefore 5x - 3x = 13 + 1$$

$$\therefore 2x = 14$$

$$\therefore x = 7$$

$$\therefore \text{The S.S.} = \{7\}$$

[b] ① $a^2 b^2 = \left(\frac{2}{3}\right)^2 \times \left(-\frac{1}{2}\right)^2 = \frac{4}{9} \times \frac{1}{4} = \frac{1}{9}$

$$\begin{aligned} \text{② } (a-b)^{-1} &= \left[\frac{2}{3} - \left(-\frac{1}{2}\right)\right]^{-1} = \left[\frac{2}{3} + \frac{1}{2}\right]^{-1} \\ &= \left[\frac{4+3}{6}\right]^{-1} = \left[\frac{7}{6}\right]^{-1} = \frac{6}{7} \end{aligned}$$

4

[a] $\frac{1}{4} \times \frac{9}{5} \times \frac{4}{3} = \frac{3}{5}$

[b] $\frac{x^5 \times x^3}{x^7 \times x^4} = x^{5+3-7-4} = x^{-3} = \frac{1}{x^3}$
when $x = 4$

$$\therefore \frac{1}{x^3} = \frac{1}{4^3} = \frac{1}{64}$$

5

[a] $\therefore 4x + 3 < 27$

$$\therefore 4x < 24$$

$$\therefore x < 6$$

$$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x < 6\}$$

[b] ① The probability that the drawn ball is red = $\frac{5}{15} = \frac{1}{3}$

② The probability that the drawn ball is not blue = $\frac{4+5}{15} = \frac{9}{15} = \frac{3}{5}$

③ The probability that the drawn ball is white or blue = $\frac{4+6}{15} = \frac{10}{15} = \frac{2}{3}$

④ The probability that the drawn ball is black = $\frac{0}{15} = 0$

11 El-Menia

1

① c

② a

③ b

④ d

⑤ a

⑥ c

2

① 20

② 1

③ 75

④ 0

⑤ 1

⑥ 2

3

[a] $1 \times \left(-\frac{5}{2}\right)^2 \times \sqrt{\frac{16}{625}} = 1 \times \frac{25}{4} \times \frac{4}{25} = 1$

[b] $\frac{5^{-4} \times 5^{-3}}{5^{-7}} = 5^{-4-3+7} = 5^0 = 1$

4

[a] $\sqrt{x^2 + y^2} = \sqrt{(3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

[b] ① $S = \{1, 2, 3, 4, 5, 6\}$

② The probability of appearing an odd number = $\frac{3}{6} = \frac{1}{2}$

③ The probability of appearing an even prime number = $\frac{1}{6}$

5

① $\therefore 5x + 6 = -9$

$$\therefore 5x = -15 \quad \therefore x = -3$$

$$\therefore \text{The S.S.} = \{-3\}$$

② $\therefore 2x - 5 < 3$

$$\therefore 2x < 8 \quad \therefore x < 4$$

$$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x < 4\}$$

12 South Sinai

1

① d

② b

③ a

④ a

⑤ c

⑥ d

2

① 0

② a^8

③ 5

④ 3.3×10^4

⑤ 1

⑥ $\{0\}$

3

$$\begin{aligned} \text{[1]} \quad & \because 5X - 2 = 13 & \therefore 5X = 15 \\ & \therefore X = 3 & \therefore \text{The S.S.} = \{3\} \end{aligned}$$

$$\begin{aligned} \text{[2]} \quad & \because 3X + 2 \leq 8 & \therefore 3X \leq 6 \\ & \therefore X \leq 2 \\ & \therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X \leq 2\} \end{aligned}$$

4

$$\text{[1]} \quad \frac{2^3 \times 2^6}{2^7} = 2^{3+6-7} = 2^2 = 4$$

$$\text{[2]} \quad 1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$$

5

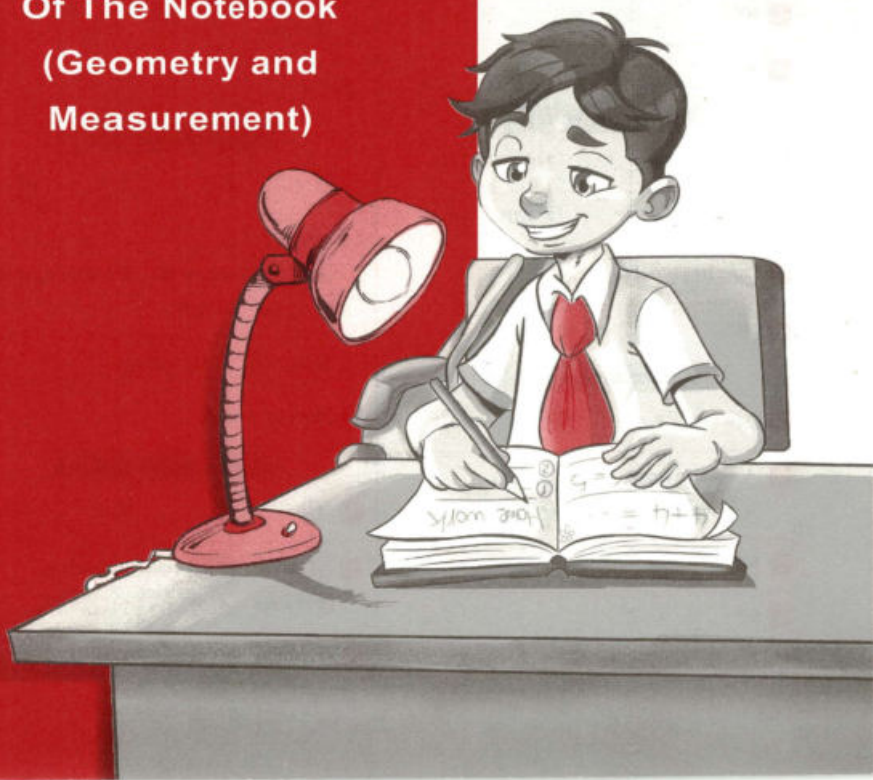
$$\text{[a]} \quad (X \vee Z)^2 = \left(-\frac{3}{2} \times \frac{1}{2} \times \frac{-4}{3} \right)^2 = (1)^2 = 1$$

$$\text{[b]} \quad \text{[1]} \quad \text{The probability of getting an even number} \\ = \frac{3}{6} = \frac{1}{2}$$

$$\text{[2]} \quad \text{The probability of getting an odd number less than 4} = \frac{2}{6} = \frac{1}{3}$$

Guide Answers

Of The Notebook
(Geometry and
Measurement)



Answers of the accumulative tests on Geometry and Measurement

Accumulative test 1

1 [a] 1 d 2 d [b] 4 C°

2 80°

3 Prove by yourself.

4 Prove by yourself.

Accumulative test 2

1 1 c 2 b 3 b 4 d

2 1 9 2 108° 3 8 sides 4 720°

3 60°

4 90°

Accumulative test 3

1 1 a 2 b 3 d 4 d

2 1 160° 2 70° 3 120° 4 16

3 Prove by yourself.

4 140°

Accumulative test 4

1 1 c 2 d 3 c 4 c

2 1 equal in length 2 6
3 a trapezium 4 1 20°

3 50°

4 65°

Accumulative test 5

1 1 c 2 c 3 b 4 c

2 1 180° 2 108° 3 120°

4 a parallelogram

3 $m(\angle BAC) = 90^\circ$, $m(\angle ABD) = 125^\circ$

4 100°

Accumulative test 6

1 1 b 2 b 3 b 4 a

2 1 parallel 2 12
3 60° 4 bisects

3 1 Prove by yourself.

2 2 cm.

4 120°

Accumulative test 7

1 1 a 2 c 3 a 4 a

2 1 $(AC)^2$
2 of the squares on the sides of the right angle
3 12 cm^2 4 120°

3 $BD = 5 \text{ cm}$, $AC = 20 \text{ cm}$.

4 Prove by yourself.

Accumulative test 8

1 1 b 2 a 3 d 4 c

2 1 obtuse 2 are equal in measure

3 8 sides 4 55°

3 1 (0, 3) 2 (7, 2)



4 [a] Prove by yourself.

[b] 20 cm.

Accumulative test 9

1 1 d 2 b 3 c 4 b

2 1 12 2 4 3 (3 - 4) 4 2

3 [a] Draw by yourself.

[b] Draw by yourself.

4 22 cm. → a parallelogram.

Accumulative test 10

1 1 a 2 a 3 b 4 d

2 1 (-3, -2) 2 (-3, 4)

3 equal in length 4 concave

3 Draw by yourself.

4 BD = 12 cm. → BC = 13 cm.

Accumulative test 11

1 1 b 2 d 3 b 4 a

2 1 6 sides 2 (4, -4)

3 (0, -4) 4 7 cm.

3 [a] 1 Δ BXM 2 Δ MYC

[b] Draw by yourself.

4 18 cm.

Accumulative test 12

1 1 c 2 c 3 a 4 b

2 1 110° 2 (-3, -2)

3 (1, -4) 4 ± 360°

3 Draw by yourself.

4 [a] m (∠ A) = 60°

[b] 1 Δ LNB 2 Δ DOM 3 Δ BON

Answers of monthly tests
on Geometry and Measurement

Answers of March tests

Answers of Test 1

1

1 a

2 b

3 c

2

1 360°

2 120°

3 40°

3

$\therefore \overline{XY} \parallel \overline{BC}$, \overline{XB} is a transversal to them

$\therefore m(\angle B) + m(\angle X) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle B) = 180^\circ - 136^\circ = 44^\circ$

$\therefore \overline{DE} \parallel \overline{BC}$, \overline{DC} is a transversal to them

$\therefore m(\angle C) + m(\angle D) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle C) = 180^\circ - 144^\circ = 36^\circ$

\therefore In $\triangle ABC$: $m(\angle BAC) = 180^\circ - (36^\circ + 44^\circ) = 100^\circ$

(The req.)

4

$\therefore B \in \overline{AC}$

$\therefore m(\angle CBE) = 180^\circ - 120^\circ = 60^\circ$

\therefore from the quadrilateral BCDE:

$\therefore m(\angle E) = 360^\circ - (60^\circ + 90^\circ + 140^\circ) = 70^\circ$

(The req.)

Answers of Test 2

1

1 b

2 b

3 a

2

1 360°

2 110°

3 5

3

In $\triangle ABC$: $m(\angle ACB) = 180^\circ - (30^\circ + 50^\circ) = 100^\circ$

$\therefore \overline{AD} \cap \overline{BF} = \{C\}$

$\therefore m(\angle ACB) = m(\angle FCD) = 100^\circ$ (V.O.A.)

\therefore The sum of measures of the interior angles of the quadrilateral CDEF = 360°

$\therefore m(\angle D) = 360^\circ - (130^\circ + 70^\circ + 100^\circ) = 60^\circ$

(The req.)

4

In the figure ABCD:

$\therefore MA = MC$ (given), $MB = MD$ (given)

\therefore Its two diagonals bisect each other

\therefore ABCD is a parallelogram (First req.)

\therefore In $\triangle MBA$:

$m(\angle AMB) = 110^\circ$, $m(\angle MBA) = 25^\circ$

$\therefore m(\angle MAB) = 180^\circ - (110^\circ + 25^\circ) = 45^\circ$

\therefore ABCD is a parallelogram

$\therefore \overline{AB} \parallel \overline{CD}$

$\therefore \overline{CA}$ is a transversal to them

$\therefore m(\angle ACD) = m(\angle CAB) = 45^\circ$ (alternate angles)

(Second req.)

Answers of April tests

Answers of Test 1

1

1 c

2 c

3 c

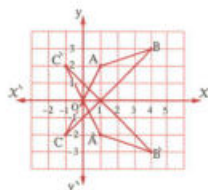
2

1 bisects the third side

2 $(2 + 3)$

3 $(-2 + -1)$

3



4

In $\triangle XYZ$: $\therefore m(\angle XYZ) = 90^\circ$

$\therefore (XZ)^2 = (XY)^2 + (YZ)^2$
 $= (5)^2 + (12)^2 = 169$

$\therefore XZ = \sqrt{169} = 13$ cm. (The req.)



Answers of Test 2

1

1 a

2 a

3 d

2

1 The sum of areas of the two squares drawn on the two sides of the right angle

2 $(2, -9)$

3 half the length of the third side

3

In $\triangle ABC$:

$\therefore X$ is the midpoint of \overline{AB}

$\therefore Y$ is the midpoint of \overline{AC}

$\therefore XY = \frac{1}{2} BC$

$\therefore Y$ is the midpoint of \overline{AC}

$\therefore Z$ is the midpoint of \overline{BC}

$\therefore YZ = \frac{1}{2} AB$

(1)

(2)

$\therefore X$ is the midpoint of \overline{AB}

$\therefore Z$ is the midpoint of \overline{BC}

$\therefore XZ = \frac{1}{2} AC$

(3)

Adding (1), (2), (3):

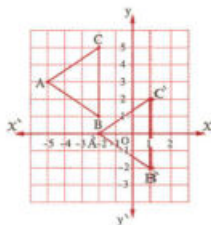
$\therefore XY + YZ + XZ = \frac{1}{2} BC + \frac{1}{2} AB + \frac{1}{2} AC$

$\therefore XY + YZ + XZ = \frac{1}{2} (BC + AB + AC)$

\therefore the perimeter of $\triangle XYZ$

$= \frac{1}{2}$ the perimeter of $\triangle ABC$ (Q.E.D.)

4



Answers of important questions on Geometry and Measurement

First Answers of multiple choice questions

- 1 (d) 2 (c) 3 (d) 4 (b) 5 (a)
 6 (c) 7 (a) 8 (c) 9 (a) 10 (b)
 11 (b) 12 (c) 13 (c) 14 (b) 15 (c)
 16 (b) 17 (c) 18 (b) 19 (b) 20 (a)
 21 (b) 22 (b) 23 (a) 24 (c) 25 (b)
 26 (a) 27 (a) 28 (a) 29 (a) 30 (c)
 31 (c) 32 (a) 33 (c) 34 (b) 35 (c)
 36 (d) 37 (d) 38 (a) 39 (d) 40 (a)
 41 (a) 42 (a) 43 (a) 44 (c) 45 (a)
 46 (d)

Second Answers of complete questions

- 1 360° 2 8 3 60°
 4 right 5 \overline{AC} , \overline{BD} 6 a rhombus
 7 a square 8 a rhombus 9 a trapezium
 10 8 11 6 12 120°
 13 120° 14 70° 15 parallel
 16 half 17 $(AB)^2 + (BC)^2$
 18 $(AC)^2 - (BC)^2$ 19 obtuse-angle-d
 20 100 21 7 cm. 22 90°
 23 $(-3, 4)$ 24 $(2, -1)$ 25 $(3, -5)$
 26 $(-2, 0)$ 27 zero
 28 the axis of symmetry of the figure 29 1
 30 $(2, -3)$ 31 $(3, 1)$
 32 equal in length and parallel 33 $(5, -1)$
 34 $(8, 2)$ 35 $(-4, 1)$ 36 $(5, -3)$
 37 $(-5, 3)$ 38 $(-3, 0)$ 39 $(-2, 1)$
 40 $(-3, 5)$

Third Answers of the essay questions

- 1 $\because \overline{BE}$ bisects $\angle CBD$
 $\therefore m(\angle CBE) = m(\angle EBD) = 65^\circ$
 $\because m(\angle ABC) + m(\angle CBE) + m(\angle EBD)$
 $+ m(\angle ABD) = 360^\circ$
 $\therefore m(\angle ABD) + 90^\circ + 65^\circ + 65^\circ = 360^\circ$
 $\therefore m(\angle ABD) = 360^\circ - 220^\circ = 140^\circ$ (The req.)

- 2 In $\triangle ABC$:
 $m(\angle B) = 180^\circ - (100^\circ + 30^\circ) = 50^\circ$
 $\because \overline{ED} \parallel \overline{BC}$, \overline{BE} is a transversal to them
 $\therefore m(\angle E) = m(\angle B) = 50^\circ$
 (Alternate angles) (The req.)

- 3 $\because \overline{EF} \parallel \overline{CD}$, \overline{EC} is a transversal to them.
 $\therefore m(\angle ECD) = m(\angle CEF) = 95^\circ$
 (Alternate angles)

- $\therefore m(\angle ACD) = 95^\circ - 30^\circ = 65^\circ$
 $\therefore m(\angle ACD) + m(\angle A) = 65^\circ + 115^\circ = 180^\circ$
 and they are interior angles in the same side of the transversal
 $\therefore \overline{AB} \parallel \overline{CD}$, $\because \overline{CD} \parallel \overline{EF}$
 $\therefore \overline{AB} \parallel \overline{EF}$ (The req.)

- 4 $\because \overline{AB} \parallel \overline{CD}$, \overline{AC} is a transversal to them
 $\therefore m(\angle ACD) = m(\angle A) = 50^\circ$
 (Alternate angles)
 $\because m(\angle ACE) = 90^\circ$
 $\therefore m(\angle DCE) = 90^\circ - 50^\circ = 40^\circ$
 $\therefore m(\angle DCE) = m(\angle E)$ and they are alternate angles.
 $\therefore \overline{CD} \parallel \overline{EF}$ (The req.)

- 5 $\because \overline{AE} \parallel \overline{BC}$, \overline{AB} is a transversal to them.
 $\therefore m(\angle A) + m(\angle B) = 180^\circ$
 (Two interior angles in the same side of the transversal)
 $\therefore m(\angle A) = 180^\circ - 70^\circ = 110^\circ$
 \because the sum of the measures of the interior angles of the pentagon $ABCDE = 540^\circ$
 $\therefore m(\angle E) = 540^\circ - (70^\circ + 150^\circ + 80^\circ + 110^\circ)$
 $= 130^\circ$ (The req.)



6 $\therefore E \in \overline{AB}$

$\therefore m(\angle ABC) = 180^\circ - 130^\circ = 50^\circ$

\therefore the sum of the measures of the interior angles of the quadrilateral ABCD = 360°

$\therefore m(\angle C) = 360^\circ - (50^\circ + 80^\circ + 120^\circ) = 110^\circ$
(The req.)

7 \therefore The sum of the measures of the interior angles of the pentagon = 540°

$\therefore x + 2x + 2x + x + 2x = 540^\circ$

$\therefore 8x = 540^\circ$

$\therefore x = \frac{540^\circ}{8} = 67.5^\circ$ (The req.)

8 \therefore The sum of the measures of the interior angles of the pentagon = 540°

$\therefore a + a + a + 90^\circ + 90^\circ = 540^\circ$

$\therefore 3a + 180^\circ = 540^\circ$

$\therefore 3a = 540^\circ - 180^\circ = 360^\circ$

$\therefore a = \frac{360^\circ}{3} = 120^\circ$ (The req.)

9 $\therefore \overline{DE} \parallel \overline{BC}$, \overline{AB} is a transversal to them

$\therefore m(\angle B) = m(\angle BAE) = 120^\circ$
(Alternate angles)

$\therefore m(\angle B) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$

and they are interior angles in the same side of the transversal

$\therefore \overline{AB} \parallel \overline{CD}$ $\therefore \overline{AD} \parallel \overline{BC}$

\therefore ABCD is a parallelogram. (Q.E.D.)

10 \therefore ABCD is a parallelogram

$\therefore m(\angle A) + m(\angle B) = 180^\circ$

$\therefore m(\angle B) = 180^\circ - 50^\circ = 130^\circ$

$\therefore m(\angle C) = m(\angle A) = 50^\circ$ (First req.)

$\therefore AB = CD = 5$ cm, $\therefore AD = BC = 7$ cm.

\therefore The perimeter of the parallelogram
ABCD = $(5 + 7) \times 2 = 24$ cm. (Second req.)

11 \therefore ABCD is a rhombus, \overline{BD} is a diagonal

$\therefore m(\angle ABC) = 2m(\angle ABD) = 2 \times 62^\circ = 124^\circ$

$\therefore m(\angle A) = 180^\circ - 124^\circ = 56^\circ$ (The req.)

12 $\therefore F \in \overline{AB}$

$\therefore m(\angle ABC) = 180^\circ - 60^\circ = 120^\circ$

\therefore ABCD is a parallelogram

$\therefore m(\angle D) = m(\angle ABC) = 120^\circ$

$\therefore AD = BC = 5$ cm. (The req.)

13 In $\triangle ABE$: $m(\angle B) = 180^\circ - (45^\circ + 70^\circ) = 65^\circ$

$\therefore m(\angle D) + m(\angle C) = 65^\circ + 115^\circ = 180^\circ$

and they are interior angles in the same side of the transversal

$\therefore \overline{AD} \parallel \overline{BC}$ (1)

$\therefore m(\angle B) + m(\angle C) = 65^\circ + 115^\circ = 180^\circ$

and they are interior angles in the same side of the transversal

$\therefore \overline{AB} \parallel \overline{CD}$ (2)

\therefore from (1) and (2):

\therefore ABCD is a parallelogram. (Q.E.D.)

14 $\therefore M \in \overline{AC}$

$\therefore m(\angle BMC) = 180^\circ - 70^\circ = 110^\circ$

\therefore In $\triangle MBC$:

$m(\angle BCM) = 180^\circ - (110^\circ + 40^\circ) = 30^\circ$

$\therefore m(\angle BCM) = m(\angle CAD)$

and they are alternate angles

$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore \overline{AB} \parallel \overline{DC}$

\therefore ABCD is a parallelogram. (Q.E.D.)

15 $\therefore \overline{AD} \parallel \overline{BC}$ (Two opposite sides in the square)

$\therefore E \in \overline{BC}$ $\therefore \overline{AD} \parallel \overline{CE}$

$\therefore \overline{AC} \parallel \overline{DE}$ (Given)

\therefore ACED is a parallelogram. (Q.E.D.)

16 In $\triangle FCE$:

$m(\angle FCE) = 180^\circ - (60^\circ + 50^\circ) = 70^\circ$

$\therefore \overline{BE} \cap \overline{DF} = \{C\}$

$\therefore m(\angle DCB) = m(\angle ECF) = 70^\circ$ (V.O.A.)

\therefore the sum of the measures of the interior angles of the quadrilateral ABCD = 360°

$\therefore m(\angle A) = 360^\circ - (80^\circ + 70^\circ + 110^\circ) = 100^\circ$
(The req.)

- 17** In $\triangle ABC$:
 $m(\angle ACB) = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$
 $\therefore \overline{BD} \cap \overline{AF} = \{C\}$
 $\therefore m(\angle FCD) = m(\angle ACB) = 50^\circ$ (V.O.A.)
 \therefore The sum of the measures of the interior angles of the quadrilateral $CFED = 360^\circ$
 $\therefore m(\angle E) = 360^\circ - (50^\circ + 90^\circ + 130^\circ) = 90^\circ$
 (The req.)

- 18** $\therefore \overline{BD} \parallel \overline{CA}$, \overline{AB} is a transversal to them.
 $\therefore m(\angle A) = m(\angle ABD) = 75^\circ$ (Alternate angles)
 \therefore In $\triangle ABC$:
 $m(\angle ABC) = 180^\circ - (75^\circ + 45^\circ) = 60^\circ$
 (The req.)

- 19** \therefore ABCD is a square and \overline{BD} is a diagonal in it.
 $\therefore m(\angle BDC) = 45^\circ$, $m(\angle C) = 90^\circ$
 \therefore From the quadrilateral DEFC
 $m(\angle EFC) = 360^\circ - (45^\circ + 90^\circ + 102^\circ) = 123^\circ$
 $\therefore y = 123^\circ$, $\therefore F \in \overline{BC}$
 $\therefore m(\angle AFB) = 180^\circ - 123^\circ = 57^\circ$
 \therefore In $\triangle ABF$ which is right-angled at B
 $m(\angle BAF) = 180^\circ - (90^\circ + 57^\circ) = 33^\circ$
 $\therefore x = 33^\circ$
 (The req.)

- 20** \therefore ABCDE is a regular pentagon.
 $\therefore m(\angle EAB) = \frac{(5-2) \times 180^\circ}{5} = 108^\circ$
 \therefore ABM is a regular triangle
 $\therefore m(\angle MAB) = \frac{(3-2) \times 180^\circ}{3} = 60^\circ$
 $\therefore m(\angle EAM) = m(\angle EAB) - m(\angle MAB)$
 $= 108^\circ - 60^\circ = 48^\circ$ (The req.)

- 21** $\therefore \overline{ED} \parallel \overline{BC}$, \overline{EC} is a transversal to them.
 $\therefore m(\angle C) + m(\angle E) = 180^\circ$
 (Two interior angles in the same side of the transversal)
 $\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ$
 \therefore In $\triangle ABC$:
 $m(\angle BAC) = 180^\circ - (40^\circ + 60^\circ) = 80^\circ$ (The req.)

- 22** $\therefore \overline{AB} \parallel \overline{MN}$, \overline{LM} is a transversal to them
 $\therefore m(\angle M) = m(\angle ALM) = 70^\circ$
 (Alternate angles)

- $\therefore \overline{AB} \parallel \overline{MN}$, \overline{LN} is a transversal to them
 $\therefore m(\angle N) = m(\angle BLN) = 50^\circ$
 (Alternate angles)

- In $\triangle LMN$:
 $\therefore m(\angle MLN) = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$
 (The req.)

- 23** $\therefore \overline{DE} \parallel \overline{BC}$, \overline{DB} is a transversal to them
 $\therefore m(\angle B) = m(\angle D) = 60^\circ$ (Alternate angles)
 $\therefore \overline{AY} \cap \overline{BX} = \{C\}$
 $\therefore m(\angle ACB) = m(\angle XCY) = 40^\circ$ (V.O.A.)
 \therefore In $\triangle ABC$:
 $m(\angle BAC) = 180^\circ - (40^\circ + 60^\circ) = 80^\circ$ (The req.)

- 24** $\therefore \overline{DF} \parallel \overline{BC}$, \overline{CD} is a transversal to them
 $\therefore m(\angle C) + m(\angle D) = 180^\circ$
 (Two interior angles in the same side of the transversal)
 $\therefore m(\angle C) = 180^\circ - 144^\circ = 36^\circ$
 $\therefore \overline{XY} \parallel \overline{BC}$, \overline{BX} is a transversal to them
 $\therefore m(\angle B) + m(\angle X) = 180^\circ$
 (Two interior angles in the same side of the transversal)
 $\therefore m(\angle B) = 180^\circ - 136^\circ = 44^\circ$
 \therefore In $\triangle ABC$:
 $m(\angle BAC) = 180^\circ - (36^\circ + 44^\circ) = 100^\circ$
 (The req.)

- 25** In $\triangle ABC$:
 $\therefore X$ is the midpoint of \overline{AB} , $\overline{XY} \parallel \overline{BC}$
 $\therefore Y$ is the midpoint of \overline{AC}
 $\therefore XY = \frac{1}{2} BC$
 $\therefore BC = 2 \times 3 = 6$ cm. (First req.)
 $\therefore AY = \frac{1}{2} AC = \frac{1}{2} \times 8 = 4$ cm. (Second req.)

- 26** In $\triangle ABC$:
 $\therefore D$ and F are the midpoints of \overline{AB} and \overline{BC}
 $\therefore \overline{DF} \parallel \overline{AC}$ (First req.)
 $\therefore m(\angle BDF) = m(\angle A) = 48^\circ$
 (corresponding angles) (second req.)



27 In $\triangle ABC$:

$$\therefore D \text{ is the midpoint of } \overline{AB}$$

$$\therefore AD = \frac{1}{2} \times 5 = 2.5 \text{ cm.} \quad (1)$$

$$\therefore E \text{ is the midpoint of } \overline{AC}$$

$$\therefore AE = \frac{1}{2} \times 6 = 3 \text{ cm.} \quad (2)$$

$$\therefore D \text{ is the midpoint of } \overline{AB} \text{ and } E \text{ is the midpoint of } \overline{AC}$$

$$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 7 = 3.5 \text{ cm.} \quad (3)$$

From (1) , (2) and (3) :

$$\therefore \text{The perimeter of } \triangle ADE = 2.5 + 3 + 3.5 = 9 \text{ cm.}$$

(The req.)

28 In $\triangle ABC$:

$$\therefore D \text{ is the midpoint of } \overline{AB} \text{ and } F \text{ is the midpoint of } \overline{AC}$$

$$\therefore DF = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6 \text{ cm.} \quad (1)$$

$$\therefore D \text{ is the midpoint of } \overline{AB} \text{ and } E \text{ is the midpoint of } \overline{BC}$$

$$\therefore DE = \frac{1}{2} AC = \frac{1}{2} \times 8 = 4 \text{ cm.} \quad (2)$$

$$\therefore E \text{ is the midpoint of } \overline{BC} \text{ and } F \text{ is the midpoint of } \overline{AC}$$

$$\therefore EF = \frac{1}{2} AB = \frac{1}{2} \times 10 = 5 \text{ cm.} \quad (3)$$

From (1) , (2) and (3) :

$$\therefore \text{The perimeter of } \triangle DEF = 6 + 4 + 5 = 15 \text{ cm.}$$

(The req.)

29 In $\triangle ABC$:

$$\therefore D \text{ is the midpoint of } \overline{AB} \text{ and } F \text{ is the midpoint of } \overline{AC}$$

$$\therefore BC = 2 DF = 2 \times 5 = 10 \text{ cm.} \quad (1)$$

$$\therefore D \text{ is the midpoint of } \overline{AB} \text{ and } E \text{ is the midpoint of } \overline{BC}$$

$$\therefore AC = 2 DE = 2 \times 7 = 14 \text{ cm.} \quad (2)$$

$$\therefore E \text{ is the midpoint of } \overline{BC} \text{ and } F \text{ is the midpoint of } \overline{AC} \quad (3)$$

$$\therefore AB = 2 EF = 2 \times 6 = 12 \text{ cm.}$$

From (1) , (2) and (3) :

$$\therefore \text{The perimeter of } \triangle ABC = 10 + 14 + 12 = 36 \text{ cm.} \quad (\text{The req.})$$

30 In $\triangle ABC$:

$$\therefore D \text{ is the midpoint of } \overline{AB} \text{ and } E \text{ is the midpoint of } \overline{AC}$$

$$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6 \text{ cm.}$$

$$\therefore \overline{DE} \parallel \overline{BC}$$

In $\triangle FDE$:

$$\therefore X \text{ is the midpoint of } \overline{FD} \text{ and } \overline{XY} \parallel \overline{DE}$$

$$\therefore Y \text{ is the midpoint of } \overline{FE}$$

$$\therefore XY = \frac{1}{2} DE = \frac{1}{2} \times 6 = 3 \text{ cm.} \quad (\text{The req.})$$

31 \therefore ABCD is a parallelogram whose diagonals intersect at M

$$\therefore M \text{ is the midpoint of } \overline{AC}$$

\therefore in $\triangle ABC$:

$$\therefore \overline{MX} \parallel \overline{CB} \text{ and } M \text{ is the midpoint of } \overline{AC}$$

$$\therefore X \text{ is the midpoint of } \overline{AB}$$

$$\therefore AX = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3 \text{ cm.} \quad (\text{The req.})$$

32 In $\triangle ABC$:

$$\therefore X \text{ is the midpoint of } \overline{AB} \text{ and } \overline{XY} \parallel \overline{BC}$$

$$\therefore Y \text{ is the midpoint of } \overline{AC}$$

In $\triangle ACD$:

$$\therefore Z \text{ is the midpoint of } \overline{AD} \text{ and } Y \text{ is the midpoint of } \overline{AC}$$

$$\therefore YZ = \frac{1}{2} CD = \frac{1}{2} \times 10 = 5 \text{ cm.} \quad (\text{The req.})$$

33 In $\triangle ABC$:

$$\therefore D \text{ is the midpoint of } \overline{AB} \text{ and } F \text{ is the midpoint of } \overline{AC}$$

$$\therefore DF = \frac{1}{2} BC = EC = 5 \text{ cm.}$$

$$\therefore D \text{ is the midpoint of } \overline{AB} \text{ and } E \text{ is the midpoint of } \overline{BC}$$

$$\therefore DE = \frac{1}{2} AC = FC = 4 \text{ cm.}$$

$$\therefore \text{The perimeter of the figure DECF} = 5 + 4 + 5 + 4 = 18 \text{ cm.} \quad (\text{The req.})$$

34 In $\triangle DBC$:

$$\therefore E \text{ is the midpoint of } \overline{BD} \text{ and } F \text{ is the midpoint of } \overline{DC}$$

$$\therefore \overline{EF} \parallel \overline{BC}$$

$$\therefore \overline{AD} \parallel \overline{BC} \quad \therefore \overline{EF} \parallel \overline{AD} \quad (1)$$

$$\therefore EF = \frac{1}{2} BC$$

$$\therefore \because 2 AD = BC \quad \therefore AD = \frac{1}{2} BC$$

$$\therefore EF = AD \quad (2)$$

From (1) and (2) :

$\therefore AEFD$ is a parallelogram. (Q.E.D.)

35 In $\triangle ABC$:

$\therefore D$ is the midpoint of \overline{AB} and E is the midpoint of \overline{AC}

$$\therefore \overline{DE} \parallel \overline{BC}$$

$$\therefore \because F \in \overline{BC} \quad \therefore \overline{DE} \parallel \overline{CF} \quad (1)$$

$$\therefore DE = \frac{1}{2} BC$$

$$\therefore \because CF = \frac{1}{2} BC \quad \therefore DE = CF \quad (2)$$

From (1) and (2) :

$\therefore DCFE$ is a parallelogram. (Q.E.D.)

36 $\therefore ABCD$ is a rectangle

$$\therefore m(\angle B) = 90^\circ$$

$$\therefore \text{In } \triangle ABC : (BC)^2 = (AC)^2 - (AB)^2 \\ = 100 - 36 = 64$$

$$\therefore BC = \sqrt{64} = 8 \text{ cm.} \quad (\text{First req.})$$

The area of the rectangle $ABCD$

$$= AB \times BC = 6 \times 8 = 48 \text{ cm}^2 \quad (\text{Second req.})$$

37 $\therefore ABC$ is a right-angled triangle at B

$$\therefore \text{its area} = 24 \text{ cm}^2$$

$$\therefore \frac{1}{2} \times AB \times BC = 24$$

$$\therefore \frac{1}{2} \times 6 \times BC = 24$$

$$\therefore BC = 8 \text{ cm.}$$

$$\therefore \because (AC)^2 = (AB)^2 + (BC)^2 = 36 + 64 = 100$$

$$\therefore AC = \sqrt{100} = 10 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle ABC = 6 + 8 + 10 = 24 \text{ cm.}$$

(The req.)

38 In $\triangle ABC$:

$$\therefore m(\angle CAB) = 90^\circ$$

$$\therefore (AB)^2 = (BC)^2 - (AC)^2 = 625 - 225 = 400$$

$$\therefore AB = \sqrt{400} = 20 \text{ cm.} \quad (\text{First req.})$$

$$\text{In } \triangle CDA : \therefore m(\angle CDA) = 90^\circ$$

$$\therefore (AD)^2 = (AC)^2 - (DC)^2 = 225 - 81 = 144$$

$$\therefore AD = \sqrt{144} = 12 \text{ cm.} \quad (\text{Second req.})$$

39 In $\triangle ADC$:

$$\therefore m(\angle D) = 90^\circ$$

$$\therefore (AD)^2 = (AC)^2 - (CD)^2 = 169 - 25 = 144$$

$$\therefore AD = \sqrt{144} = 12 \text{ cm.} \quad (\text{First req.})$$

\therefore in $\triangle ADB$:

$$\therefore m(\angle D) = 90^\circ$$

$$\therefore (AB)^2 = (AD)^2 + (BD)^2 = 144 + 81 = 225$$

$$\therefore AB = \sqrt{225} = 15 \text{ cm.} \quad (\text{Second req.})$$

40 In $\triangle XYZ$:

$$\therefore m(\angle Y) = 90^\circ$$

$$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = 49 + 576 = 625$$

$$\therefore XZ = \sqrt{625} = 25 \text{ cm.} \quad (\text{First req.})$$

\therefore in $\triangle XLZ$:

$$\therefore m(\angle L) = 90^\circ$$

$$\therefore (LZ)^2 = (XZ)^2 - (XL)^2 = 625 - 225 = 400$$

$$\therefore LZ = \sqrt{400} = 20 \text{ cm.} \quad (\text{Second req.})$$

41 In $\triangle ADB$: $\therefore m(\angle ADB) = 90^\circ$

$$\therefore (AD)^2 = (AB)^2 - (BD)^2 = 225 - 81 = 144$$

$$\therefore AD = \sqrt{144} = 12 \text{ cm.} \quad (\text{First req.})$$

\therefore in $\triangle ADC$: $\therefore m(\angle ADC) = 90^\circ$

$$\therefore (DC)^2 = (AC)^2 - (AD)^2 = 169 - 144 = 25$$

$$\therefore DC = \sqrt{25} = 5 \text{ cm.} \quad (\text{Second req.})$$

$$\therefore \text{The area of } \triangle ABC = \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times 14 \times 12 = 84 \text{ cm}^2 \quad (\text{Third req.})$$

42 $\therefore D$ is the image of B by reflection in E

$\therefore C$ is the image of A by reflection in E

$\therefore DC$ is the image of BA by reflection in E

$\therefore \therefore$ the reflection in a point reserves the lengths

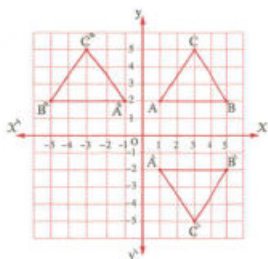
$$\therefore AB = CD \quad \therefore x + 5 = 2x - 3$$

$$\therefore 5 + 3 = 2x - x \quad \therefore x = 8$$

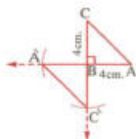
$$\therefore CD = 2 \times 8 - 3 = 13 \text{ cm.} \quad (\text{The req.})$$



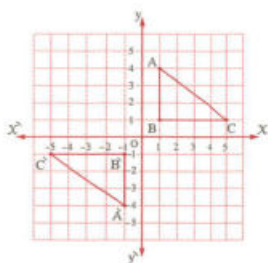
43



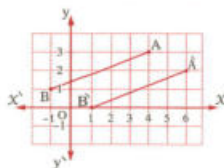
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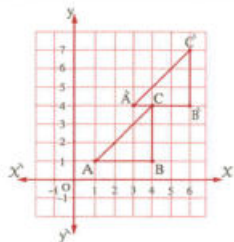
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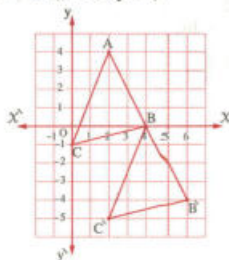
46



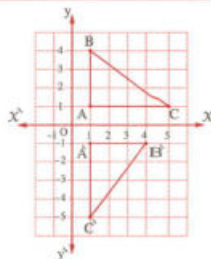
47



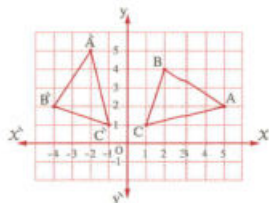
48 $(x, y) \rightarrow (x + 2, y - 4)$



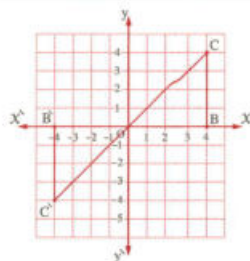
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50



51



52

- 1 $\triangle DBE$
- 2 $\triangle EDF$
- 3 $\triangle FDE$

Answers of school book models on Geometry and Measurement

Model 1

- 1 (c) 2 (a) 3 (d) 4 (b) 5 (b) 6 (b)

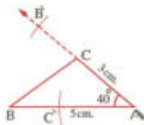
2

- 1 (2 + 1) 2 80° 3 4
4 160° 5 180°

3

- [a] $\therefore \angle ACD$ is an exterior angle of $\triangle ABC$
 $\therefore m(\angle ACD) = m(\angle A) + m(\angle B) = 25^\circ + 25^\circ = 50^\circ$
(The req.)

[b]



4

- [a] $\therefore M \in \overline{AC}$
 $\therefore m(\angle BMC) = 180^\circ - 70^\circ = 110^\circ$
 \therefore In $\triangle BMC$:
 $m(\angle MCB) = 180^\circ - (110^\circ + 40^\circ) = 30^\circ$
 $\therefore m(\angle MCB) = m(\angle MAD)$
and they are alternate angles
 $\therefore \overline{AD} \parallel \overline{BC}$, $\therefore \overline{AB} \parallel \overline{DC}$
 $\therefore ABCD$ is a parallelogram (Q.E.D.)

- [b] The point is (0, 0)

5

- [a] In $\triangle ABD$: $\therefore m(\angle ADB) = 90^\circ$
 $\therefore (BD)^2 = (AB)^2 - (AD)^2 = 676 - 576 = 100$
 $\therefore BD = \sqrt{100} = 10$ cm.
In $\triangle ADC$: $\therefore m(\angle ADC) = 90^\circ$
 $\therefore (CD)^2 = (AC)^2 - (AD)^2 = 900 - 576 = 324$
 $\therefore CD = \sqrt{324} = 18$ cm.

- $\therefore BC = 10 + 18 = 28$ cm. (First req.)
 \therefore The area of $\triangle ABC = \frac{1}{2} BC \times AD$
 $= \frac{1}{2} \times 28 \times 24 = 336$ cm²
(Second req.)

- [b] $\therefore ABCD$ is a square

- $\therefore \overline{AD} \parallel \overline{BC}$, $\therefore E \in \overline{BC}$
 $\therefore \overline{AD} \parallel \overline{EC}$, $\therefore \overline{AC} \parallel \overline{DE}$
 $\therefore ACED$ is a parallelogram (Q.E.D.)

Model 2

1

- 1 (a) 2 (c) 3 (b) 4 (c) 5 (c) 6 (d)

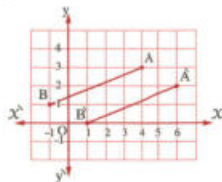
2

- 1 44 2 (5, 5) 3 1728000
4 bisects the third side. 5 ZYC

3

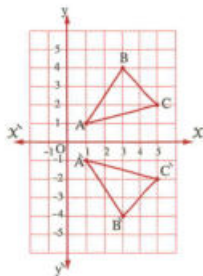
- [a] In $\triangle XYZ$: $\therefore m(\angle Y) = 90^\circ$
 $\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = 49 + 576 = 625$
 $\therefore XZ = \sqrt{625} = 25$ cm. (First req.)
In $\triangle LXZ$: $\therefore m(\angle L) = 90^\circ$
 $\therefore (LZ)^2 = (XZ)^2 - (LX)^2 = 625 - 225 = 400$
 $\therefore LZ = \sqrt{400} = 20$ cm. (Second req.)

[b]



4

[a]





[b] In $\triangle ABC$: $m(\angle ACB) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$

$\therefore BD \cap AO = \{C\}$

$\therefore m(\angle ACB) = m(\angle OCD) = 60^\circ$ (V.O.A)

$\therefore m(\angle E) = 360^\circ - (60^\circ + 120^\circ + 90^\circ) = 90^\circ$

(The req.)

5

[a] $\therefore \overleftrightarrow{EO} \parallel \overleftrightarrow{CD}$, \overleftrightarrow{EB} is a transversal

$\therefore m(\angle CBA) = m(\angle E) = 50^\circ$ (alternate angles)

From $\triangle ABC$:

$m(\angle BAC) = 180^\circ - (50^\circ + 30^\circ) = 100^\circ$

$\therefore \angle ABD$ is an exterior angle of $\triangle ABC$

$\therefore m(\angle ABD) = 30^\circ + 100^\circ = 130^\circ$ (The req.)

[b] $\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$, $\therefore AX = XB$

$\therefore DY = YC$

$\therefore Y$ is the midpoint of \overleftrightarrow{CD}

In $\triangle CDE$: $\therefore \overleftrightarrow{ZY} \parallel \overleftrightarrow{DE}$

$\therefore Y$ is the midpoint of \overleftrightarrow{CD}

$\therefore Z$ is the midpoint of \overleftrightarrow{CE}

$\therefore CZ = ZE$

(Q.E.D.)

Model examination for the merge students

1 1 c 2 b 3 b

4 a 5 b

2 1 half 2 right 3 6

4 $(3 + -2)$ 5 70°

3 1 ✗ 2 ✗ 3 ✓

4 ✗ 5 ✓

4 1 360° 2 120° 3 $(4 + 0)$

4 $(-1 + -3)$ 5 45°

5 Fig (1) : $X = 8$

Fig (2) : $X = 90^\circ$

**Answers of the schools examinations
on Geometry and Measurement**

1 Cairo

1

- 1 d 2 c 3 d 4 c 5 b 6 c

2

- 1 is parallel 2 120° 3 $(2, -1)$
4 $\pm 360^\circ$ 5 90° 6 10 cm.

3

- [a] $\because \overline{DF} \parallel \overline{BC}$, \overline{CD} is a transversal to them.
 $\therefore m(\angle C) = m(\angle D) = 80^\circ$ (Alternate angles)
 \therefore In $\triangle ABC$:
 $m(\angle BAC) = 180^\circ - (40^\circ + 80^\circ) = 60^\circ$ (The req.)
 [b] $\because C \in \overline{BE}$ $\therefore m(\angle BCD) = 180^\circ - 110^\circ = 70^\circ$
 From the quadrilateral ABCD
 $\therefore m(\angle A) = 360^\circ - (70^\circ + 80^\circ + 90^\circ) = 120^\circ$
 (The req.)

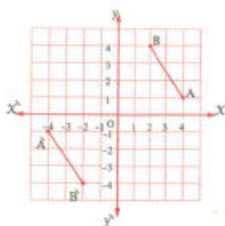
4

- [a] In $\triangle XYZ$:
 $\because D$ is the midpoint of \overline{XY}
 $\therefore F$ is the midpoint of \overline{XZ}
 $\therefore DF = \frac{1}{2} YZ = 6$ cm.
 $\because D$ is the midpoint of \overline{XY}
 $\therefore E$ is the midpoint of \overline{YZ}
 $\therefore DE = \frac{1}{2} XZ = 4$ cm.
 $\because F$ is the midpoint of \overline{XZ}
 $\therefore E$ is the midpoint of \overline{YZ}
 $\therefore FE = \frac{1}{2} XY = 3$ cm.
 \therefore The perimeter of $\triangle DEF$
 $= 6 + 4 + 3 = 13$ cm. (The req.)
 [b] In $\triangle ABC$: $\because m(\angle B) = 90^\circ$
 $\therefore (BC)^2 = (AC)^2 - (AB)^2 = (15)^2 - (9)^2 = 144$
 $\therefore BC = \sqrt{144} = 12$ cm. (The req.)

5

- [a] Write by yourself

[b]



2 Cairo

1

- 1 b 2 c 3 b 4 a 5 c 6 d

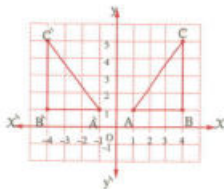
2

- 1 $(5, 3)$ 2 half 3 90°
4 360° 5 540° 6 120°

3

- [a] In $\triangle XYZ$: $\because m(\angle Y) = 90^\circ$
 $\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = (7)^2 + (24)^2 = 625$
 $\therefore XZ = \sqrt{625} = 25$ cm. (First req.)
 \therefore In $\triangle XLZ$: $\because m(\angle L) = 90^\circ$
 $\therefore (LZ)^2 = (XZ)^2 - (XL)^2 = (25)^2 - (15)^2 = 400$
 $\therefore LZ = \sqrt{400} = 20$ cm. (Second req.)

[b]



4

- [a] In the quadrilateral ABCD:
 $\because m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360^\circ$
 $\therefore 90^\circ + 2x + 5x + 3x = 360^\circ$
 $\therefore 90^\circ + 10x = 360^\circ$
 $\therefore 10x = 360^\circ - 90^\circ = 270^\circ$
 $\therefore x = \frac{270^\circ}{10} = 27^\circ$ (The req.)



[b] $\because \overline{AD} \parallel \overline{BC}$, \overline{AB} is a transversal to them.

$$\therefore m(\angle B) = m(\angle DAB) = 65^\circ \text{ (alternate angles)}$$

$$\therefore \overline{AF} \cap \overline{BE} = \{C\}$$

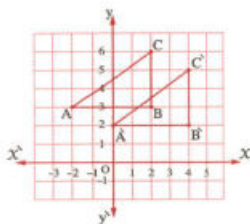
$$\therefore m(\angle ACB) = m(\angle ECF) = 55^\circ \text{ (V.O.A)}$$

\therefore In $\triangle ABC$:

$$m(\angle BAC) = 180^\circ - (65^\circ + 55^\circ) = 60^\circ \text{ (The req.)}$$

5

[a]



[b] In $\triangle ABC$:

$\therefore X$ is the midpoint of \overline{AB}

$\therefore Y$ is the midpoint of \overline{AC}

$$\therefore XY = \frac{1}{2} BC = 5 \text{ cm.}$$

$$\therefore XB = \frac{1}{2} AB = 3 \text{ cm.}$$

$$\therefore YC = \frac{1}{2} AC = 3.5 \text{ cm.}$$

\therefore The perimeter of the figure XBCY

$$= 5 + 3 + 10 + 3.5 = 21.5 \text{ cm. (The req.)}$$

3

Giza

1

[1] c [2] c [3] a [4] d [5] c [6] c

2

[1] 90° [2] the square [3] $(4 + 1)$
[4] 360° [5] parallel [6] 360°

3

[a] $\because \overline{AB} \cap \overline{CD} = \{M\}$

$$\therefore m(\angle AMD) = m(\angle CMB) = 50^\circ \text{ (V.O.A)}$$

$\therefore \overline{MD}$ bisects $\angle AME$

$$\therefore m(\angle DME) = m(\angle AMD) = 50^\circ$$

$$\therefore m(\angle EMB) = 180^\circ - (50^\circ + 50^\circ) = 80^\circ \text{ (The req.)}$$

[b] $\because \overline{DE} \parallel \overline{BC}$, \overline{BD} is a transversal to them

$$\therefore m(\angle B) = m(\angle D) = 50^\circ \text{ (alternate angles)}$$

(First req.)

\therefore In $\triangle ABC$:

$$m(\angle BAC) = 180^\circ - (50^\circ + 35^\circ) = 95^\circ$$

(Second req.)

4

[a] In $\triangle ABC$:

$\therefore E$ is the midpoint of \overline{AB}

$\therefore F$ is the midpoint of \overline{AC}

$$\therefore EF = \frac{1}{2} BC = 5 \text{ cm.}$$

$$\therefore AF = \frac{1}{2} AC = 8 \text{ cm.}$$

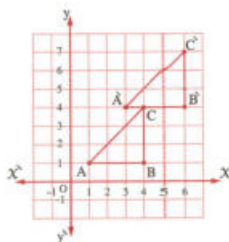
$$\therefore AE = \frac{1}{2} AB = 6 \text{ cm.}$$

\therefore The perimeter of the triangle AEF

$$= 5 + 8 + 6 = 19 \text{ cm.}$$

(The req.)

[b]



5

[a] $\because ABCD$ is a parallelogram

$$\therefore m(\angle B) + m(\angle C) = 180^\circ$$

$$\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ \text{ (First req.)}$$

$$\therefore AB = CD = 5 \text{ cm.}$$

$$\therefore AD = BC = 8 \text{ cm.}$$

\therefore The perimeter of the parallelogram ABCD

$$= (5 + 8) \times 2 = 26 \text{ cm. (Second req.)}$$

[b] $\because \overline{AB} \parallel \overline{CD}$, \overline{AC} is a transversal to them

$$\therefore m(\angle ACD) = m(\angle A) = 50^\circ \text{ (alternate angles)}$$

$$\therefore \overline{AC} \perp \overline{CH} \therefore m(\angle ACH) = 90^\circ$$

$$\therefore m(\angle DCH) = 90^\circ - 50^\circ = 40^\circ$$

$$\therefore m(\angle H) = m(\angle DCH) = 40^\circ$$

and they are alternate angles

$$\therefore \overline{CD} \parallel \overline{HO} \therefore \overline{AB} \parallel \overline{CD}$$

$$\therefore \overline{AB} \parallel \overline{HO} \text{ (Q.E.D.)}$$

4

Giza

1

1 d

2 a

3 c

4 d

5 d

6 d

2

1 parallel

2 180°

3 (1, 3)

4 360°

5 70°

6 bisects the third side

3

[a] In $\triangle ABC$:

$\therefore X$ is the midpoint of \overline{AB}

$\therefore Z$ is the midpoint of \overline{AC}

$\therefore XZ = \frac{1}{2} BC = 4$ cm.

$\therefore X$ is the midpoint of \overline{AB}

$\therefore Y$ is the midpoint of \overline{BC}

$\therefore XY = \frac{1}{2} AC = 5$ cm.

$\therefore Z$ is the midpoint of \overline{AC}

$\therefore Y$ is the midpoint of \overline{BC}

$\therefore ZY = \frac{1}{2} AB = 3$ cm.

\therefore The perimeter of $\triangle XYZ = 4 + 5 + 3 = 12$ cm.

(The req.)

[b] In $\triangle XYZ$: $\therefore m(\angle Y) = 90^\circ$

$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = (7)^2 + (24)^2 = 625$

$\therefore XZ = \sqrt{625} = 25$ cm. (First req.)

In $\triangle XZL$: $\therefore m(\angle L) = 90^\circ$

$\therefore (LZ)^2 = (XZ)^2 - (XL)^2 = (25)^2 - (15)^2 = 400$

$\therefore LZ = \sqrt{400} = 20$ cm. (Second req.)

4

[a] $\therefore \overline{BA} \parallel \overline{CD}$, \overline{BE} is a transversal to them

$\therefore m(\angle BEC) = m(\angle ABE) = 80^\circ$

(alternate angles)

\therefore In $\triangle BCE$:

$m(\angle ECB) = 180^\circ - (40^\circ + 80^\circ) = 60^\circ$ (The req.)

[b] $\therefore ABCD$ is a parallelogram

$\therefore m(\angle B) + m(\angle A) = 180^\circ$

$\therefore m(\angle B) = 180^\circ - 75^\circ = 105^\circ$ (First req.)

$\therefore m(\angle C) = m(\angle A) = 75^\circ$ (Second req.)

5

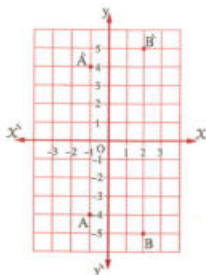
[a] $\therefore ABCD$ is a quadrilateral

$\therefore m(\angle A) = m(\angle C)$

$\therefore 2m(\angle C) = 360^\circ - (120^\circ + 60^\circ) = 180^\circ$

$\therefore m(\angle C) = \frac{180^\circ}{2} = 90^\circ$ (The req.)

[b]



5

Alexandria

1

1 a

2 a

3 c

4 a

5 b

6 a

2

1 48

2 540°

3 a rectangle

4 150°

5 $(-1, -4)$

6 half

3

[a] $\therefore \overline{DF} \parallel \overline{CB}$, \overline{BD} is a transversal to them

$\therefore m(\angle B) = m(\angle D) = 50^\circ$

(alternate angles) (First req.)

$\therefore \angle DAC$ is an exterior angle of $\triangle ABC$

$\therefore m(\angle DAC) = 35^\circ + 50^\circ = 85^\circ$ (Second req.)

[b] $\therefore ABCD$ is a rhombus, \overline{BD} is a diagonal

$\therefore m(\angle ABC) = 2m(\angle ABD) = 2 \times 65^\circ = 130^\circ$

$\therefore m(\angle C) = 180^\circ - 130^\circ = 50^\circ$ (First req.)

\therefore the perimeter of the rhombus $ABCD$

$= 6 \times 4 = 24$ cm. (Second req.)

4

[a] $\therefore ABCD$ is a parallelogram

$\therefore m(\angle B) + m(\angle C) = 180^\circ$



$$\therefore m(\angle C) = 180^\circ - 125^\circ = 55^\circ \quad (\text{First req.})$$

$$\therefore AB = CD = 5 \text{ cm.}$$

$$\therefore AD = BC = 8 \text{ cm.}$$

$$\therefore \text{The perimeter of the parallelogram ABCD} \\ = (5 + 8) \times 2 = 26 \text{ cm.} \quad (\text{Second req.})$$

[b] In $\triangle XYZ$:

$$\therefore D \text{ is the midpoint of } \overline{XY}$$

$$\therefore E \text{ is the midpoint of } \overline{XZ}$$

$$\therefore DE = \frac{1}{2} YZ = 5 \text{ cm.}$$

$$\therefore D \text{ is the midpoint of } \overline{XY}$$

$$\therefore O \text{ is the midpoint of } \overline{YZ}$$

$$\therefore DO = \frac{1}{2} XZ = 3 \text{ cm.}$$

$$\therefore E \text{ is the midpoint of } \overline{XZ}$$

$$\therefore O \text{ is the midpoint of } \overline{YZ}$$

$$\therefore EO = \frac{1}{2} XY = 4 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle DOE \\ = 5 + 3 + 4 = 12 \text{ cm.} \quad (\text{The req.})$$

5

[a] In $\triangle ABC$:

$$m(\angle ACB) = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$$

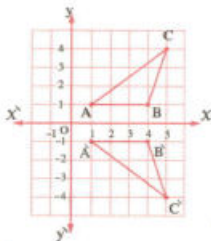
$$\therefore \overline{BD} \cap \overline{AF} = \{C\}$$

$$\therefore m(\angle DCF) = m(\angle ACB) = 60^\circ \text{ (V.O.A)}$$

From the quadrilateral CDEF:

$$\therefore m(\angle E) = 360^\circ - (90^\circ + 120^\circ + 60^\circ) = 90^\circ \\ (\text{The req.})$$

[b]



6

El-Kalyoubia

1

- [1] d [2] c [3] b [4] b [5] d [6] c

2

- [1] $(EF)^2$ [2] $(2 + 3)$ [3] 90°
[4] 12 [5] 45° [6] 120°

3

[a] $\therefore ABCD$ is a rhombus, \overline{BD} is a diagonal

$$\therefore m(\angle ABC) = 2m(\angle ABD) = 2 \times 65^\circ = 130^\circ$$

$$\therefore m(\angle A) = 180^\circ - 130^\circ = 50^\circ \quad (\text{The req.})$$

[b] $\therefore \overline{AB} \parallel \overline{OE}$, \overline{BO} is a trans versal to them

$$\therefore m(\angle O) = m(\angle B) \text{ (alternate angles)}$$

$$\therefore \overline{AD} \cap \overline{BO} = \{C\}$$

$$\therefore m(\angle DCO) = m(\angle ACB) \text{ (V.O.A)}$$

$$\therefore \text{in } \triangle ABC:$$

$$m(\angle B) + m(\angle ACB) = 180^\circ - 30^\circ = 150^\circ$$

$$\therefore m(\angle O) + m(\angle DCO) = 150^\circ$$

From the quadrilateral CDEO:

$$\therefore m(\angle D) = 360^\circ - (150^\circ + 80^\circ) = 130^\circ \\ (\text{The req.})$$

4

[a] In $\triangle BCD$: $\therefore m(\angle C) = 90^\circ$

$$\therefore (BC)^2 = (BD)^2 - (CD)^2 = (13)^2 - (12)^2 = 25$$

$$\therefore BC = \sqrt{25} = 5 \text{ cm.} \quad (\text{First req.})$$

In $\triangle ACD$: $\therefore m(\angle C) = 90^\circ$

$$\therefore (AD)^2 = (AC)^2 + (CD)^2 = (16)^2 + (12)^2 = 400$$

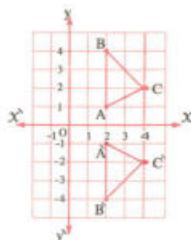
$$\therefore AD = \sqrt{400} = 20 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle ABD \\ = 11 + 13 + 20 = 44 \text{ cm.} \quad (\text{Second req.})$$

[b] [1] $\triangle DZM$ [2] $\triangle DLM$

5

[a]



[b] In $\triangle ABC$:

$$\therefore X \text{ is the midpoint of } \overline{AB}, \overline{XY} \parallel \overline{BC}$$

$$\therefore Y \text{ is the midpoint of } \overline{AC}$$

In $\triangle ACD$:

$$\therefore Y \text{ is the midpoint of } \overline{AC}$$

$$\therefore Z \text{ is the midpoint of } \overline{AD}$$

$$\therefore YZ = \frac{1}{2} CD = 3 \text{ cm.} \quad (\text{The req.})$$

7 El-Monofia

1

- 1 d 2 b 3 c 4 a 5 d 6 b

2

- 1 (2 + -1) 2 50° 3 160°
4 half 5 5 6 ± 360°

3

[a] ∵ D ∈ BC

∴ ∠ACD is the exterior angle of ΔABC

$$\begin{aligned}\therefore m(\angle ACD) &= m(\angle A) + m(\angle B) \\ &= 25^\circ + 25^\circ = 50^\circ \quad (\text{The req.})\end{aligned}$$

[b] In ΔABD: ∵ m(∠ADB) = 90°

$$\begin{aligned}\therefore (BD)^2 &= (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100 \\ \therefore BD &= \sqrt{100} = 10 \text{ cm.} \quad (1)\end{aligned}$$

∴ In ΔADC: ∵ m(∠ADC) = 90°

$$\begin{aligned}\therefore (CD)^2 &= (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324 \\ \therefore CD &= \sqrt{324} = 18 \text{ cm.} \quad (2)\end{aligned}$$

From (1) & (2):

$$\therefore BC = BD + CD = 10 + 18 = 28 \text{ cm.} \quad (\text{The req.})$$

4

[a] In ΔABC:

$$m(\angle ACB) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

$$\therefore \overline{BD} \cap \overline{AO} = \{C\}$$

$$\therefore m(\angle DCO) = m(\angle ACB) = 60^\circ \quad (\text{V.O.A})$$

From the quadrilateral CDEO

$$\begin{aligned}\therefore m(\angle E) &= 360^\circ - (90^\circ + 60^\circ + 120^\circ) = 90^\circ \\ &\quad (\text{The req.})\end{aligned}$$

[b] ∵ $\overline{ED} \parallel \overline{CB}$, \overline{AB} is a transversal to them.

$$\begin{aligned}\therefore m(\angle B) &= m(\angle ADE) = 70^\circ \\ &\quad (\text{Corresponding angles})\end{aligned}$$

∴ In ΔABC:

$$m(\angle A) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ \quad (\text{The req.})$$

5

[a] ∵ M ∈ AC

$$\therefore m(\angle BMC) = 180^\circ - 70^\circ = 110^\circ$$

∴ In ΔBMC:

$$m(\angle MCB) = 180^\circ - (110^\circ + 40^\circ) = 30^\circ$$

$$\therefore m(\angle MCB) = m(\angle MAD) = 30^\circ$$

and they are alternate angles

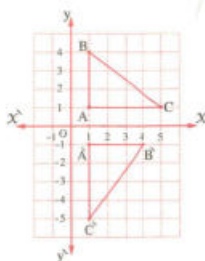
$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore \overline{AB} \parallel \overline{DC}$$

∴ ABCD is a parallelogram.

(Q.E.D.)

[b]



8 El-Dakhlia

1

- 1 a 2 b 3 b 4 a 5 c 6 b

2

- 1 (2 + -7) 2 parallel to
3 a rectangle 4 two
5 acute 6 900°

3

[a] In ΔABC:

∵ D is the midpoint of AB

∴ E is the midpoint of AC

$$\therefore DE = \frac{1}{2} BC = 4.5 \text{ cm.}$$

∴ E is the midpoint of AC

∴ F is the midpoint of BC

$$\therefore EF = \frac{1}{2} AB = 3.5 \text{ cm.}$$

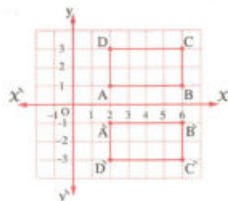
∴ F is the midpoint of BC

∴ D is the midpoint of AB

$$\therefore FD = \frac{1}{2} AC = 4 \text{ cm.}$$

$$\begin{aligned}\therefore \text{The perimeter of } \Delta DEF &= 4.5 + 3.5 + 4 \\ &= 12 \text{ cm.} \quad (\text{The req.})\end{aligned}$$

[b]





4

[a] In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (9)^2 + (12)^2 = 225$$

$$\therefore AC = \sqrt{225} = 15 \text{ cm.} \quad (\text{First req.})$$

In $\triangle ACD$: $\therefore m(\angle ACD) = 90^\circ$

$$\therefore (AD)^2 = (AC)^2 + (CD)^2 = (15)^2 + (20)^2 = 625$$

$$\therefore AD = \sqrt{625} = 25 \text{ cm.} \quad (\text{Second req.})$$

[b] $\therefore \triangle ABC$ is an equilateral triangle

$$\therefore m(\angle ACB) = m(\angle A) = m(\angle B) = \frac{180^\circ}{3} = 60^\circ$$

$$\therefore \overline{BD} \cap \overline{AE} = \{C\}$$

$$\therefore m(\angle DCE) = m(\angle ACB) = 60^\circ (\text{V.O.A.})$$

From the quadrilateral DCEX

$$\therefore m(\angle X) = 360^\circ - (130^\circ + 60^\circ + 90^\circ) = 80^\circ \quad (\text{The req.})$$

5

[a] $\therefore \overline{DE} \parallel \overline{BC}$, \overline{CD} is a transversal to them

$$\therefore m(\angle C) + m(\angle D) = 180^\circ$$

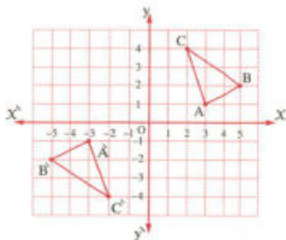
(interior angles on the same side of the transversal)

$$\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ$$

 $\therefore \angle BAD$ is an exterior angle of $\triangle ABC$

$$\therefore m(\angle BAD) = m(\angle B) + m(\angle C) = 40^\circ + 60^\circ = 100^\circ \quad (\text{The req.})$$

[b]



9

Port Said

1

- 1 b 2 b 3 d 4 a 5 c 6 c
 7 d 8 b 9 b 10 b 11 c 12 c
 13 a 14 d 15 b 16 d 17 a 18 a
 19 b 20 c 21 a

2

In $\triangle DHO$: $\therefore X$ is the midpoint of \overline{DH} $\therefore Y$ is the midpoint of \overline{DO}

$$\therefore XY = \frac{1}{2} HO = \frac{1}{2} \times 10 = 5 \text{ cm.} \quad (\text{The req.})$$

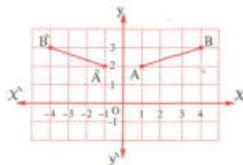
3

 $\therefore \triangle XYZ$ is right-angled at X

$$\therefore (YZ)^2 = (XY)^2 + (XZ)^2 = (12)^2 + (5)^2 = 169$$

$$\therefore YZ = \sqrt{169} = 13 \text{ cm.} \quad (\text{The req.})$$

4



10

Kafr El-Sheikh

1

- 1 c 2 b 3 a 4 c 5 b 6 d

2

- 1 $(-5, 3)$ 2 180° 3 5
 4 The rhombus 5 $(5, 0)$ 6 bisects

3

[a] In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (6)^2 + (8)^2 = 100$$

$$\therefore AC = \sqrt{100} = 10 \text{ cm.} \quad (\text{The req.})$$

[b] $\therefore \overline{CB} \parallel \overline{DE}$, \overline{BD} is a transversal to them

$$\therefore m(\angle B) = m(\angle D) = 40^\circ \quad (\text{alternate angles}) \quad (\text{First req.})$$

 \therefore In $\triangle ABC$:

$$m(\angle CAB) = 180^\circ - (40^\circ + 50^\circ) = 90^\circ \quad (\text{Second req.})$$

4

[a] $\therefore \triangle DFE$ is an equilateral triangle

$$\therefore m(\angle EDF) = m(\angle F) = m(\angle E) = \frac{180^\circ}{3} = 60^\circ$$

$$\therefore \overline{AE} \cap \overline{FC} = \{D\}$$

$$\therefore m(\angle ADC) = m(\angle EDF) = 60^\circ (\text{V.O.A.})$$

From the quadrilateral ABCD

$$\therefore m(\angle B) = 360^\circ - (60^\circ + 105^\circ + 120^\circ) = 75^\circ \quad (\text{The req.})$$

[b] In $\triangle ABC$:

$\therefore X$ is the midpoint of \overline{AB}

$\therefore Y$ is the midpoint of \overline{AC}

$$\therefore XY = \frac{1}{2} BC = 3 \text{ cm.}$$

$\therefore X$ is the midpoint of \overline{AB}

$\therefore Z$ is the midpoint of \overline{BC}

$$\therefore XZ = \frac{1}{2} AC = 3.5 \text{ cm.}$$

$\therefore Y$ is the midpoint of \overline{AC}

$\therefore Z$ is the midpoint of \overline{BC}

$$\therefore YZ = \frac{1}{2} AB = 2.5 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle XYZ = 3 + 3.5 + 2.5 = 9 \text{ cm.}$$

(The req.)

5

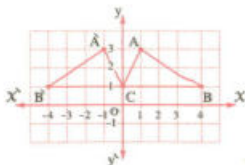
[a] $\therefore ABCD$ is a parallelogram

$$\therefore m(\angle B) = 180^\circ - 70^\circ = 110^\circ \quad (\text{First req.})$$

$$\therefore BC = AD = 8 \text{ cm.}$$

$$\therefore CD = AB = 5 \text{ cm.} \quad (\text{Second req.})$$

[b]



11 Souhag

1

- 1 c 2 a 3 b 4 b 5 d 6 c

2

- 1 $(-5, -2)$ 2 half 3 30°
4 15 5 2 6 120°

3

[a] $\therefore ABCD$ is a parallelogram

$$\therefore m(\angle D) = m(\angle B) = 135^\circ \quad (\text{First req.})$$

$$\therefore m(\angle C) + m(\angle B) = 180^\circ$$

$$\therefore m(\angle C) = 180^\circ - 135^\circ = 45^\circ \quad (\text{Second req.})$$

$$\therefore AB = CD = 5 \text{ cm.}$$

$$\therefore AD = BC = 8 \text{ cm.}$$

\therefore The perimeter of the parallelogram $ABCD$

$$= (5 + 8) \times 2 = 26 \text{ cm.} \quad (\text{Third req.})$$

[b] $\therefore \overline{EO} \parallel \overline{CD}$, \overline{BE} is a transversal to them

$$\therefore m(\angle EBC) = m(\angle E) = 50^\circ$$

(alternate angles)

\therefore In $\triangle ABC$:

$$m(\angle BAC) = 180^\circ - (50^\circ + 30^\circ) = 100^\circ$$

$$\therefore m(\angle C) = 30^\circ \quad (\text{First req.})$$

$\therefore \angle ABD$ is an exterior angle of $\triangle ABC$

$$\therefore m(\angle ABD) = m(\angle BAC) + m(\angle C) \\ = 100^\circ + 30^\circ = 130^\circ \quad (\text{Second req.})$$

4

[a] In $\triangle ABC$:

$\therefore X$ is the midpoint of \overline{AB}

$\therefore Y$ is the midpoint of \overline{BC}

$$\therefore AC = 2 XY = 8 \text{ cm.}$$

$\therefore X$ is the midpoint of \overline{AB}

$\therefore Z$ is the midpoint of \overline{AC}

$$\therefore BC = 2 XZ = 10 \text{ cm.}$$

$\therefore Y$ is the midpoint of \overline{BC}

$\therefore Z$ is the midpoint of \overline{AC}

$$\therefore AB = 2 YZ = 12 \text{ cm.}$$

\therefore The perimeter of $\triangle ABC$

$$= 12 + 10 + 8 = 30 \text{ cm.} \quad (\text{The req.})$$

[b] In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (4)^2 + (3)^2 = 25$$

$$\therefore AC = \sqrt{25} = 5 \text{ cm.} \quad (\text{First req.})$$

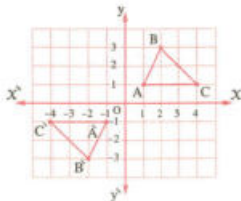
In $\triangle ACD$: $\therefore m(\angle ACD) = 90^\circ$

$$\therefore (DC)^2 = (AD)^2 - (AC)^2 = (13)^2 - (5)^2 = 144$$

$$\therefore DC = \sqrt{144} = 12 \text{ cm.} \quad (\text{Second req.})$$

5

[a]





[b] $A(1, 1) \rightarrow \hat{A}(3, -2)$

$B(2, 3) \rightarrow \hat{B}(4, 0)$

12

Aswan

1

[1] b

[2] b

[3] d

[4] b

[5] b

[6] b

2

[1] $(-4, -7)$

[2] acute-angled

[3] the hypotenuse

[4] obtuse

[5] a rhombus

[6] 120°

3

[a] In $\triangle ABC$:

$\therefore D$ is the midpoint of \overline{AB}

$\therefore O$ is the midpoint of \overline{AC}

$\therefore DO = \frac{1}{2} BC = 6 \text{ cm.}$

$\therefore D$ is the midpoint of \overline{AB}

$\therefore E$ is the midpoint of \overline{BC}

$\therefore DE = \frac{1}{2} AC = 5 \text{ cm.}$

$\therefore CE = \frac{1}{2} BC = 6 \text{ cm.}$

$\therefore CO = \frac{1}{2} AC = 5 \text{ cm.}$

\therefore The perimeter of the shape DECO

$= 5 + 6 + 5 + 6 = 22 \text{ cm.}$ (The req.)

[b] $m(\angle EBD) = 360^\circ - (140^\circ + 110^\circ + 35^\circ) = 75^\circ$

(The req.)

4

[a] In $\triangle XYZ$: $\therefore m(\angle Y) = 90^\circ$

$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = (7)^2 + (24)^2 = 625$

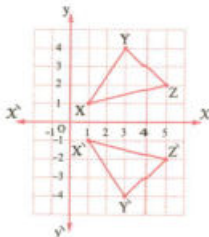
$\therefore XZ = \sqrt{625} = 25 \text{ cm.}$ (First req.)

In $\triangle XZL$: $\therefore m(\angle L) = 90^\circ$

$\therefore (LZ)^2 = (XZ)^2 - (XL)^2 = (25)^2 - (15)^2 = 400$

$\therefore LZ = \sqrt{400} = 20 \text{ cm.}$ (Second req.)

[b]



5

[a] In $\triangle ABC$:

$m(\angle ACB) = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$

$\therefore \overline{BD} \cap \overline{AO} = \{C\}$

$\therefore m(\angle DCO) = m(\angle ACB) = 60^\circ$ (V.O.A.)

From the quadrilateral CDEC:

$\therefore m(\angle E) = 360^\circ - (90^\circ + 60^\circ + 120^\circ) = 90^\circ$ (The req.)

[b] Let $A(x, y)$

$\therefore x + 2 = 2$

$\therefore x = 0$

$\therefore y + 3 = 3$

$\therefore y = 0$

$\therefore A(0, 0)$

(The req.)